Note: The text below is essentially what will be on the midterm. Your midterm will only have 10 questions.

This exam contains 22 pages (including this cover page) and 20 problems. Check to see if any pages are missing.

You may only use SOA-approved calculators and a pencil or pen on this exam.

You are required to show your work on each problem on this exam.

Grade calculation errors: If I made an arithmetic mistake (I miscounted your total points) please come and see me and I will fix it.

Regrade requests: I make every effort to grade your test (and those of your classmates) fairly. If you feel I graded a portion of your test too harshly, please write an explanation on the back of the test. Please note that to maintain fairness your entire test will be regraded, potentially resulting in a lower overall grade.

Problem	Points	Score
1	10	
2	10	
3	10	
4	10	
5	10	
6	10	
7	10	
8	10	
9	10	
10	10	
11	10	
12	10	
13	10	
14	10	
15	10	
16	10	
17	10	
18	10	
19	10	
20	10	
Total:	200	

1. [10 pts] Anne pays Brandon 200 at time 0, Brandon pays Anne 80 at times 1, 2, 3, and 10. Each time Anne receives a payment, she invests it in a separate account earning 5% effective interest and then withdraws all the money at time 10. Calculate the difference between Anne's yield rate and the interest rate Brandon pays (subtract the smaller value from the larger value).

2. [10 pts] What is the present value of an annuity-immediate which pays 10 per year for the first 20 years and 30 per year for the next 20 years at an annual effective interest rate of 5%?

3. [10 pts] Jim deposits 100 into a bank account paying a nominal interest rate of 6% convertible quarterly. At the same time, Gail deposits 50 into an account paying a force of interest of δ. After 15.25 years, the accounts have the same balance. Calculate δ.

4. [10 pts] You deposit 125 into an account which pays an annual effective discount rate of d. If the balance of the account is less than 150, $d = 4\%$. If the account balance is between 150 and 200, the effective rate of discount is $d = 5\%$. If the account balance is greater than 200, the effective rate of discount is $d = 6\%$. How many years does it take for the account balance to grow to 225?

5. [10 pts] You buy an annuity which pays 100 at time 1, $100(1 + g)$ at time 2, $100(1 + g)^{k-1}$ at time k, through $100(1 + g)^{49}$ at time 50. You could have instead purchased a perpetuity paying 110 at the end of each year for the same price. If $i = 0.04$, calculate g.

6. [10 pts] If $\delta_t = 0.01 + 0.001t + 0.0001t^3$, how much will 100 at time 2 grow to become at time 7?

7. [10 pts] You put 20 into a savings account at the end of every other month (if t is in months, at times 1, 3, 5, ...). Assuming the annual effective interest rate is 2%, how many months will it take for you to accumulate 3000 in your account?

8. [10 pts] You buy a perpetuity paying 20 per year, at the end of each year. Immediately after the 7th payment, you sell the remaining payments in your perpetuity and buy a 10-year annuity immediate. Assuming the annual effective interest rate is 6%, how large will each of your 10 annuity payments be?

9. [10 pts] At time 0, you buy an annuity which pays 10 at times 1, 2, 3, ..., 29, and 30. You do not have to pay for this annuity until time 45. Assuming the annual effective interest rate is 4%, what will you need to pay for the annuity (at time 45)?

10. [10 pts] You borrow some money and can either repay it by making (a) two payments of 20 at times 5 and 15 or (b) a single payment of 45 at time 15. Calculate the annual effective interest rate.

11. [10 pts] To be fair to your two children, you give them a gift. One gets two payments of 100, at times 0 and 1. The other receives a perpetuity paying 1 at times 1, 2, 3, ... Calculate the annual effective interest rate.

12. [10 pts] Alice has a very flexible banker who allows her to deposit 2000 into account A with an annual effective interest rate of 7%. In five years she takes half of the interest earned in that account and deposits it into a new account B earning 2% convertible quarterly. At time 16 she takes all the interest earned in account A between times 10 and 16 and transfers it to account B. Now, at year 20, what is the absolute difference between accounts A and B?

13. [10 pts] Always the worrier, you want to make sure you have enough money for your wife's birthday presents every year. You want to have 100 each year forever in an account that earns 8% interest. You and your future wife both turned 15 today, assume you will need the money starting on her 23rd birthday. How much do you need to invest today in order to have the necessary amount on her 23rd birthday?

14. [10 pts] A perpetuity-immediate pays X per year. Ernie receives the first n payments, Colleen receives the next n payments, and Jeff receives the remaining payments. Ernie's share of the present value of the original perpetuity is 40%. Calculate Jeff's share.

15. [10 pts] You can buy a deferred annuity for 21,092.04. The annuity pays twelve annual payments of X, the first of which is 12 years from today. The annuity yields 7.8%. Calculate X.

16. [10 pts] You purchase a perpetuity-immediate. It pays 1,000 at the end of each of the first 11 years and then the payments increase by 180 each year. To be clear, the payment at time 11 is 1,000, and at time 12 the payment is 1,180. Calculate the purchase price assuming a 5% annual effective interest rate.

17. [10 pts] To accumulate 8000 at the end of $3n$ years, deposits of 98 are made at the end of each of the first n years and 196 at the end of each of the next $2n$ years. The annual effective rate of interest is i. You are given $(1 + i)^n = 2.0$. Determine i.

18. [10 pts] You can receive one of the following two payment streams:

    (i) 100 at time 0, 200 at time n, and 300 at time $2n$

   (ii) 600 at time 10

At an annual effective interest rate of i, the present values of the two streams are equal. Given $v^n = 0.76$, determine i.

19. [10 pts] You invest X into an account earning an annual effective interest rate of i. The interest earned in the 2nd year is $X/2$. Calculate i.

20. [10 pts] You borrow 3000 at an annual effective rate of 8% and pay it back with one payment of 1000 at the end of the first year, 500 at the end of the second, and another payment of X at the end of the third. Calculate X.

Solutions

1. 7.433

2. 265.53

3. 0.105

4. 11.98

5. 0.01378

6. 114.125

7. 243.046

8. 45.289

9. 1010.06

10. 0.022565

11. 0.0050125

12. 2046.53

13. 729.37

14. 0.36

15. 6328

16. 64201.75

17. 0.1225

18. 0.035017

19. 0.366

20. 2072.74