Stat 274 - Hartman **Name:** ______________________

Practice Final Exam - New Material Only

Time Limit: 75 Minutes

Note: The text below is essentially what will be on the final. Your final will have 20 questions. This practice final only contains new material, but the actual final is cumulative. The breakdown will be similar to the SOA breakdown: 7 questions on material from the midterm, 4 on loans, 4 on bonds, and 5 on yield curves, duration, convexity, approximations, and immunization.

This exam contains 38 pages (including this cover page) and 36 problems. Check to see if any pages are missing.

You may only use SOA-approved calculators and a pencil or pen on this exam.

You are required to show your work on each problem on this exam.

Grade calculation errors: If I made an arithmetic mistake (I miscounted your total points) please come and see me and I will fix it.

Regrade requests: I make every effort to grade your test (and those of your classmates) fairly. If you feel I graded a portion of your test too harshly, please write an explanation on the back of the test. Please note that to maintain fairness your entire test will be regraded, potentially resulting in a lower overall grade.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
19	10	
20	10	
21	10	
22	10	
23	10	
24	10	
25	10	
26	10	
27	10	
28	10	
29	10	
30	10	
31	10	
32	10	
33	10	
34	10	
35	10	
36	10	
Total:	360	

1. [10 pts] You pay off a loan with monthly payments of 90 for 10 years. The annual effective interest rate is 5%. What is the outstanding loan balance immediately after the 15th payment?

Solution:

$\mathbf{N} = 12(10) = 120$; $\mathbf{I} = 100[(1.05)^{\frac{1}{12}} - 1] = 0.4074$; $\mathbf{PV} = \text{CPT}$; $\mathbf{PMT} = -90$; $\mathbf{FV} = 0$
Then... **AMORT** P1 = 1; P2 = 15; **BAL** = **7,676.06**

2. [10 pts] You take out a loan of 5000 at a nominal rate of 5.4% convertible monthly. Your first 59 monthly payments are 96, and the last payment is smaller. You miss the 3rd and 17th payments. What is the outstanding loan balance immediately after the 12th payment?

   Solution:

   N $= 60$; **I** $= \frac{5.4}{12} = 0.45$; **PV** $= 5000$; **PMT** $= -96$; **FV** $=$ CPT
   Then... **AMORT** P1 $= 1$; P2 $= 12$; **BAL** $= 4095.84$
   $96(1.0045)^{12-3} = 99.96$
   $4095.84 + 99.96 =$ <u>**4195.80**</u>

3. [10 pts] You borrow 200 at an annual effective rate of 3%. After 3 end-of-year payments of 20, how much interest have you paid? (Note that there are still many payments to be made)

Solution:

N = CPT; **I** = 3; **PV** = −200; **PMT** = 20; **FV** = 0
AMORT P1 = 1; P2 = 3; **INT** = **16.7274**

4. [10 pts] You take out a loan of 100,000 and will repay it with 35 monthly payments of Q and a last (36th) monthly payment of R. Assuming the annual effective interest rate is 4.5%, calculate the difference, $Q - R$ (Note that each payment must be in whole cents).

   Solution:

   $\mathbf{N} = 36$; $\mathbf{I} = 100[(1.045)^{\frac{1}{12}} - 1] = 0.36748$; $\mathbf{PV} = -100,000$;
   $\mathbf{PMT} = \text{CPT} = 2970.6662 \rightarrow \text{Q} = 2970.67$; $\mathbf{FV} = 0$
   Step 2... $\mathbf{N} = 36$; $\mathbf{PMT} = 2970.67$; $\mathbf{FV} = \text{CPT} = \underline{\mathbf{0.32}} = Q - R$

5. [10 pts] You purchase a home for 356,000, make a down payment of 140,000, then finance the rest with a thirty year mortgage at an annual effective interest rate of 6.5%. After exactly 8 years, you sell the home for 282000. Closing costs are 3% of the sale price. How much did you receive at closing?

Solution:

$\mathbf{N} = 12(30) = 360$; $\mathbf{I} = 100[(1.065)^{\frac{1}{12}} - 1] = 0.526$;
$\mathbf{PV} = 356,000 - 140,000 = 216,000$; $\mathbf{PMT} = \text{CPT} = -1338.9577$; $\mathbf{FV} = 0$
Then... $\mathbf{PMT} = -1338.96$; $\mathbf{AMORT}$ $\text{P1} = 1$; $\text{P2} = 12(8) = 96$;
$\mathbf{BAL} = 190,800.243$; $282,000(1 - 0.03) - 190,800.243 = \underline{\mathbf{82{,}739.76}}$

6. [10 pts] You purchase a home for 300,000, make a down payment of 140,000, then finance the rest with a thirty year mortgage at an annual effective interest rate of 6.5%. After exactly 8 years, you sell the home for 282000. Closing costs are 3% of the sale price. How much did you pay in interest over the 8 years?

Solution:

$\mathbf{N} = 12(30) = 360$; $\mathbf{I} = 100[(1.065)^{\frac{1}{12}} - 1] = 0.526$;
$\mathbf{PV} = 300,000 - 140,000 = 160,000$; $\mathbf{PMT} = \text{CPT}$; $\mathbf{FV} = 0$
AMORT $\text{P1} = 1$; $\text{P2} = 12(8) = 96$; $\mathbf{INT} = -\underline{\mathbf{76{,}548.49}}$

7. [10 pts] You take out a loan which will be repaid with 10 annual payments at an annual effective interest rate of 6%. The first payment will be at time 1. The first four payments are 850 each and the last six are 1250 each. What is the outstanding loan balance immediately after the 6th payment?

Solution:

Prospective: $1250a_{\overline{4}|.06} =$ **4,331.38**

8. [10 pts] A 10-year loan of 2,000 is to be repaid with payments at the end of each year. It can be repaid under the following two options:

   (i) Equal annual payments at an annual effective rate of 8.07%.

   (ii) Installments of 200 each year plus interest on the unpaid balance at an annual effective rate of i.

   The sum of the payments under option (i) equals the sum of the payments under option (ii). Determine i.

   Solution:

   $2000 = \mathrm{X}a_{\overline{10}|.0807}$; $\mathrm{X} = 299$; $10\mathrm{X} = 2990$
   (ii) $2990 = 10(200) + (2000 + 1800 + ... + 400 + 200)i$; $i = \underline{\mathbf{0.09}}$

9. [10 pts] You have decided to invest in an n-year 1,000 bond with semi-annual coupons. The ratio of the semi-annual coupon rate to the desired semi-annual yield rate, r/i, is 1.03125. The present value of the redemption value is 381.50. Given $v^n = 0.5889$, what is the price of the bond?

   Solution:

   $P = 1000r(\frac{1-v^{2n}}{i}) + 381.5 = 1000(1.03125)(1 - .5889^2) + 381.5 = \underline{\mathbf{1{,}055.109}}$

10. [10 pts] You borrow X for four years at an annual effective interest rate of 8%, to be repaid with equal payments at the end of each year. The outstanding loan balance at the end of the third year is 559.12. Calculate the principal repaid in the first payment.

Solution:

N = 4; **I** = 8; **PV** = CPT; **PMT** = X = 559.12(1.08) = 603.85; **FV** = 0
AMORT P1 = P2 = 1; **PRN** = **443.85**

11. [10 pts] A twelve-year bond with semiannual level coupons is bought at a premium to yield 7.5% convertible semiannually. If the amortization of premium in the fourth to the last payment is 8.02, find the amount of premium.

Solution:

$n = 12(2) = 24;\ j = \frac{0.075}{2} = 0.0375;\ P_{21} = B_{20} - B_{21} = 8.02$

$B_{20} = C(g-j)a_{\overline{4}|j} + C;\ B_{21} = C(g-j)a_{\overline{3}|j} + C$

$B_{20} - B_{21} = C(g-j)(a_{\overline{4}|} - a_{\overline{3}|}) \leftarrow$ (draw it out)

$8.02 = C(g-j)v_j^4 \ \rightarrow\ C(g-j) = 9.292$

$P - C = C(g-j)a_{\overline{24}|j} = 9.292(\frac{1-v_{.0375}^{24}}{0.0375}) = \underline{\mathbf{145.38}}$

12. [10 pts] A 7.5 year 14% bond with a face value of 2,500 has semiannual coupons and is sold to yield 7.2% convertible semiannually. The discount on this bond is 282.12. Find the price of this bond.

Solution:

$C - P = 282.12 = C(g - j)a_{\overline{15}|j} = (0.036C - Cg)a_{\overline{15}|j}$
$Cg = Fr = 2500(.07) = 175 \;\therefore\; C - P = (0.036g - 175)a_{\overline{15}|.036}$
$C = 5546.38;\; C - P = 282.12 \rightarrow P =$ **5,264.26**

13. [10 pts] A seven-year bond with quarterly coupon payments and redemption value 2100 is selling for 2450. You are given the modified coupon rate, $g = 0.027833$. If the annual effective rate on this bond were twice what it is at the current pricing level, what would be the absolute difference in the price and the base amount?

Solution:

$\mathbf{N} = 28$; $\mathbf{I} = \text{CPT} = 2.00 = \frac{i^{(4)}}{4}$; $\mathbf{PV} = -2450$;
$\mathbf{PMT} = 2100 \; * \; 0.027833 = 58.45$; $\mathbf{FV} = 2100$
$i = 1.02^4 - 1 = 0.082432$; $i' = 2i = 0.164864$; $\frac{i^{(4)}}{4}' = 1.664864^{1/4} - 1 = 0.038888253$;
Step 2... $\mathbf{N} = 28$; $\mathbf{I} = 3.8888253$; $\mathbf{PV} = \text{CPT} = 1708.15$; $\mathbf{PMT} = 58.45$; $\mathbf{FV} = 2100$
Base Amt $= 58.45/0.038888 = 1503.02; 1708.15 - 1503.02 =$ **<u>205.13</u>**

14. [10 pts] You take out a 10,000 loan repaid with annual payments for twelve years. The annual effective interest rate is 2% for the first two years and 6% for the remainder. What is the loan balance immediately after the fifth payment?

Solution:

$10,000 = \mathrm{X}a_{\overline{2}|.02} + \mathrm{X}a_{\overline{10}|.06}(1.02)^{-2};\ \mathrm{X} = 1109.15815$
$1109.16a_{\overline{7}|.06} = \underline{\mathbf{6{,}191.75}}$

15. [10 pts] A 2,000 bond has annual coupons and is redeemable at the end of fourteen years for 2,260. It has a base amount of 1,845 when purchased to yield 6%. Find the base amount if it were purchased to yield 7%.

Solution:

$Gj = Fr;\ 1845(.06) = 2000r;\ r = 0.055;\ G = \frac{2000(.055)}{.07} = \underline{\mathbf{1{,}571.43}}$

16. [10 pts] A 6,000 ten-year 12% par-value bond with semiannual coupons is priced using a nominal yield rate of 6% convertible semiannually. What is the amount of premium on this bond?

Solution:

$P - C = C(g-j)a_{\overline{n}|j} = (Cg - Cj)a_{\overline{n}|j} = (6000(.06) - 6000(.03))a_{\overline{20}|.03} = \underline{\mathbf{2{,}677.95}}$
(Remember that $Cg = Fr$)

17. [10 pts] A 3,000 10% bond with quarterly coupons is redeemable after an unspecified number of years for 2,500. The bond is priced to yield 8% convertible quarterly. The present value of the redemption value is 892.75. Find the purchase price of the bond.

Solution:

$2500(1.02)^{-n} = 892.75 \rightarrow n = 52;\ 3000(.025)a_{\overline{52}|.02} = \underline{\mathbf{3{,}303.62}}$

18. [10 pts] Kate purchased a Ferrari for 180,800. She paid a down payment of 38,000 and the rest is paid with a 20 end-of-the-month payments with a nominal interest rate of 3.36% convertible monthly. What portion of the loan's total interest is incurred between the 7th and 18th payments?

Solution:

$\mathbf{N} = 20$; $\mathbf{I} = \frac{3.36}{12} = 0.28$; $\mathbf{PV} = 180,800 - 38,000 = 142,800$;
$\mathbf{PMT} = \text{CPT} = -7351.7745 \to -7351.78$; $\mathbf{FV} = 0$
(After changing PMT to new value $\to$ $\mathbf{FV} = \text{CPT} = 0.1118$)
AMORT $\text{P1} = 8$; $\text{P2} = 18$; $\mathbf{INT} = -1785.77$; $\text{P1} = 1$; $\text{P2} = 20$; $\mathbf{INT} = -4235.49$;
$1785.77/4235.49 =$ **0.422**

19. [10 pts] A loan of L at a nominal rate of 3% convertible monthly is repaid with 180 monthly payments of X. The ratio, $L/X = 144.8$ and the amount of interest in the first mortgage payment is 626.42. Calculate X.

Solution:

$L * i = 626.42 = 0.0025L \rightarrow L = 250,568;\ 250,568 = \text{X}a_{\overline{180}|.0025};\ \text{X} = \underline{\textbf{1,730.38}}$

20. [10 pts] A loan is to be repaid by monthly payments of 800 for 36 months, each paid at the end of the month. The interest contained in the 27th payment is 15.78 more than the interest in the 32nd payment. Find the effective monthly rate of interest. (Note that the interest rate is less than 100%)

Solution:

$I_{27} - 15.78 = I_{32};\ 800(1 - v^{36-27+1}) - 15.78 = 800(1 - v^{36-32+1})$
$800(1 - v^{10}) = 800(1 - v^5) + 15.78;\ \text{sub. } v^5 = x;$
$800 - 800x^2 = 800 - 800x + 15.78 \rightarrow 0 = 800x^2 - 800x + 15.78$
$x = \frac{800+\sqrt{800^2-4(800)(15.78)}}{1600} = 0.9799 = (1+i)^{-5};\ \frac{i^{(12)}}{12} = \underline{\mathbf{0.004075}}$

21. [10 pts] The following two bonds are currently available.

	Price (per 100 face value)	Macaulay Duration
Bond 1	98.35	12.7
Bond 2	130.49	15.6

The combined face amount of the portfolio is 100 and the Macaulay duration of the portfolio is 13.5. Find the portfolio value.

Solution:

$\frac{98.35(12.7)x+130.49(15.6)(1-x)}{98.35x+130.49(1-x)} = 13.5;$

$1249.045x + 2035.64 - 2035.64x = 1327.725x + 1761.62 - 1761.62x$

$274.03 = 352.71x \to 0.7769 = x \to 98.35x + 130.49(1-x) = \underline{\mathbf{105.52}}$

22. [10 pts] A 3-year annual coupon bond has coupons of 20 per year starting one year from now and matures in 3 years for amount 100. The annual effective yield on the bond is 11.8%. Find the Macaulay duration for the bond.

Solution:

$v = \frac{1}{1.118}$; $\frac{20(1)v+20(2)v^2+120(3)v^3}{20v+20v^2+120v^3} = \underline{\mathbf{2.568}}$

23. [10 pts] The duration of a level perpetuity-immediate paying 100 per year is 13.5. Calculate the annual effective interest rate.

Solution:

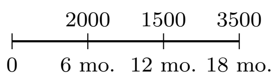
$13.5 = \frac{\text{Ia}_{\overline{\infty}|i}}{\text{a}_{\overline{\infty}|i}} = \frac{\frac{1}{i}+\frac{1}{i^2}}{\frac{1}{i}} = 1 + \frac{1}{i} \rightarrow 12.5 = \frac{1}{i};\ i = \underline{\mathbf{0.08}}$

24. [10 pts] You have to pay 2,000 in six months, 1,500 in 12 months, and 3,500 in 18 months. The bonds available for purchase (at any face amount) are

- Six-month zero-coupon bonds, sold to yield 6% nominal interest convertible semiannually.
- 12-month 6% par-value bonds with semiannual coupons.
- 18-month 5% par-value bonds with semiannual coupons.

You purchase bonds to exactly match your liabilities to your assets. What is the sum of the face amounts of those bonds?

Solution:



$F_{18} = \frac{3500}{1+\frac{0.05}{2}} = 3414.63$; $F_{12} = \frac{1500-3414.63(0.025)}{1+\frac{0.06}{2}} = 1373.43$

$F_6 = 2000 - 3414.63(0.025) - 1373.43(0.03) = 1873.43$; $F_6 + F_{12} + F_{18} = \underline{\mathbf{6{,}661.49}}$

25. [10 pts] Calculate the Macaulay duration for a 30-year mortgage for 300,000 at a nominal interest rate of 4.5% convertible monthly.

Solution:

$\mathbf{N} = 360;\ \mathbf{I} = 4.5/12;\ \mathbf{PV} = -300,000;\ \mathbf{PMT} = \text{CPT} = 1520.06 = Q;\ \mathbf{FV} = 0$

$\text{MacD} = \frac{Qa_{\overline{360|}} + \frac{Q}{i}(a_{\overline{360|}} - 360v^{360})}{300{,}000} = 141.248 \text{ months} \rightarrow \underline{\mathbf{11.77}}$

26. [10 pts] You receive the following cash flows:

t	C_t
1	-2,500
2	-3,500
4	2,000
6	7,000

Assuming that the current interest rate is 0.02, calculate the second-order Taylor approximation of the present value after a 100 basis point increase.

Solution:

$P(i) = -2500(1+i)^{-1} - 3500(1+i)^{-2} + 2000(1+i)^{-4} + 7000(1+i)^{-6};\ P(0.02) = 2248.42$
$P'(i) = 2500v^2 + 7000v^3 - 8000v^5 - 42000v^7;\ P'(0.02) = -34,810.20$
$P''(i) = -5000v^3 - 21000v^4 + 40000v^6 + 294000v^8;\ P''(0.02) = 262,332.66$
$P(i) \approx P(0.02) + P'(0.02)(i-0.02) + \frac{P''(0.02)}{2}(i-0.02)^2$
$= 2248.42 - 34810.20(0.03-0.02) + \frac{262,332.66}{2}(0.03-0.02)^2 = \underline{\mathbf{1,913.43}}$

27. [10 pts] Assume you are going to receive the following cash flows:

t	C_t
1	-1,500
2	-3,000
4	2,000
6	7,000

Assuming that the current interest rate is 0.05, calculate the absolute difference between the tangent approximation and the first-order Macaulay approximation of the present value after a 100 basis point increase.

Solution: $P(i) = -1500v - 3000v^2 + 2000v^4 + 7000v^6$; $P(0.05) = 2719.25$
$P'(i) = 1500v^2 + 6000v^3 - 8000v^5 - 42000v^7$; $P'(0.05) = -29,573.26$
$\text{ModD} = \frac{-P'(0.05)}{P(0.05)} = \frac{29{,}573.26}{2719.25} = 10.88$; $\text{MacD} = \text{ModD}(1.05) = 11.42$
$P(i) \approx P(0.05)[\frac{1.05}{1.06}]^{11.42} = 2440.29$
$P(i) \approx P(0.05) + P'(0.05)(0.06 - 0.05) = 2423.52$; $2440.29 - 2423.52 =$ **16.7692**

28. [10 pts] The interest rate on your bond has increased by 0.0035. The redemption value of the bond is 14,000 with a modified coupon rate of 3.4%. The bond pays coupons annually for 3 years. What is the price estimate using the first order Macaulay duration approximation, given the original interest rate was 0.045.

Solution:

$i_0 = 0.045;\ i = 0.045 + 0.0035 = 0.0485;\ C = 14000;\ g = 0.034;\ Cg = Fr = 476$

$\text{MacD} = \frac{476(1.045)^{-1}+2(476)(1.045)^{-2}+3(14476)(1.045)^{-3}}{476(1.045)^{-1}+476(1.045)^{-2}+14476(1.045)^{-3}} = 2.9$

$P(i) \approx 13,576.66(\frac{1.045}{1.0485})^{2.9} = \underline{\mathbf{13,445.61}}$

29. [10 pts] Assume you are going to receive the following cash flows:

t	C_t
2	X
4	-6,500
8	Y

Assuming $i = 0.05$, calculate $X + Y$ such that you will never lose money, no matter the shift in interest rates, under the constraint that the present value of the surplus is 0 when $i = 0.05$.

Solution:

$X = 6500v^2 - Yv^6$

Derivative: $0 = -13000v^3 + 6Yv^7$; $Y = \frac{13000}{6v^4} = 2633.60$; $X = 6500v^2 - 2633.60v^6 = 3930.46$; $X + Y =$ **6,564.06**

30. [10 pts] Assume you are going to receive the following cash flows:

t	C_t
0	1,500
1	-1,000
2	-X
4	2,500
7	-Y
9	3,500

Assuming $i = 0.03$, calculate $X + Y$ such that the cash flows are Redington immunized.

Solution:

$0 = 1500(1.03)^2 - 1000(1.03) - X + 2500v^2 - Yv^5 + 3500v^7$
Derivative: $3000(1.03) - 1000 - 5000v^3 + 5Yv^6 - 24500v^8 = 0$
$Y = 5212.33$; $X = 1267.46$; $X + Y =$ **6,479.79**

31. [10 pts] You know the following:

- A 6000 7.8% bond where g=0.052 and pays annual coupons for 3 years costs 8975.
- A two-year 1700 6.7% bond that matures for 2150 and pays annual coupons costs 2100.
- A one year zero-coupon bond matures for 4500 initially costs 4250.

What is the three-year spot rate?

Solution:

$\frac{4500}{4250} - 1 = y_1 = 0.05882$; $2100 = \frac{113.90}{1+y_1} + \frac{2263.90}{(1+y_2)^2} \rightarrow y_2 = 0.06595$;

−2100	113.90	2263.90
0	1	2

−8975	468	468	9468
0	1	2	3

$8975 = \frac{456}{1+y_1} + \frac{468}{(1+y_2)^2} + \frac{9468}{(1+y_3)^3} \rightarrow y_3 = \underline{\mathbf{0.05248}}$

32. [10 pts] Using an annual effective interest rate of 0.05, calculate the Macaulay duration of a level perpetuity that pays at the end of each year.

Solution:
$\frac{\frac{x}{i}+\frac{x}{i^2}}{\frac{x}{i}} = 1 + \frac{1}{i} = 1 + \frac{1}{0.05} = \underline{\mathbf{21}}$

33. [10 pts] (Modified from SOA 59) A liability consists of a series of 15 payments of 35,000 with the first payment to be made a year from now. The assets available to immunize this liability are five-year and ten-year zero-coupon bonds. The annual effective interest rate used to value the assets and the liability is 6.2%. The liability has the same present value and duration as the asset portfolio. Calculate the amount invested in the ten-year zero-coupon bonds.

Solution:

Liabilities:
Price: $35000a_{\overline{15}|0.062} = 335,530.30$; Duration $= \frac{35000a_{\overline{15}|} + \frac{35000}{0.062}(a_{\overline{15}|} - 15v^{15})}{335{,}530.30} = 6.89214$

Assets:
Price: $= A + B = 335,530.30 \to A = 335,530.30 - B$
Duration $= \frac{5A+10B}{A+B} = \frac{5A+10B}{335{,}530.30} = \frac{5(335{,}530.30-B)+10B}{335{,}530.30}$
$6.89214 = \frac{1{,}677{,}651.5+5B}{335{,}530.30} \to B =$ **126,974.09**

34. [10 pts] (Modified from SOA 68) You buy a ten-year 8000 par bond with an annual coupon rate of 5%, paid annually. The bond sells for 8000. Let d_1 be the Macaulay duration just before the first coupon is paid. Let d_2 be the Macaulay duration just after the first coupon is paid. Calculate d_1/d_2.

Solution:

$Fr = 8000(0.05) = 400;\ n = 10;\ C = 8000 = P$

$d_1 = \frac{400(0)+400(\text{Ia})_{\overline{9}|0.05}+9(8000)v^9}{400+400a_{\overline{9}|0.05}+8000v^9} = \frac{59705.70}{8400};\ d_2 = \frac{400(\text{Ia})_{\overline{9}|0.05}+9(8000)v^9}{400a_{\overline{9}|0.05}+8000v^9} = \frac{59705.70}{8000}$

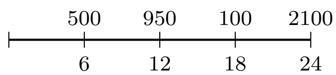
$\frac{d_1}{d_2} = \frac{8000}{8400} = \underline{\mathbf{0.95238}}$

35. [10 pts] You must pay liabilities of 25 and 15 at times 10 and 14 respectively. You will have asset cash flows of X, 20 and 10 at times t, 12, and 16 respectively. What must X be in order to achieve full immunization ($i = 0.02$)?

Solution:
At time 10,
PV of Assets $= 20v^2 + 10v^6 + Xv^{t-10}$
PV Liabilities $= 25 + 15v^4 = 38.858$
Derivative of PV Assets $= -(t-10)xv^(t-10)v - 40v^3 - 60v^7$
Derivative of PV Liabilities $= -60v^5 = -54.344$
Set the PVs equal to each other and solve for Xv^{t-10}
$Xv^{t-10} = 25 + 15v^4 - 20v^2 - 10v^6 = 10.75459$
Now set the derivatives equal to each other and solve for t
$t = \frac{60v^5 - 40v^3 - 60v^7}{10.75459v} + 10 = 6.6252$
Finally, solve for X$X = \frac{10.75459}{v^{t-10}} = 10.059359$

36. [10 pts] You have liabilities of 2100 in 24 months, 100 in 18 months, 950 in 12 months, and 500 in 6 months. There are 3 bonds available for purchase in any face amount. Bond A is a 2-year 10% bond with semi-annual coupons. Bond B is a 1-year 8% bond with semi-annual coupons. Bond C is a 6-month zero-coupon bond with a yield of 2% convertible semi-annually. What face value of bond C must you purchase to exactly match the liability cash flows?

Solution:



$F_A = \frac{2100}{1.05} = 2000$; $F_B = \frac{950-2000(0.05)}{1.04} = 817.31$

$F_C = 500 - 2000(0.05) - 817.31(0.04) = \underline{\mathbf{367.31}}$

Solutions

1. 7676.06
2. 4195.80
3. 16.7274
4. 0.32
5. 82,739.76
6. 76,548.49
7. 4331.38
8. 0.09
9. 1055.11
10. 443.85
11. 145.38
12. 5264.26
13. 205.13
14. 6191.75
15. 1571.43
16. 2677.95
17. 3303.62
18. 42.2%
19. 1730.38
20. 0.408%
21. 105.52
22. 2.568
23. 0.08
24. 6661.49
25. 11.77
26. 1913.43
27. 16.77
28. 13445.61
29. 6564.06
30. 6479.79
31. 0.0525
32. 21
33. 126,974.09
34. 0.95238
35. 10.059359
36. 367.31