Stat 274

Homework Assignment 5

1. You purchase an annuity for 1000. It pays X at time 1, $X(1+g)$ at time 2, $X(1+g)^{t-1}$ at time t, through $X(1+g)^{29}$ at time 30. $i = 0.05$.

   (a) Assuming $g = 0.03$, calculate X. [45.62]
   Answer:

$$\begin{aligned} 1000 &= X\left(\frac{1-\left(\frac{1.03}{1.05}\right)^{30}}{.05-.03}\right) \\ 1000 &= 21.9193X \\ 45.62 &= X \end{aligned}$$

   (b) Assuming $g = 0.05$, calculate X. [35.00]
   Answer:
   When $i = g$,

$$\begin{aligned} PV &= \frac{nX}{1+i} \\ 1000 &= \frac{30X}{1.05} \\ 35.0 &= X \end{aligned}$$

   (c) Assuming $g = 0.07$, calculate X. [26.27]
   Answer:

$$\begin{aligned} 1000 &= X\left(\frac{1-\left(\frac{1.07}{1.05}\right)^{30}}{.05-.07}\right) \\ X &= 26.27 \end{aligned}$$

   (d) Assuming $X = 40$, calculate g. [4.014%]
   Answer:

$$\begin{aligned} 1000 &= \frac{X\ddot{a}_{\overline{30|}j}}{1.05} \\ 26.25 &= \ddot{a}_{\overline{30|}j} \end{aligned}$$

   Using your calculator in **begin mode**:
   30 [N] 26.25 [PV] -1 [PMT] [CPT] [I/Y]

The calculator will tell you $j = .00948376$

$$\begin{aligned}
j &= \frac{i-g}{1+g} \\
(1+g)j + g &= i \\
j + jg + g &= i \\
g(j+1) &= i - j \\
g &= \frac{i-j}{1+j} \\
&= \frac{.05 - .00948376}{1.00948376} \\
g &= 0.0401356
\end{aligned}$$

2. Otis just retired and will start receiving monthly pension payments at the end of this month. The first payment is 5,000 and will increase by 100 each month. What would be the price of this perpetuity if the effective annual rate is 6%? [5,247,857.10]
   Answer:
   The monthly effective interest rate is:

$$\begin{aligned}
(1.06)^{1/12} - 1 = .004867551 \\
(I_{P,Q}a)_{\overline{\infty}|i} &= \frac{P}{i} + \frac{Q}{i^2} \\
&= \frac{5000}{.00487} + \frac{100}{.00487^2} \\
&= 5,247,857.10
\end{aligned}$$

3. Tyler and Eunice just got married. As a wedding present Eunice's father purchased an annuity that will give them payments every other year starting one year from today (at times 1, 3, 5, ...,19). The effective annual rate is 4%. The first payment is for 3000 and increases by 100 each payment. How much did Eunice's father pay for this annuity? [23,459.20]
   Answer:
   For this problem we need the effective two-year interest rate:

$$\begin{aligned}
(1.04)^2 - 1 = .0816 \\
(I_{P,Q}a)_{\overline{n}|i} &= Pa_{\overline{n}|i} + Q\frac{a_{\overline{n}|i} - nv^n}{i} \\
&= 3000a_{\overline{10}|.0816} + 100\frac{a_{\overline{10}|.0816} - 10v^{10}}{.0816} \\
&= 22556.92
\end{aligned}$$

This answer is the value of a series of increasing payments starting at **time 2**, so to get the present value of payments starting at time 1, we move it forward by one year's interest.

$$= 22556.92(1.04) = 23459.20$$

4. Herbert buys an annuity for 2,500. He will receive payments of 500 in one year, with payments increasing by 3.5% each year for six years. Compute i. [8.05%]
   Answer:
   The simplest way to do this problem is to use the cash flow worksheet on your calculator

$$\begin{aligned} CF_0 &= -2500 \\ C01 &= 500 \\ C02 &= 500(1.035) \\ C03 &= 500(1.035)^2 \\ C04 &= 500(1.035)^3 \\ C05 &= 500(1.035)^4 \\ C06 &= 500(1.035)^5 \end{aligned}$$

   Push [IRR] [CPT] and you get 8.05%

5. Myrtle puts money in the bank at the end of each year, increasing her deposits by 4% each year. The first year she puts in X. Clyde starts putting money into his own account the same day. He puts in 4,000 the first year, with his deposits increasing by 100 each year. Assuming the effective annual interest rate for both accounts is 5%, and the accounts will have an equal balance in 7.5 years, calculate X [3,822.81]
   Answer: The last payment occurs at time 7, after which the balances in each account will grow by $(1.05)^{0.5}$. That means that both accounts have the same amount at time 7.

$$\begin{aligned} X\left(\frac{1-\left(\frac{1.04}{1.05}\right)^7}{.05-.04}\right)(1.05)^7 &= 4000s_{\overline{7}|.05} + 100\frac{s_{\overline{7}|.05}-7}{.05} \\ X(6.479185)(1.4071) &= 4000(8.142) + 100(22.84017) \\ X &= 3,822.81 \end{aligned}$$

6. Jim deposits 100 into his account at time 0. At time 1, he deposits 50. At time 2, deposits 25. Jim continues to make deposits half as large every year. Calculate the account balance at time 30 assuming $i = 0.02$. [355.31]
   Answer:

$$\left(\frac{1-\left(\frac{1-.5}{1.02}\right)^{31}}{.02+.5}\right)(1.02)^{31} = 355.305$$

   We have to compound one year to get the present value of an annuity-due, then 30 more to move to the future value at time 30.

7. What is $(Ia)_{\overline{n}|i} + (Da)_{\overline{n}|i}$?
Answer:

$$\begin{aligned}
&\frac{\ddot{a}_{\overline{n}|i} - nv^n}{i} + \frac{n - a_{\overline{n}|i}}{i} \\
&\frac{\ddot{a}_{\overline{n}|i} - a_{\overline{n}|i}}{i} + \frac{n - nv^n}{i} \\
&\frac{ia_{\overline{n}|i}}{i} + \frac{n(1-v^n)}{i} \\
&a_{\overline{n}|i} + na_{\overline{n}|i} \\
&(1+n)a_{\overline{n}|i}
\end{aligned}$$

8. You have two options (which are of equal value at time 0).

   - A geometrically increasing 30 year annuity, paying 100 at time 1, and increasing by 3% for each future payment.
   - An arithmetically decreasing annuity, starting at X at time 1 and decreasing by 1 each year for 30 years.

   Assuming $i = 0.04$, calculate X. [157.15]
   Answer:

$$\begin{aligned}
100\left(\frac{1 - \left(\frac{1.03}{1.04}\right)^{30}}{.04 - .03}\right) &= Xa_{\overline{30}|.04} - \frac{a_{\overline{30}|.04} - 30v^{30}}{.04} \\
2516.297 &= X(17.292) - 201.062 \\
X &= 157.15
\end{aligned}$$

9. On his 30th birthday ($t = 30$), Julio begins saving his money for retirement. He puts 1000 into an account earning 2% interest. He gets a raise most years, so will add 100 to each subsequent payment (when $t = 31$, the payment is 1100, $t = 32$, payment equals 1200). He makes his last deposit on his 65th birthday. He immediately purchases a 25-year annuity immediate, with a first payment of X and each additional payment increasing by 2%. The price of the annuity was calculated using $i = 0.02$. Calculate X. [5384.22]
   Answer:

$$\begin{aligned}
1000s_{\overline{36}|.02} + 100\frac{s_{\overline{36}|.02} - 36}{.02} &= \frac{25X}{1.02} \\
1000(51.994367) + 100(799.71836) &= \frac{25X}{1.02} \\
X &= 5384.22
\end{aligned}$$

10. An investment earns yield rates according to the following table.

Year of Investment	Year 1 Rate	Year 2 Rate	Year 3 Rate	Year 4 Rate	First Ultimate Rate
1965	0.05	0.055	0.0475	0.05	0.055
1966	0.06	0.0525	0.05	0.06	0.0575
1967	0.05	0.05	0.0625	0.06	0.0625
1968	0.0675	0.07	0.07	0.0675	0.0615

You deposit 2000 at the beginning of 1965, how much is it worth at the end of 1972? [3066.17]
Answer:

$$2000(1.05)(1.055)(1.0475)(1.05)(1.055)(1.0575)(1.0625)(1.0615) = 3066.17$$