Stat 274 - Homework Assignment 3 Solution

1. Bruce and Robbie each open up new bank accounts at time 0. Bruce deposits 100 into his bank account, and Robbie deposits 50 into his. Each account earns the same annual effective interest rate. The amount of interest earned in Bruce's account during the 11th year is equal to X. The amount of interest earned in Robbie's account during the 17th year is also equal to X. Calculate X. [38.88]
   Answer:

$$
\begin{aligned}
100(1+i)^{10} * i &= 50(1+i)^{16} * i \\
2 &= (1+i)^6 \\
i &= 0.12246 \\
X &= 100(1.12246)^{10} * 0.12246 \\
X &= 38.88
\end{aligned}
$$

2. David can receive one of the following two payment streams: (1) 100 at time 0, 200 at time n, and 300 at time $2n$ or (2) 600 at time 10. At an annual effective interest rate of i, the present values of the two streams are equal. Given $v^n = 0.76$, determine i. [0.0350]
   Answer:

$$
\begin{aligned}
100 + 200v^n + 300v^{2n} &= 600v^{10} \\
100 + 200(.76) + 300(.5776) &= 600v^{10} \\
0.7088 &= v^{10} \\
i &= .0350
\end{aligned}
$$

3. Jeff deposits 10 into a fund today and 20 fifteen years later. Interest is credited at a nominal discount rate of d compounded quarterly for the first 10 years, and at a nominal interest rate of i compounded semiannually thereafter. If the account balance is less than 90, $i = 0.06$. If the account balance is greater than 90, $i = 0.07$. The accumulated balance in the fund at the end of 30 years is 100. Calculate d. [0.04245]
   Answer:
   First, let t_1 be the time it takes for the account to grow from 90 to 100. Then,

$$
90\left(1 + \frac{.07}{2}\right)^{2t_1} = 100
$$

$$
t_1 = \frac{log(\frac{100}{90})}{log(1.035)} * \left(\frac{1}{2}\right) = 1.5313
$$

So the account grows at 6% semiannually until time 28.4687 (30 - 1.5313). Now,

$$\left[10\left(1-\frac{d^{(4)}}{4}\right)^{-10(4)}\left(1+\frac{.06}{2}\right)^{5(2)}+20\right]\left(1+\frac{.06}{2}\right)^{13.4687(2)}=90$$
$$d=0.04245$$

4. Adam opens an account paying 3% effective annually with an initial deposit (at time 0) of 1000. At time 2, he adds another 1500 to the account. Bridget opens a savings account at time 1 with a deposit of 2000 earning i% annually. At time 5, Adam and Bridget have the same amount in their accounts. Calculate i. [0.08760]
Answer:

$$\begin{aligned}\left(1000(1.03)^2+1500\right)(1.03)^3 &= 2000(1+i)^4\\ 1.39918 &= (1+i)^4\\ i &= 0.08760\end{aligned}$$

5. You are given the choice of investing in an account which will double your money in 6 years or one which will triple your money in nine years. Which has a better annual interest rate? [0.12246; 0.12983]
Answer:

$$\begin{aligned}(1+i_1)^6 &= 2\\ i_1 &= 0.12246\end{aligned}$$

$$\begin{aligned}(1+i_2)^9 &= 3\\ i_2 &= 0.12983\end{aligned}$$

6. Assuming compound interest, the effective rate for $[3,15]$ is one third the effective rate for $[18,42]$. Find the annual interest rate. [0.05946]
Answer:

$$\begin{aligned}3[\frac{(1+i)^{15}-(1+i)^3}{(1+i)^3}] &= \frac{(1+i)^{42}-(1+i)^{18}}{(1+i)^{18}}\\ 3[(1+i)^{12}-1] &= (1+i)^{24}-1\\ \text{let } x &= (1+i)^{12}\\ 0 &= x^2-3x+2\\ x &= 2=(1+i)^{12}\\ i &= 0.05946\end{aligned}$$

7. Jim received a trust fund on his tenth birthday. When he turned 21, the trust fund was valued at 18750. If the trust fund grew at an annual effective rate of 5.8%, how much was it

worth on his 16th birthday? [14,144.02]
Answer:

$$18750v^5 = 14,144.02$$

8. Suppose $a(t) = 1 + 0.02t + 0.001t^2$:

   - Find d, the effective annual discount rate, as a function of t.
     Answer:

$$\begin{aligned} d_t &= \frac{a(t) - a(t-1)}{a(t)} \\ &= \frac{1 + 0.02t + 0.001t^2 - \left(1 + 0.02(t-1) + 0.001(t-1)^2\right)}{1 + 0.02t + 0.001t^2} \\ &= \frac{0.019 + 0.002t}{1 + 0.02t + 0.001t^2} \end{aligned}$$

   - Find d_3, the effective discount rate for the third year. [0.023386]
     Answer:

$$\begin{aligned} d_3 &= \frac{a(3) - a(2)}{a(3)} \\ &= \frac{1.069 - 1.044}{1.069} = 0.023386 \end{aligned}$$

   - Find $d_{[2,6]}$, the effective discount rate from 2 to 6. [0.096886]
     Answer:

$$\begin{aligned} d_{[2,6]} &= \frac{a(6) - a(2)}{a(6)} \\ &= \frac{1.156 - 1.044}{1.156} = 0.096886 \end{aligned}$$

   - Given an initial deposit of 25, how much interest is earned from 2 to 6? [2.80]
     Answer:

$$I_{[2,6]} = 25 * a(2) \left(\frac{1}{(1 - 0.096996)} - 1 \right) = 2.80$$

9. (SOA Study Note FM-09-05-3) Eric deposits 100 into a savings account at time 0, which pays interest at a nominal rate of i, compounded semiannually. Mike deposits 200 into a different savings account at time 0, which pays simple interest at an annual rate of i. Eric and Mike earn the same amount of interest during the last 6 months of the 8th year. Calculate i.

[0.09459]
Answer:

$$100(1+\frac{i}{2})^{15} * \frac{i}{2} = 200(0.5)i$$
$$(1+\frac{i}{2})^{15} = 2$$
$$i = 0.09459$$

10. Using the table below:

Hypothetical Repayment Amounts & Interest Rates for
a Loan of Amount 1,000 for Various Terms

Term	1	2	3	4
Repayment assuming no defaults	1018	1050	1080	1111
Exp. Defaults /1000	3	5	8	15

(a) Assume the lender doesn't recover anything from those who default on their loan. What is the updated repayment amount and effective annual interest rate for a 4-year loan? [1127.91,3.055%]
Answer:

$$1000(1111) = (1000-15)X$$
$$X = 1127.91$$
$$\left(\frac{1127.91}{1000}\right)^{0.25} - 1 = 0.03055 = i$$

(b) Now assume that when default occurs, the lender is able to recover 25% of the amount owed. What is the updated repayment amount and effective annual interest rate for a 3-year loan? [1086.52, 2.8%]
Answer:

$$1000(1080) = (1000-8)X + 8(0.25)X$$
$$X = 1086.52$$
$$\left(\frac{1086.52}{1000}\right)^{0.33} - 1 = 0.0280457 = i$$

11. Using the table below

Repayment Amounts & Interest Rates Before Inflation Adjustment
for a Loan of 1,000 with Inflation Protection

Term	1	2	3	4
Repayment amount before inflation adjustment	990	996	1002	1011

Suppose that a particular loan contract is for a term of 3 years and that the reference index specified in the contract increases 2% during the first year of the loan, 3.5% during the second, and 1% in the third. What is the amount the borrower must repay at the end of the loan term? [1068.39]

Answer:

$$1002(1.02)(1.035)(1.01) = 1068.39$$