

Stat 274 — Fall 2023
Homework Assignment 6

Due: Thursday, November 16th on Learning Suite at 9:30 am

1. You take out a loan for 10000. You pay off the loan with monthly payments of 90 for 10 years.

- (a) What is the monthly effective rate? What is the annual effective rate? [0.128937%, 1.55827%]

Answer:

$$\begin{aligned}10000 &= 90a_{\overline{120}|j} \\111.11 &= \frac{1 - v^{120}}{j} \\j &= .00128937\end{aligned}$$

$$\begin{aligned}i &= (1 + j)^{12} \\i &= .0155827\end{aligned}$$

- (b) What is the outstanding loan balance immediately after the 7th payment? Calculate using both the retrospective and prospective formulas. [9458.16]

Answer:

$$\begin{aligned}OLB_k &= 10000(1.00129)^7 - 90s_{\overline{7}|.00129} \\&= 10090.60 - 632.44 = 9458.16\end{aligned}$$

$$\begin{aligned}OLB_k &= 90\left(a_{\overline{112}|.00129} + (1.00129)^{-113}\right) \\&= 9458.16\end{aligned}$$

- (c) Assume you miss the 13th and 53rd payments, what will be the outstanding loan balance after the 71st payment? [4460.01]

Answer:

$$\begin{aligned}OLB_{71} &= 10000(1.00129)^{71} - 90s_{\overline{71}|.00129} + 90\left((1.00129)^{58} + (1.00129)^{18}\right) \\OLB_{71} &= 4460.01\end{aligned}$$

2. You take out a 30-year mortgage for 250,000 to be repaid with end-of-month payments beginning in one month. The interest rate is 4.8% compounded monthly. Find:
- (a) The monthly payment amount [1311.67]
Answer: 360 [N] 0.4 [I/Y] 250000 [PV] [CPT] [PMT]
 - (b) The balance after 10 years [202,117.53]
Answer: [2ND] [AMORT] P1: 1 P2: 120
 - (c) The balance after 20 years [124,809.94]
Answer: [2ND] [AMORT] P1: 1 P2: 240
 - (d) The total interest paid in the 6th year (payments 61-72)[10,881.84]
Answer: [2ND] [AMORT] P1: 61 P2: 72
 - (e) The total principal paid in the 6th year [4,858.20]
Answer: [2ND] [AMORT] P1: 61 P2: 72
3. For a 25-year loan of 12,000 at 6% nominal annual interest (compounded semiannually) with payments at the end of each half year find the following:
- (a) The semiannual payment amount [466.39]
Answer: 50 [N] 3 [I/Y] 12000 [PV] [CPT] [PMT]
 - (b) The outstanding balance after the 20th payment [9,141.26]
Answer: [2ND] [AMORT] P1: 1 P2: 20
 - (c) The amount of interest in the 20th payment [279.83]
Answer: [2ND] [AMORT] P1: 20 P2: 20
 - (d) The amount of principal in the 20th payment [186.56]
Answer: [2ND] [AMORT] P1: 20 P2: 20
 - (e) The amount of interest in the 30th payment [215.67]
Answer: [2ND] [AMORT] P1: 30 P2: 30
4. You take out a 7-year loan of L to buy a new car. You pay off the loan with monthly payments of Q at a yearly effective interest rate of i . The amount of interest paid off in the first payment is \$180.10 and the amount of principle paid off in the 43rd payment is \$438.50. Calculate the loan amount L . (Note: Using the prospective formula when all the payments are equal, the amount of principle paid off in the k^{th} payment is $P_k = Qv^{n-k+1}$). [\$36999.89]

Answer:

$$\begin{aligned}P_{43} &= 438.50 = Qv^{84-43+1} \\438.50 &= Qv^{42} \\438.50(1+i)^{42} &= Q\end{aligned}$$

$$\begin{aligned}P_1 &= Q - 180.10 = Qv^{84-1+1} \\438.50(1+i)^{42} - 180.10 &= \frac{438.5(1+i)^{42}}{(1+i)^{84}} \\438.50(1+i)^{84} - 180.10(1+i)^{42} - 438.50 &= 0\end{aligned}$$

We can now use the quadratic equation by substituting x for $(1+i)^{42}$.

$$x = \frac{-(-180.10) \pm \sqrt{180.10^2 - 4(438.50)(-438.50)}}{2(438.50)}$$

$$\begin{aligned}x &= 1.226227629 \\(1+i)^{42} &= 1.226227629 \\i &= .004867582\end{aligned}$$

$$\begin{aligned}I_1 &= L * i \\ \frac{180.10}{.004867582} &= L \\ 36999.89 &= L\end{aligned}$$

5. Find and work 5 more practice problems on loans. You can find those:

- In the online practice problems
- In the study manuals
- In the book
- Ask the TA's to write one
- In your purchased software (Infinite Actuary, Coaching Actuaries, Actex, etc.)