Stat 274

Homework Assignment 2 Solution

1. Suppose you deposit 10000 in an account with annually compounding interest of $i = 0.05$ and withdraw the money when the account reaches 15000.

   (a) How long will it take? [8.3104]
   Answer:

$$\begin{aligned} 10000(1.05)^t &= 15000 \\ (1.05)^t &= 1.5 \\ t &= 8.3104 \end{aligned}$$

   (b) Find I_2 the amount of interest earned during the second year. [525]
   Answer:

$$10000[(1.05)^2 - 1.05] = 525$$

   (c) Find $I_{[7,9]}$ the amount of interest earned from time 7 to time 9. [928.996]
   Answer:
   Since \$15000 is reached before time 9, we calculate the amount of interest earned from time 7 until there is \$15000 in the account.

$$15000 - (1.05)^7 * 10000 = 928.996$$

   (d) What is the total interest paid on this transaction? [5000]
   Answer:

$$15000 - 10000 = 5000$$

2. Find the accumulated value of 2480 at the end of twelve years if the nominal interest rate was 2% monthly for the first three years, the nominal rate of discount was 3% semiannually for the next two years and the rate of interest (convertible semiannually) was 4.2% for the next four years, and the annual effective rate of discount was 0.058 for the last three years. [3951.81]
   Answer:

$$2480\left(1+\frac{.02}{12}\right)^{3*12}\left(1-\frac{.03}{2}\right)^{-2*2}\left(1+\frac{.042}{2}\right)^{4*2}(1-.058)^{-3} = 3951.81$$

3. Given equivalent rates $i^{(m)} = 0.0469936613$ and $d^{(m)} = 0.046773854$, find m. [10]
   Answer:

$$\begin{aligned} i^{(m)} &= \frac{d^{(m)}}{1-\frac{d^{(m)}}{m}} \\ .0469936613 &= \frac{.046773854}{1-\frac{.046773854}{m}} \\ m &= 10.0 \end{aligned}$$

4. A savings account starts with 1000 and a level annual effective discount rate of 6.4%. Find the accumulated value at time 5. [1391.94]
Answer:

$$1000(1 - .064)^{-5} = 1391.94$$

5. The amount of (compound) interest on X for two years is 320. The amount of discount on X for one year is 148 (meaning that $X - 148$ at time 0 turns into X at time 1). Find the effective interest rate i and the value of X. [0.05311; 2934.68]
Answer:

$$\begin{aligned} Xd &= 148 \\ X\frac{i}{1+i} &= 148 \\ X &= 148\frac{1+i}{i} \qquad (*) \end{aligned}$$

$$\begin{aligned} X((1+i)^2 - 1) &= 320 \\ X(i^2 + 2i) &= 320 \qquad (**) \end{aligned}$$

Substitute (*) into (**)

$$\begin{aligned} 148\frac{1+i}{i}(1^2 + 2i) &= 320 \\ i^2 + 3i - \frac{24}{148} &= 0 \\ \frac{-3 \pm \sqrt{3^2 - 4(1)(\frac{-24}{148})}}{2} &= 0 \\ i &= .05311 \qquad \text{plug into (*)} \\ X &= 2934.68 \end{aligned}$$

6. Given that $\delta_t = \frac{3t^2}{(1+t^3)}$

   (a) Find $a(t)$
   Answer:

$$\begin{aligned} \delta_t &= \frac{a^{'}(t)}{a(t)} \\ a(t) &= 1 + t^3 \end{aligned}$$

(b) Assuming an initial deposit of 2500, find $I_{[4,7]}$. [697500]
Answer:

$$\begin{aligned} I_{[4,7]} &= k(a(7) - a(4)) \\ &= 2500(344 - 65) = 697500 \end{aligned}$$

7. Given $a(t) = e^{0.04t+0.002t^2}$ find δ_3. [0.052]
Answer:

$$\begin{aligned} \delta_t &= \frac{d}{dt} log\ a(t) \\ &= \frac{d}{dt} 0.04t + 0.002t^2 \\ &= 0.04 + 0.004t \\ \delta_3 &= .052 \end{aligned}$$

8. Alicia goes to the bank to finance a car. The banker gives her the option of an annual effective interest rate of 0.047 or an annual effective discount rate of 0.045. Which option should she choose? [The interest rate.]
Answer:

$$d = 0.045$$

converting the discount rate to an interest rate we get:

$$\begin{aligned} i &= \frac{0.045}{1 - 0.045} \\ &= 0.0471 \end{aligned}$$

0.0471 > 0.047 so we choose the interest rate of 0.047, not the discount rate of 0.045. Because we are borrowing the money, we want the lower interest rate.

9. Find and work 5 more practice problems. These will be graded, so be sure to include them in your submitted assignment. You can find them:

- In the online practice problems
- In the study manuals in the library
- In the book
- Ask the TA's to write one
- In your purchased software (Infinite Actuary, Coaching Actuaries, Actex, etc.)