Stat 274

Homework Assignment 8

Due: Tuesday, December 6th in class

1. A 1,500 7% bond with semiannual coupons is redeemable for 1,700 in fifteen years. The bond is purchased to yield 9% per year compounded semiannually. Find

   (a) The price of the bond [1309.07]
   Answer:

$$\begin{aligned} PV &= 52.50a_{\overline{30}|.045} + 1700v_{.045}^{30} \\ &= 1309.07 \end{aligned}$$

   (b) The amount of interest in the 10th coupon [62.02]
   Answer:

$$\begin{aligned} I_{10} &= BV_9 * i \\ &= \left(52.50a_{\overline{21}|.045} + 1700v_{.045}^{21}\right)i \\ &= 62.02 \end{aligned}$$

   (c) The accumulation of discount in the 10th coupon [9.52]
   Answer:

$$\begin{aligned} D_{10} &= I_{10} - Fr \\ &= 62.02 - 52.50 = 9.52 \end{aligned}$$

2. A 6,000 12% bond with semiannual coupons will mature in 10 years at par. If the bond is priced to nominally yield 6% convertible semiannually, find

   (a) The premium paid for this bond [2677.95]
   Answer:

$$\begin{aligned} Premium &= P - C \\ P &= 360a_{\overline{20}|.03} + 6000v_{.03}^{20} \\ P &= 8677.95 \\ P - C &= 2677.95 \end{aligned}$$

   (b) The interest due in the 7th coupon [241]
   Answer:

$$\begin{aligned} I_7 &= BV_6 * i \\ &= \left(360a_{\overline{14}|.03} + 6000v_{.03}^{14}\right)i \\ &= 241 \end{aligned}$$

(c) The amortization of premium in the 7th coupon [119]
Answer:

$$\begin{aligned} P_7 &= Fr - I_7 \\ &= 360 - 241 = 119 \end{aligned}$$

3. An n-year 2,000 par-value 9% bond with annual coupons has an annual effective yield of i ($i > 0$). The book value of the bond at the end of the fifth year is 1,902.63 and the book value at the end of the seventh year is 1,924.18. Find the price of this bond. [1863.73]
Answer:
Using your calculator
2 [N] -1902.63 [PV] 180 [PMT] 1924.18 [FV] [CPT] [I/Y] = 10%

$$\begin{aligned} P &= 180a_{\overline{5}|.10} + 1902.63v^5_{.10} \\ &= 1863.73 \end{aligned}$$

4. Joseph purchases a 10,000 4-year 10% par-value bond with annual coupon payments that is priced to yield 5% annually. Calculate the absolute difference between the exact new price of the bond, and the approximation for the new price of the bond using the following methods, if the yield rate were to increase by 0.5%.

(a) The first-order Macaulay approximation. [0.11639]
Answer:

$$\begin{aligned} v &= (1.05)^{-1} \\ w &= (1.055)^{-1} \\ D(0.05,\infty) &= \frac{1000v + 2000v^2 + 3000v^3 + 44000v^4}{1000(v + v^2 + v^3) + 11000v^4} = 3.52985 \\ Approximation &= [1000(v + v^2 + v^3) + 11000v^4][\frac{1.05}{1.055}]^{D(0.05,\infty)} = 11577.20116 \\ Actual &= 1000(w + w^2 + w^3) + 11000w^4 = 11577.31755 \\ Actual - Approximation &= 0.11639 \end{aligned}$$

(b) The tangent approximation. [2.232]

Answer:

$$
\begin{aligned}
i_0 &= 0.05 \\
i &= 0.055 \\
P(i_0) &= 1000(1+i_0)^{-1} + 1000(1+i_0)^{-2} + 1000(1+i_0)^{-3} + 11000(1+i_0)^{-4} \\
&= 11772.98 \\
P'(i_0) &= -1000(1+i_0)^{-2} - 2000(1+i_0)^{-3} - 3000(1+i_0)^{-4} - 44000(1+i_0)^{-5} \\
&= -39577.96 \\
\frac{P'(i_0)}{P(i_0)}(i-i_0) &= \frac{-39577.96}{11772.98}(0.055-0.05) = -0.0168 \\
Approximation &= [1+(-0.0168)](11772.98) = 11575.0854 \\
Actual - Approximation &= 2.232
\end{aligned}
$$

(c) The quadratic approximation. [0.02547]
Answer:

$$
\begin{aligned}
P''(i_0) =& 2000(1+i_0)^{-3} + 6000(1+i_0)^{-4} + 12000(1+i_0)^{-5} + \\
& 220000(1+i_0)^{-6} = 180233.59 \\
\frac{P'(i_0)}{P(i_0)}(i-i_0) + \frac{P''(i_0)}{P(i_0)}\frac{(i-i_0)^2}{2} =& -0.0168 + \frac{180233.59}{11772.98}\frac{(0.055-0.05)^2}{2} = -0.016617 \\
Approximation =& [1+(-0.016617)](11772.98) = 11577.34302 \\
Approximation - Actual =& 0.02547
\end{aligned}
$$

5. Calculate the convexity of a stream of cashflows with interest rate 5% that pays 10 at time 1, 15 at time 2, and 20 at time 3. [6.9140]
Answer:

$$
\begin{aligned}
P &= 10(1+i)^{-1} + 15(1+i)^{-2} + 20(1+i)^{-3} = 40.406 \\
P' &= -10(1+i)^{-2} - 30(1+i)^{-3} - 60(1+i)^{-4} \\
P'' &= 20(1+i)^{-3} + 90(1+i)^{-4} + 240(1+i)^{-5} = 279.36 \\
\frac{P''}{P} &= \frac{279.36}{40.406} = 6.914
\end{aligned}
$$

6. C.J. has assets at time 2 of A and at time 9 of B. He has a liability of 95000 at time 5. C.J. has achieved Redington immunization in his portfolio using an annual effective interest rate of 4%. Calculate A and B. [48259.80, 47629.96]

Answer: At $t = 2$,

$$\begin{aligned}
PV(Assets) &= A + Bv^7 \\
PV(Liabilities) &= 95000v^3 \\
PV'(Assets) &= -7Bv^8 \\
PV'(Liabilities) &= (-3)95000v^4 \\
-7Bv^8 &= -285000v^4 \\
B &= \frac{285000v^4}{7v^8} = 47629.96 \\
PV(Assets) &= PV(Liabilities) \\
A + Bv^7 &= 95000v^3 \\
A &= 95000v^3 - Bv^7 \\
A &= 95000v^3 - 47629.96v^7 = 48259.80
\end{aligned}$$

7. Geovani has a liability of 12,000 due in eight years. This liability will be met with payments of 5000 in five years and Q in 8 + m years. Geovani is employing a full immunization strategy using an annual effective interest rate of 3%. Calculate Q and m. [7039.26, 2.5076]
Answer: At $t = 5$,

$$\begin{aligned}
PV(Assets) &= 5000 + Qv^{3+m} \\
PV(Liabilities) &= 12000v^3 \\
5000 + Qv^{3+m} &= 12000v^3 \\
Qv^3v^m &= 12000v^3 - 5000 \\
Qv^m &= (12000v^3 - 5000) * 1/v^3 = 6536.37 \\
PV'(Assets) &= -Q(3+m)v^{4+m} \\
PV'(Liabilities) &= -36000v^4 \\
-Q(3+m)v^{4+m} &= -36000v^4 \\
Q(3+m)v^4v^m &= 36000v^4 \\
3 + m &= \frac{36000v^4}{Qv^mv^4} \\
m &= \frac{36000v^4}{6536.37v^4} - 3 = 2.5076 \\
Qv^m &= 6536.37 \\
Q &= 6536.37/v^{2.5076} = 7039.26
\end{aligned}$$