Stat 344 Homework Assignment 2 Solutions

General Notes:

- Please hand in Part I on paper in class on the due date.
- Upload the Excel spreadsheet you create to answer the questions in Part II to Learning Suite. Your spreadsheet should be neatly organized and labeled; each answer should be highlighted in some manner, and it should be very clear how each of your answers was obtained.

Part I

1. Explain the purpose of the underwriting done by insurance companies prior to issuing life insurance policies. Include an explanation of how a life insurance company could potentially lose money if it decided not to perform underwriting prior to issuing its life insurance policies. (Your answer should be roughly one or two paragraphs in length.)

   Solution:

   Life insurance is intended to hedge the unforeseen mortality risk of the insured by spreading it among a large number of people. Being able to spread this risk depends on getting a representative cross-section of insureds from the population. One purpose of underwriting in insurance to be able to assess the level of risk posed by each individual; once this has been done, the individual can be put into an appropriate risk category (i.e., into a group of other people with similar levels of risk, in terms of health, lifestyle, family history, etc.) and each category can be charged an appropriate premium that reflects their level of risk. A second reason for underwriting is to prevent anti-selection, or adverse selection, where an insured can use their superior knowledge of their own risk to their advantage against the insurer.

   For example, suppose that a life insurance company decided to no longer underwrite its policies. Then an individual might wait until they knew that they had a high risk of dying soon (perhaps after being diagnosed with a terminal illness) to purchase a life insurance policy. Because the life insurance company would not know which applicants were doing this, they would have to either (a) charge a very high premium to all applicants, which wouldn't allow them to be competitive in the marketplace, or (b) charge a premium that is too low for the risk being undertaken by the company, leading to poor financial results for the company.

2. Explain why a life insurance company might request each of the following items from a life insurance applicant: (Each of your answers should be roughly one to three sentences in length.)

   (a) Health records and history of hospitalizations

(b) Driving records

(c) Credit reports

(d) List at least two other types of information that you think it might be helpful for a life insurance company to collect from a life insurance applicant. Briefly explain why each would be useful to the insurer.

Solution:

(a) A life insurance company may request health records from an individual in order to assess the overall health of the applicant and to determine how likely they are to die in the future, or to be able to accurately assess the distribution of the future lifetime of the person.

(b) Because many deaths (particularly at younger ages) are the result of accidental causes, driving records may help the insurance company to get a better idea of the level of overall risk posed by the applicant. That is, a person with a record of many driving accidents or risky driving behaviors (e.g., DUI arrests) may have a larger risk of accidental death in the future than a person with a safer driving history.

(c) A life insurance policy is a long-term financial agreement between the insurance company and the policyholder, usually requiring the policyholder to make regular payments to the insurer over the life of the policy. The insurance company may want to assess the likelihood of the policyholder being able to make these payments prior to entering this type of agreement.

(d)
- Information about applicant's job / career: Some jobs are much more dangerous than others; knowing more information about the applicant's career can help determine job-related risks to the insured.
- Information about hobbies: If the insured engages in dangerous hobbies (e.g., private plane flying, skydiving, cliff jumping, etc.) their risk of death could increase substantially.

Other possibilities might include family history of medical problems, genetic information about the insured, information about the assets of the insured and information about other life insurance policies owned by the insured.

3. You are given the following information:

$$ {}_3p_{51} = 0.9126, \qquad {}_2q_{50} = 0.0298, \qquad q_{52} = 0.0300, \qquad {}_2p_{52} = 0.9312, \qquad q_{54} = 1 $$

(a) Using the above information, fill in the blank entries of the following life table:

(b) Using the UDD assumption, write the pmf of $K_{52}^{(2)}$. (The curtate future lifetime random variable, where instead of rounding down to the nearest year, you round down to the nearest 6 months)

$\mathbf{x}$	$\boldsymbol{\ell_x}$	$\mathbf{d_x}$
50		
51		
52		
53		
54	9,034.50	—
⋮	⋮	⋮

Solution:

(a)

$\mathbf{x}$	$\boldsymbol{\ell_x}$	$\mathbf{d_x}$
50	$\mathbf{10,000.00}$	**100.00**
51	$\mathbf{9,900.00}$	**198.00**
52	$\mathbf{9,702.00}$	**291.06**
53	$\mathbf{9,410.94}$	**376.44**
54	9,034.50	$\mathbf{9,034.50}$
⋮	⋮	⋮

(b) First, we can calculate some values:

$$\ell_{52.5} = 9556.47, \qquad \ell_{53.5} = 9222.72, \qquad \ell_{54.5} = 4517.25$$

Then $P\left[K_{52}^{(2)} = 0\right] = {}_{0.5}q_{52} = \frac{9{,}702 - 9556.47}{9{,}702}$,
$P\left[K_{52}^{(2)} = 0.5\right] = {}_{0.5|0.5}q_{52} = \frac{9556.47 - 9{,}410.94}{9{,}702}$, etc.

k	$P\left[K_{52}^{(2)} = k\right]$
0	0.015
0.5	0.015
1	0.0194
1.5	0.0194
2	0.4656
2.5	0.4656

4. You are given the following life table excerpt (Table 3.1 from Dickson et al.):

x	ℓ_x	d_x
30	$10,000.00$	34.78
31	$9,965.22$	38.10
32	$9,927.12$	41.76
33	$9,885.35$	45.81
34	$9,839.55$	50.26
$\vdots$	$\vdots$	$\vdots$

Calculate the following probabilities under (i) the UDD fractional age assumption and (ii) the Constant Force of Mortality fractional age assumption. (Be sure to carry your calculations to at least 5 decimal places for this problem.)

(a) ${}_{0.7}q_{33}$ [0.00324, 0.00325]

(b) ${}_{0.4}p_{32.5}$ [0.99831]

(c) $p_{31.3}$ [0.99606]

(d) ${}_{1.7}p_{30.8}$ [0.99339,0.99338]

(e) ${}_{0.8|2.1}q_{30.8}$ [0.00893]

Solution:

(a) (i) ${}_{0.7}q_{33} \overset{UDD}{=} (0.7)q_{33} = (0.7)\frac{45.81}{9885.35} \overset{UDD}{=} 0.00324.$

(ii) ${}_{0.7}q_{33} \overset{CF}{=} 1 - {}_{0.7}p_{33} \overset{CF}{=} 1 - (p_{33})^{0.7} \overset{CF}{=} 1 - \left(\frac{9839.55}{9885.35}\right)^{0.7} \overset{CF}{=} 0.00325.$

(b) (i) ${}_{0.4}p_{32.5} = \frac{\ell_{32.9}}{\ell_{32.5}} \overset{UDD}{=} \frac{(0.9)\ell_{33}+(0.1)\ell_{32}}{(0.5)\ell_{33}+(0.5)\ell_{32}} \overset{UDD}{=} 0.99831.$

(ii) ${}_{0.4}p_{32.5} \overset{CF}{=} (p_{32})^{0.4} \overset{CF}{=} 0.99831.$

(c) (i) $p_{31.3} = \frac{\ell_{32.3}}{\ell_{31.3}} \overset{UDD}{=} \frac{(0.3)\ell_{33}+(0.7)\ell_{32}}{(0.3)\ell_{32}+(0.7)\ell_{31}} \overset{UDD}{=} 0.99606.$

(ii) $p_{31.3} = {}_{0.7}p_{31.3} \cdot {}_{0.3}p_{32} \overset{CF}{=} (p_{31})^{0.7}\,(p_{32})^{0.3} \overset{CF}{=} 0.99606.$

(d) (i) ${}_{1.7}p_{30.8} = \frac{\ell_{32.5}}{\ell_{30.8}} \overset{UDD}{=} \frac{(0.5)\ell_{32}+(0.5)\ell_{33}}{(0.2)\ell_{30}+(0.8)\ell_{31}} \overset{UDD}{=} 0.99339.$

(ii) ${}_{1.7}p_{30.8} = {}_{0.2}p_{30.8} \cdot p_{31} \cdot {}_{0.5}p_{32} \overset{CF}{=} (p_{30})^{0.2} \cdot p_{31} \cdot (p_{32})^{0.5} \overset{CF}{=} 0.99338.$

(e) (i) ${}_{0.8|2.1}q_{30.8} = \frac{\ell_{31.6}-\ell_{33.7}}{\ell_{30.8}} \overset{UDD}{=} \frac{(0.4)\ell_{31}+(0.6)\ell_{32}-(0.3)\ell_{33}-(0.7)\ell_{34}}{(0.2)\ell_{30}+(0.8)\ell_{31}} \overset{UDD}{=} 0.00893.$

(ii) ${}_{0.8|2.1}q_{30.8} = {}_{0.8}p_{30.8} \cdot {}_{2.1}q_{31.6} = {}_{0.2}p_{30.8} \cdot {}_{0.6}p_{31} \cdot (1 - {}_{2.1}p_{31.6})$

$= {}_{0.2}p_{30.8} \cdot {}_{0.6}p_{31} \cdot (1 - {}_{0.4}p_{31.6} \cdot p_{32} \cdot {}_{0.7}p_{33})$

$\overset{CF}{=} (p_{30})^{0.2} \cdot (p_{31})^{0.6} \cdot \left(1 - (p_{31})^{0.4} \cdot p_{32} \cdot (p_{33})^{0.7}\right) = 0.00893.$

5. Below is an excerpt of a select and ultimate mortality table with a 3 year select period (Table 3 from Jordan's Life Contingencies).

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$
20	946,394	945,145	943,671	942,001	23
21	944,710	943,435	941,916	940,202	24
22	942,944	941,652	940,108	938,359	25
23	941,143	939,835	938,265	936,482	26
24	939,279	937,964	936,379	934,572	27
25	937,373	936,061	934,460	932,628	28
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$

(a) Calculate ${}_3p_{[22]}$. [0.99514]

(b) Calculate ${}_3p_{[20]+2}$. [0.99437]

(c) Explain in one or two sentences what the above two probabilities represent, and how their interpretations differ.

(d) Calculate ${}_{2|}q_{[21]+1}$. [0.00195]

(e) Calculate $e_{[21]+1:\overline{3}|}$. [2.98958]

(f) Calculate ${}_{1.6}q_{[23]+2}$ under the UDD fractional age assumption.[0.00312]

(g) Calculate ${}_{1.6}q_{[23]+2}$ under the Constant Force of Mortality fractional age assumption. [0.00312]

Solution:

(a) ${}_3p_{[22]} = \dfrac{\ell_{25}}{\ell_{[22]}} = \dfrac{938,359}{942,944} = 0.99514.$

(b) ${}_3p_{[20]+2} = \dfrac{\ell_{25}}{\ell_{[20]+2}} = \dfrac{938,359}{943,671} = 0.99437.$

(c) ${}_3p_{[22]}$ represents the probability that a person just selected at age 22 will survive at least 3 years (i.e., until at least age 25), whereas ${}_3p_{[20]+2}$ represents the probability that a person selected at age 20 and who is now 22, will survive at least three years (i.e., at least until age 25).

(d) $_{2|}q_{[21]+1} = \dfrac{\ell_{[21]+1+2} - \ell_{[21]+1+3}}{\ell_{[21]+1}} = \dfrac{\ell_{24} - \ell_{25}}{\ell_{[21]+1}} = \dfrac{940,202 - 938,359}{943,435} = 0.00195.$

(e)

$$
\begin{aligned}
e_{[21]+1:\overline{3}|} &= {}_1p_{[21]+1} + {}_2p_{[21]+1} + {}_3p_{[21]+1} \\
&= \frac{941,916}{943,435} + \frac{940,202}{943,435} + \frac{938,359}{943,435} \\
&= 2.98958
\end{aligned}
$$

(f) $_{1.6}q_{[23]+2} = 1 - \dfrac{\ell_{[23]+3.6}}{\ell_{[23]+2}} = 1 - \dfrac{\ell_{26.6}}{\ell_{[23]+2}} \overset{UDD}{=} 1 - \dfrac{(0.6) \times \ell_{27} + (0.4) \times \ell_{26}}{\ell_{[23]+2}}$

$\overset{UDD}{=} 1 - \dfrac{(0.6)(934572) + (0.4)(936482)}{938265} = 1 - 0.99688 = 0.00312.$

(g) $_{1.6}q_{[23]+2} = 1 - {}_{1.6}p_{[23]+2} = 1 - \left(p_{[23]+2}\right)\left({}_{0.6}p_{26}\right) \overset{CF}{=} 1 - \left(p_{[23]+2}\right)\left(p_{26}\right)^{0.6}$

$\overset{CF}{=} 1 - (0.99810)\,(0.99878) \overset{CF}{=} 0.00312.$

Part II

For this part, use the **Standard Select Mortality Tables**; an Excel version can be found on my website under supplemental material. We will consider the mortality of $[y]$, where y is the last two digits of your BYU ID number. If the last two digits of your BYU ID are between 00 and 19, then add 20 to the last two digits to get y. For example, if your BYU ID number is 01-234-4456, then $y = 56$; if your BYU ID number is 02-400-3403, then $y = 03 + 20 = 23$.

1. In a new tab, create a column of $\ell_{[y]+k}$, for $k = 0, 1, \ldots$.

2. Calculate the value of ${}_{10}p_{[y]}$ and write a sentence that interprets this value.

3. Calculate the value of ${}_{15|5}q_{[y]}$ and write a sentence that interprets this value.

4 - 6. For the random variable $K_{[y]}$, calculate its median,[1] mean, and standard deviation.

7. Give the age at which $[y]$ is most likely to die and the probability that $[y]$ will die at this age.

[1] The median will likely not be an exact integer, in which case there is not a universal agreement on the definition of the median. For this problem, if the median comes out between two integers, you can use either of these two integers, or any number in between them.