

# Stat 344 - Fall 2023

## Homework Assignment 1

Due: Tuesday, September 26th in class

1. Jimmy recently purchased a house for him and his family to live in with a \$300,000 30-year mortgage. He is worried that should he die before the mortgage is paid, his family will not be able to afford the mortgage payment. A friend of his suggests that he purchase a life insurance policy.
  - (a) What type of life insurance product, if any, should Jimmy purchase? (Your answer should be one or two sentences in length.)
  - (b) Before being issued the policy, Jimmy is required to go through an underwriting procedure. In your own words, briefly describe what is meant by the term **underwriting** and what it might entail for Jimmy and the insurance company. (Your answer should be two to four sentences in length.)
2. Consider a proposed survival function  $S_0(t) = \frac{1}{10}\sqrt{100-t}$ ,  $0 \leq t \leq 100$ .
  - (a) Verify that this is indeed a valid survival function. (That is, verify that it meets the three necessary conditions discussed in class.)
  - (b) Find the probability that a newborn dies between the ages of 10 and 20. [0.054]
  - (c) Find the probability that a 30-year old lives to at least age 60. [0.756]
  - (d) Find the median lifetime length for a newborn. [75]
  - (e) Find an expression for the force of mortality,  $\mu_x$ , simplifying as far as possible.
  - (f) Sketch the force of mortality  $\mu_x$  for  $x \in [0, 100]$ .
  - (g) Find the mean of the random variable  $T_{30}$ . [46.67]
  - (h) Find the variance of the random variable  $T_{30}$ . [435.55]
  - (i) Calculate  $\overset{\circ}{e}_{30:\overline{20}|}$ . [18.49]
  - (j) Find the median future lifetime of a person age 30. (Is your answer consistent with your answer to part (c)?) [52.5]
3. Assume that the force of mortality for a survival model is given by  $\mu_x = \frac{1}{110-x}$ .
  - (a) Find the survival function  $S_0(t)$  corresponding to this force of mortality, simplifying as far as possible.
  - (b) What is the limiting age  $\omega$  for this model? [110]

- (c) Sketch the survival function  $S_0(t)$ .
- (d) Find the density function  $f_0(t)$ .
- (e) Calculate the probability of a newborn dying between the ages of 40 and 60. [20/110]
- (f) Calculate the probability of a 20-year-old dying between the ages of 40 and 60. [20/90]
- (g) Do you think this survival model is suitable as a model for human mortality? Why or why not? (Your answer should be one or two sentences in length.)

4. You are given the following information:

$${}_3p_{51} = 0.9126, \quad {}_2q_{50} = 0.0298, \quad q_{52} = 0.0300, \quad {}_2p_{52} = 0.9312, \quad q_{54} = 1$$

- (a) Find the numerical values of the following quantities (and also make sure you understand the interpretation of each):
  - i.  $p_{52}$  [0.97]
  - ii.  ${}_4p_{50}$  [0.9035]
  - iii.  $p_{51}$  [0.98]
  - iv.  ${}_2|_2q_{50}$  [0.0667]
  - v.  ${}_3|q_{50}$  [0.0376]
  - vi.  ${}_2p_{53}$  [0]
  - vii.  $e_{50}$  [3.8047]
- (b) For the random variable  $K_{51}$ , find its:
  - i. pmf (probability mass function)
  - ii. mean [2.8432]
  - iii. standard deviation [0.5577]
  - iv. mode [3]
  - v. Calculate the probability that  $K_{51}$  has an odd value. [0.942]

5. (Extra Credit) Show that  $\frac{d}{dx} {}_t p_x = {}_t p_x (\mu_x - \mu_{x+t})$ .