

# Stat 274 — Winter 2024

## Homework Assignment 7

Due: Thursday, April 4th on Learning Suite at 9:30 am

1. You will pay 2,318.63 for an  $n$ -year 2,000 par-value bond with a coupon rate of 10% compounded semiannually or pay 2,531.05 for an  $n$ -year 2,000 par-value bond with a coupon rate of 11% compounded semiannually. Assuming that you get the same yield on the two bonds, find the nominal annual yield rate (compounded semiannually) and  $n$ . [8.5%, 28]

**Answer:**

$$(1) \quad 2531.05 = 110a_{\overline{2n}|j} + 2000v_j^{2n}$$

$$(2) \quad 2318.63 = 100a_{\overline{2n}|j} + 2000v_j^{2n}$$

Subtract (2) from (1) to get:

$$212.42 = 10a_{\overline{2n}|j}$$

$$21.242 = a_{\overline{2n}|j}$$

Substitute back into (1) and solve for  $v_j^{2n}$

$$2531.05 = 110(21.242) + 2000v_j^{2n}$$

$$v_j^{2n} = .097215$$

$$21.242 = \frac{1 - .097215}{j}$$

$$j = 0.0425$$

$$v_j^{2n} = \left[ \frac{1}{1.0425} \right]^{2n} = 0.097215$$

$$2n = 56$$

$$n = 28$$

2. You own a 3,000 par-value 12% bond with semiannual coupons. The bond will mature at the end of fourteen years. You decide a ten-year bond will be preferable. The current yield rate (for both bonds) is 6% convertible semiannually. You use the proceeds from the sale of the 12% bond to buy an 8% bond also with semiannual coupons maturing at par at the end of ten years. Find the face value of the 8% bond. [4081.54]

**Answer:**

$$\begin{aligned} PV &= 180a_{\overline{28}|.03} + 3000v_{.03}^{28} \\ &= 4688.77 \end{aligned}$$

$$4688.77 = F(.04)a_{\overline{20}|.03} + Fv_{.03}^{20}$$

$$4688.77 = F\left((.04)a_{\overline{20}|.03} + v_{.03}^{20}\right)$$

$$F = 4081.54$$

3. A 3,000 9% twelve-year bond with annual coupons is purchased with a discount of 57 and yields 9.1% if held to maturity. Find the price. [2997.95]

**Answer:**

$$\begin{aligned}
 C - P &= C(j - g)a_{\overline{n}|i} \\
 57 &= \left( C(.091) - 3000(.09) \right) a_{\overline{12}|.091} \\
 C &= 3054.95 \\
 P &= C - 57 = 2997.95
 \end{aligned}$$

4. You buy a newly issued 1,000 20% ten-year bond, redeemable at 1,100 and having yearly coupons, for 1,400. You immediately take a constant amount  $D$  from each coupon and deposits it in another account earning 8% effective annual interest, so as to accumulate the full amount of the premium the moment after the final deposit. Find  $D$ . [20.71]

**Answer:**

$$\begin{aligned}
 \text{Premium} &= P - C = 1400 - 1100 = 300 \\
 300 &= Ds_{\overline{10}|.08} \\
 20.709 &= D
 \end{aligned}$$

5. A 2,000 11% ten-year bond has semiannual coupons and is sold to yield 5.2% convertible semiannually. The discount on the bond is 83.28. Find the redemption amount. [4438.18]

**Answer:**

$$\begin{aligned}
 83.28 &= \left( C(.026) - 110 \right) a_{\overline{20}|.026} \\
 C &= 4438.18
 \end{aligned}$$

6. You purchase a ten-year 1,000 bond with semiannual coupons for 982. The bond has a 1,100 redemption payment at maturity, a nominal coupon rate of 7% for the first five years, and a nominal coupon rate of  $q\%$  for the final five years. Christie calculated that her annual effective yield for the ten-year period was 7.35%. Find  $q$ . [5.216%]

**Answer:**

$$\begin{aligned}
 982 &= 35a_{\overline{10}|.0361} + \left( \frac{g}{2} \right) 1000a_{\overline{10}|.0361} v_{.0361}^{10} + 1100v_{.0361}^{20} \\
 982 &= 35(8.271) + 500g(8.271)(.7014) + 1100(.4920) \\
 g &= .05216
 \end{aligned}$$

7. A 1,000 bond with a coupon rate of 8% has quarterly coupons and is redeemable after an unspecified number of years at 957. The bond is bought to yield 12% convertible quarterly. If the present value of the redemption amount is 231.59, find the purchase price of this bond.

[736.93]

**Answer:**

$$\begin{aligned}231.59 &= 957v^n \\ \frac{231.59}{957} &= 1.03^{-n} \\ \log\left(\frac{231.59}{957}\right) &= -n \log(1.03) \\ n &= -\frac{\log\left(\frac{231.59}{957}\right)}{\log(1.03)} = 48 \\ P &= 20a_{\overline{48}|} + 231.59 = 736.93\end{aligned}$$

8. A 20,000 bond has annual coupons and is redeemable at the end of fourteen years for 22,600. It has a base amount equal to 18,450 when purchased to yield 6%. Find its base amount if it were purchased to yield 7%. [15814.29]

**Answer:**

$$\begin{aligned}G &= Fr/j \\ 18450 &= 20000r/.06 \\ r &= 0.05535\end{aligned}$$

$$\begin{aligned}G &= 20000(.05535)/.07 \\ G &= 15814.29\end{aligned}$$

9. Joey purchased an  $n$ -year par-value 2,000 bond that had a coupon rate of 9% convertible quarterly. Todd purchased a par-value bond with an identical coupon rate but having a term of  $2n$  years. The coupons that Joey and Todd received in the first  $n$  years were identical and both bonds had a yield rate of 6% convertible quarterly. Todd paid 233.02 more than Joey. Calculate  $n$ . Note that  $4n$  must be an integer. [7.75]

**Answer:**

$$\begin{aligned}233.02 &= 45a_{\overline{8n}|.015} + 2000v_{.015}^{8n} - 45a_{\overline{4n}|.015} - 2000v_{.015}^{4n} \\&= 45(a_{\overline{8n}|.015} - a_{\overline{4n}|.015}) + 2000(v_{.015}^{8n} - v_{.015}^{4n}) \\&= 45a_{\overline{4n}|.015}v_{.015}^{4n} + 2000(v_{.015}^{8n} - v_{.015}^{4n}) \\&= 45\left(\frac{1 - v_{.015}^{4n}}{.015}\right)v_{.015}^{4n} + 2000(v_{.015}^{8n} - v_{.015}^{4n}) \\&= 3000(v_{.015}^{4n} - v_{.015}^{8n}) + 2000(v_{.015}^{8n} - v_{.015}^{4n}) \\233.02 &= 1000(v_{.015}^{4n} - v_{.015}^{8n}) \\ .23302 &= v_{.015}^{4n} - v_{.015}^{8n}\end{aligned}$$

$$\text{Let } X = v_{.015}^{4n}$$

$$0 = -X^2 + X - .23302$$

Using the quadratic equation,  $X = v_{.015}^{4n} = .630307$

$$n = 7.750$$

10. Find and work 5 more practice problems on bonds. You can find those:

- In the online practice problems
- In the study manuals
- In the book
- Ask the TA's to write one
- In your purchased software (Infinite Actuary, Coaching Actuaries, Actex, etc.)