*Disclaimer: This formula sheet is provided by the TAs as a helpful tool, but it does not include all formulas or topics which are covered on the exam.

Formulas:

Chapter 1:

Interest:

Simple: $A_K(t) = K\cdot(1 + i_s\cdot t)$

$a(t) = 1 + i_s\cdot t$

Compound: $A_K(t) = K\cdot(1 + i)^t$; $a(t) = (1 + i)^t$

Effective Interest Rate:

$$i_{(t_1,t_2)} = \frac{A(t_2) - A(t_1)}{A(t_1)} = \frac{a(t_2) - a(t_1)}{a(t_1)}$$

Discount:

Simple: $A_K(t) = K\cdot \frac{1}{1 - d_s t}$

Compound: $A_K(t) = K \cdot \left(\frac{1}{(1-d)^t}\right)$

Effective Discount Rate:

$$d_{(t_1,t_2)} = \frac{A(t_2) - A(t_1)}{A(t_2)} = \frac{a(t_2) - a(t_1)}{a(t_2)}$$

Nominal (convertible) Rates:

I: $A_K(t) = K\left(1 + \frac{i^{(m)}}{m}\right)^n$

D: $A_K(t) = K\frac{1}{\left(1 - \frac{d^{(m)}}{m}\right)^n}$

Nominal to Effective conversion:

$$1 + i = \left(1 + \frac{i^{(m)}}{m}\right)^m \quad (1 - d)^{-1} = \frac{1}{\left(1 - \frac{d^{(m)}}{m}\right)^m}$$

Effective Rates:

$$1 + \frac{i^{(m)}}{m} = (1 + i)^{1/m} \qquad 1 + i = (1 + \frac{i^{(m)}}{m})^m$$

$$\left(1 - \frac{d^{(m)}}{m}\right)^{-1} = (1 - d)^{-1/m} \qquad (1 - d)^{-1} = \left(1 - \frac{d^{(m)}}{m}\right)^{-m}$$

Force of Interest:

Constant δ: $A_K(t) = K \cdot e^{\delta t}$

Variable δ: $A_K(t) = K \cdot e^{\int_{t_1}^{t_2} \delta_t \, dt}$

Definitions:

K = Principal

$A_K(t)$ = Amount of K @ t

a(t) = Amount of 1 @ t

t = Time

i_s = Simple Interest

i = Compound Interest

d_s = Simple Discount

d = Compound Discount

*The interest rate used will always be an effective rate. A nominal rate is only used to get to an effective rate.

m = number of compounding periods per year

$i^{(m)}$ = nominal interest rate, compounded m times per year

$d^{(m)}$ = nominal discount rate, compounded m times per year

n = Total number of periods

*I.e., if a problem says find i effective quarterly it means $(1+i)^{1/4}$. If it says i is convertible quarterly, it means (1+i/4).

Formulas:

Definitions:

Conversions:

$$\left(1 + \frac{i^{(m)}}{m}\right)^{mt} = (1 + i)^t = e^{\delta t} = \frac{1}{(1-d)^t} = \frac{1}{\left(1 - \frac{d^{(m)}}{m}\right)^{mt}}$$

$$i = \frac{d}{1-d};\ d = \frac{i}{1+i};\ \delta_t = \frac{a`(t)}{a(t)} = \frac{d}{dt} log\, a(t);$$
$$v = \frac{1}{a(t)} = \frac{1}{1+i} = 1 - d = \frac{1}{e^{\delta}}$$

v = "value" of the accumulation function, one payment period earlier

Chapter 2:

Cash Flow Calculator: CF_0 = Initial Deposit, C_t = \$ in/out, F_t = Frequency, IRR = Internal Rate of Return ⟵ CPT

$a_{\overline{n}|i}$ = Annuity-immediate or n years at interest i

Chapter 3:

$s_{\overline{n}|i}$ = Future value of annuity

$ä_{\overline{n}|i}$ = Annuity-due (payments at beginning of the period)

$\bar{a}_{\overline{n}|}$ = Continuous annuity

$$a_{\overline{n}|i} = \frac{1 - v^n}{i}$$

$$s_{\overline{n}|i} = (1+i)^n \cdot a_{\overline{n}|i}$$

$$ä_{\overline{n}|i} = (1+i) \cdot a_{\overline{n}|i} = \frac{1-v^n}{d}$$

$$\bar{a}_{\overline{n}|} = \frac{1 - e^{-\delta t}}{\delta}$$

$$a_{\overline{\infty}|i} = \frac{1}{i}$$

$$ä_{\overline{\infty}|i} = \frac{1}{d} = \frac{1}{i}(1+i)$$

g = growth rate

Geometrically-Increasing Annuity:

$j = \frac{i-g}{1+g}$

P = payment

$$P\left(\frac{1-\left(\frac{1+g}{1+i}\right)^n}{i-g}\right) = \frac{P ä_{\overline{n}|j}}{1+i}$$

- When i = g: $\frac{nP}{1+i} \rightarrow j = 0$
- When n → ∞: $\frac{P}{i-g} \rightarrow \left(\left(\frac{1+g}{1+i}\right)^n = 0;\ i \geq g\right)$

Q = Payment amount

Arithmetically-Increasing Annuity:

$$Pa_{\overline{n}|i} + \frac{Q}{i}\left(a_{\overline{n}|i} - nv^n\right)$$

$P \quad . \quad Q$

Formulas:

Chapter 5:

$L = Qa_{\overline{n}|i}$

$OLB_k = L(1+i)^k - Qs_{\overline{k}|i}$

$OLB_k = Qa_{\overline{n-k-1}|i} + R(1+i)^{-(n-k)}$

$P_t = Qv^{n-t+1}$

$I_t = Q(1 - v^{n-t+1})$

$Q = P_t + I_t$

Chapter 6:

$P = (Fr)a_{\overline{n}|i} + K$

$K = Cv_j^n$

$Fr = Cg = Gj$

(Premium) $P - C = C(g-j)a_{\overline{n}|j}$

(Discount) $C - P = C(j-g)a_{\overline{n}|j}$

$P = (C-G)v_j^n + G$

$P = \frac{g}{j}(C-K) + K$

$Fr = I_t + P_t$

$I_t = jB_{t-1}$

$P_t = B_{t-1} - Fr$

$B_t = (1+j)\, B_{t-1} - Fr$

$B_t = C(g-j)a_{\overline{n-t}|} + C$

$B_t = Fra_{\overline{n}|} + Cv^{n-t+1}$ = OLB_t

Definitions:

L = Loan amount

OLB_k = Outstanding Loan Balance at time k

R = Final payment amount

P_t = Principal Paid at time t

I_t = Interest Paid at time t

N = number of years in bond term

m = number of coupons per year

n = number of coupons = Nm

α = nominal coupon rate

r = coupon rate per coupon period

g = modified coupon rate

I = nominal yield rate convertible m times per year

j = effective yield rate per period

F = face (par) value

C = redemption amount

G = base amount

P = price at issue

K = present value of redemption amount

Formulas: *Definitions:*

Chapters 8 & 9:

Immunization:

Reddington: $S(i) = 0,\ S`(i) = 0,\ S``(i) = 0$

S(i) = "surplus", or present value of Assets - Liabilities

$$S(i) = \sum_{t\geq 0} A_t\,(1+i)^{-t} - \sum_{t\geq 0} L_t(1+I)^{-t}$$

- Reddington immunization works for small changes in i

Full Immunization: Special case of immunization where assets occur on either side of the liability- no change in interest rate will cause a decrease in price

- Made by a single liability being matched by two assets: one paid before the liability and one after.

Sensitivity:

$P(i) = \sum C_t(1+i)^{-t}$

P(i) = Price as a function of i

$P`(i) = \sum C_t(-t)(1+i)^{-t-1}$

C_t = Value of cash flow @ time = t

$P``(i) = \sum C_t(-t)(-t-1)(1+i)^{-t-2}$

$C(i,1) = \frac{P``(i)}{P(i)}$

C(i, 1) = Convexity

Formulas:

Tangent/1st order approximation:

$$\frac{P(i) - P(i_0)}{P(i_0)} \approx \frac{P`(i_0)}{P(i_0)}(i - i_0)$$

$\frac{P(i)-P(i_0)}{P(i_0)} \approx$ is an approximation of the *percent change* of $P(i_0)$. In order to approximate $P(i)$, be sure to get $P(i)$ *alone on its side*.

Quadratic/2nd Order Approximation:

$$\frac{P(i)-P(i_0)}{P(i_0)} \approx \frac{P`(i_0)}{P(i_0)}(i-i_0) + \frac{P``(i_0)}{P(i_0)}\frac{(i-i_0)^2}{2}$$

$D(i,1) = \frac{-P`(i)}{P(i)}$

$D(i,1)$ = Modified Duration

$D(i,\infty) = D(i,1)(1+i) = \frac{-P`(i)}{P(i)}(1+i) = \frac{\sum C_t(t)(1+i)^{-t}}{\sum C_t(1+i)^{-t}}$

$D(i,\infty)$ = Macaulay Duration

Formulas:

Tangent/1st order approximation:

$$\frac{P(i)-P(i_0)}{P(i_0)} \approx \frac{P`(i_0)}{P(i_0)}(i-i_0)$$

Quadratic/2nd Order Approximation:

$$\frac{P(i)-P(i_0)}{P(i_0)} \approx \frac{P`(i_0)}{P(i_0)}(i-i_0)+\frac{P``(i_0)}{P(i_0)}\frac{(i-i_0)^2}{2}$$

$D(i,1)=\frac{-P`(i)}{P(i)}$

$D(i,\infty)=D(i,1)(1+i)=\frac{-P`(i)}{P(i)}(1+i)=$
$\frac{\sum C_t(t)(1+i)^{-t}}{\sum C_t(1+i)^{-t}}$

Macaulay Approximation (1st order):

$$P(i) \approx P(i_0)\left(\frac{1+i_0}{1+i}\right)^{D(i_0,\infty)}$$

Definitions:

$\frac{P(i)-P(i_0)}{P(i_0)} \approx$ is an approximation of the *percent change* of $P(i_0)$. In order to approximate $P(i)$, be sure to get $P(i)$ *alone on its side*.

$D(i,1)$ = Modified Duration

$D(i,\infty)$ = Macaulay Duration