

Stat 344 - Fall 2024

Homework Assignment 1

Due: Thursday, September 26th on Learning Suite at 2:00 pm

1. Jimmy recently purchased a house for him and his family to live in with a \$300,000 30-year mortgage. He is worried that should he die before the mortgage is paid, his family will not be able to afford the mortgage payment. A friend of his suggests that he purchase a life insurance policy.
 - (a) What type of life insurance product, if any, should Jimmy purchase? (Your answer should be one or two sentences in length.)
 - (b) Before being issued the policy, Jimmy is required to go through an underwriting procedure. In your own words, briefly describe what is meant by the term **underwriting** and what it might entail for Jimmy and the insurance company. (Your answer should be two to four sentences in length.)
2. Consider a proposed survival function $S_0(t) = \frac{1}{10}\sqrt{100-t}$, $0 \leq t \leq 100$.
 - (a) Verify that this is indeed a valid survival function. (That is, verify that it meets the three necessary conditions discussed in class.)
 - (b) Find the probability that a newborn dies between the ages of 10 and 20. [0.054]
 - (c) Find the probability that a 30-year old lives to at least age 60. [0.756]
 - (d) Find the median lifetime length for a newborn. [75]
 - (e) Find an expression for the force of mortality, μ_x , simplifying as far as possible.
 - (f) Sketch the force of mortality μ_x for $x \in [0, 100]$.
 - (g) Find the mean of the random variable T_{30} . [46.67]
 - (h) Find the variance of the random variable T_{30} . [435.55]
 - (i) Calculate ${}^{\circ}e_{30:\overline{20}|}$. [18.49]
 - (j) Find the median future lifetime of a person age 30. (Is your answer consistent with your answer to part (c)?) [52.5]
3. Assume that the force of mortality for a survival model is given by $\mu_x = \frac{1}{110-x}$.
 - (a) Find the survival function $S_0(t)$ corresponding to this force of mortality, simplifying as far as possible.
 - (b) What is the limiting age ω for this model? [110]

- (c) Sketch the survival function $S_0(t)$.
 - (d) Find the density function $f_0(t)$.
 - (e) Calculate the probability of a newborn dying between the ages of 40 and 60. [20/110]
 - (f) Calculate the probability of a 20-year-old dying between the ages of 40 and 60. [20/90]
 - (g) Do you think this survival model is suitable as a model for human mortality? Why or why not? (Your answer should be one or two sentences in length.)
4. You are given the following information:
- $${}_3p_{51} = 0.9126, \quad {}_2q_{50} = 0.0298, \quad q_{52} = 0.0300, \quad {}_2p_{52} = 0.9312, \quad q_{54} = 1$$
- (a) Find the numerical values of the following quantities (and also make sure you understand the interpretation of each):
 - i. p_{52} [0.97]
 - ii. ${}_4p_{50}$ [0.9035]
 - iii. p_{51} [0.98]
 - iv. ${}_2|_2q_{50}$ [0.0667]
 - v. ${}_3|q_{50}$ [0.0376]
 - vi. ${}_2p_{53}$ [0]
 - vii. e_{50} [3.8047]
 - (b) For the random variable K_{51} , find its:
 - i. pmf (probability mass function)
 - ii. mean [2.8432]
 - iii. standard deviation [0.5577]
 - iv. mode [3]
 - v. Calculate the probability that K_{51} has an odd value. [0.942]
5. (Extra Credit) Show that $\frac{d}{dx} {}_t p_x = {}_t p_x (\mu_x - \mu_{x+t})$.