Stat 274
Theory of Interest

Chapter 5: Loan Repayment

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Amortized Loan

Each time a payment is made, the interest due is paid first.

Examples:

- You borrow 2000 at 5% annual interest, and pay back 800 in one year and 1000 in three years. Calculate the interest paid on each payment and the outstanding loan balance at time 3. [100, 133.25, 433.25]
- Assume you make a final payment at time 5 which completely pays off the loan. Calculate the final payment amount. [477.66]

How to compute the payment schedule of a loan.

1. Compute $Q_1 = L/a_{\overline{n}|i}$ (rounded to the nearest penny).
2. Round $Q_1 a_{\overline{n}|i}$ to the nearest penny, if equal to L then all payments are equal to Q_1.
3. Else, $Q_2 = \lceil Q_1 \rceil$ and the first $n-1$ payments are equal to Q_2.
4. Compute the over-payment $E = Q_2 a_{\overline{n}|i} - L$.
5. The final payment is $Q_2 - E(1+i)^n$.

1. You wish to borrow 4400 using a two-year loan with monthly payments and monthly effective interest rate of 0.25%. Calculate the payment amounts. [189.12, 189.05]
2. You wish to have 200000 at time 18. You will make payments at times 1, 2, . . ., 18. The fund has an annual effective discount rate of 5%. How large do your contributions need to be? [6936.47, 6936.40]
3. You put 100 in a savings account at the end of each month, earning an annual interest rate of 5.4% convertible monthly. How many months will it take to accumulate 3000? [29]

To calculate the outstanding loan balance on a loan of L with $n-1$ payments of Q and one payment of R at time n:

- Retrospective: $OLB_k = L(1+i)^k - Qs_{\overline{k}|i} = La(k) - Qs_{\overline{k}|}$
- Prospective (no drop payment): $OLB_k = Qa_{\overline{n-k}|i}$
- Prospective (drop payment): $OLB_k = Qa_{\overline{n-k-1}|i} + R(1+i)^{-(n-k)}$

How can you calculate the amounts of interest and principal paid?

Examples

1. You are repaying a loan by paying 80 at times 1, 2, ..., 30 at an effective rate of 0.4% per period. What is your loan balance immediately after your 12th payment? [1,386.71]
2. A loan of 2000 is repaid by payments of 250 at the end of each year and a final smaller payment at the end of the last year. With an annual effective rate of 8%, find the loan balance immediately after the 6th payment. [1,339.77]
3. You purchase a home for 456,000, make a down payment of 80,000, then finance the rest with a thirty-year mortgage (mortgages have monthly payments) at a monthly nominal interest rate of 6.6%. After exactly 8 years, you sell the home for 482,000. Closing costs are 3% of the sale price. How much do you receive at closing and how much interest did you pay? [133,548.61; 188,521.95]

1. You have a 60 month car loan with a nominal rate of interest of 3% convertible monthly. You make (end of) monthly payments of 252.65. During the first three years, you forgot to make the 14th and 30th payments. Calculate the outstanding balance at the end of the third year. [6,401.53]
2. You have a loan to be repaid by 20 end of quarter payments of 1000. The interest rate for the first two years is 6% convertible quarterly, and for the last three is 8% convertible quarterly. Calculate the outstanding loan balances after the sixth and fifteenth payments. [12,220.96; 4,713.46]

We know that the outstanding loan balance at time $k-1$ is

$$Qa_{\overline{n-k+1}|i} \qquad \text{or} \qquad Qa_{\overline{n-k}|i} + Rv^{n-k+1}$$

Then the interest due at time k is

$$Q(1-v^{n-k+1}) \qquad \text{or} \qquad Q(1-v^{n-k}) + Riv^{n-k+1}$$

and the principal paid at time k is

$$Qv^{n-k+1} \qquad \text{or} \qquad Qv^{n-k} - Riv^{n-k+1}$$

You purchase a home for 308,000. Down payment of 46,000 and the rest is financed with a 30-year mortgage with a nominal interest rate of 5.55% convertible monthly. How much interest did you pay between the 57th and 67th payments (inclusive, 11 total payments)? [12,312.93]

An amortized loan is repaid by 15 end-of-year payments of 1,800 at 6.6% effective annual interest. What portion of the loan's total interest is paid in the first 5 years? [49.7%]

SOA #75

A borrower takes out a 15-year loan for 400,000, with level end-of-month payments, at an annual nominal interest rate of 9% convertible monthly. Immediately after the 36th payment, the borrower decides to refinance the loan at an annual nominal interest rate of j, convertible monthly. The remaining term of the loan is kept at twelve years, and level payments continue to be made at the end of the month. However, each payment is now 409.88 lower than each payment from the original loan.

Calculate j. [0.069]

SOA #86

A loan of 10,000 is repaid with a payment made at the end of each year for 20 years. The payments are 100, 200, 300, 400, and 500 in years 1 through 5, respectively. In the subsequent 15 years, equal annual payments of X are made. The annual effective interest rate is 5%.

Calculate X. [1075]

SOA #107

Tim takes out an n-year loan with equal annual payments at the end of each year. The interest portion of the payment at time $(n-1)$ is equal to 0.5250 of the interest portion of the payment at time $(n-3)$ and is also equal to 0.1427 of the interest portion of the first payment.

Calculate n. [22]