Stat 344 Homework Assignment 3 Solutions

General Notes:

- Please hand in Part I on paper in class on the due date.
- Upload the Excel spreadsheet you create to answer the questions in Part II to Learning Suite. Your spreadsheet should be neatly organized and labeled; each answer should be highlighted in some manner, and it should be very clear how each of your answers was obtained.

Part I

1. Suppose that Joe, age 50, wants to purchase a whole life insurance policy with a death benefit of \$100,000. Assume an annual effective interest rate of $i = 8\%$ and that Joe's future lifetime is described by a random variable with density

$$f_{50}(t) = \frac{1}{70} \quad \text{for} \quad 0 < t < 70$$

(a) Explain what is meant by the term **mortality risk** and how this insurance policy might help Joe mitigate this risk.
Answer: Mortality risk for Joe is the risk that Joe or his dependents will suffer financial loss as a result of Joe dying before expected. Buying a life insurance policy – whole life insurance in this case – can help to mitigate this risk by making a payment to his beneficiaries upon his death.

(b) Assuming that this insurance pays at the moment of death, find the expected value and variance of the present value of this benefit, making sure to use the appropriate actuarial notation. [0.18477, 0.05867]

Answer:

$$\begin{aligned}
\bar{A}_{50} &= \int_0^{70} \frac{1}{70}\, e^{-\delta t}\, dt \\
&= \frac{1}{70\,\delta}\left[-e^{-\delta t}\right]_0^{70} \\
&= \frac{1}{70\,\delta}\left[1 - e^{-\delta\, 70}\right] \qquad (1)
\end{aligned}$$

Plugging in $\delta = \ln(1.08) = 0.07696$ yields $\bar{A}_{50} = 0.1847736$. The second moment can be calculated by plugging in twice the force of interest into (1) above:

$${}^{2}\bar{A}_{50} = \frac{1}{70\,(2*0.07696)}\left[1 - e^{-(2*0.07696)\,70}\right] = 0.0928094.$$

Then the variance is

$$V(Z) = {}^2\bar{A}_{50} - \left(\bar{A}_{50}\right)^2 = 0.05866813.$$

The expected value of the benefit is $100,000\,E[Z] = 18,477.36$ and the variance of the benefit is $(100,000)^2\,V(Z) = 586,681,280$.

(c) Find the 2.5^{th} and 97.5^{th} percentiles for the present value of the benefit. [523.39, 87399.40]

Answer: Letting Z denote the present value of the benefit, we know that $Z = 100,000\,e^{-\delta T_{50}}$. Then the 2.5^{th} percentile is the value of k such that $Pr(Z < k) = 0.025$:

$$\begin{aligned} Pr(Z < k) &= 0.025 \\ Pr\left(100,000\,e^{-\delta T_{50}} < k\right) &= 0.025 \\ Pr\left(T_{50} > -\frac{\ln(k/100,000)}{\delta}\right) &= 0.025 \end{aligned}$$

Letting $q = -\dfrac{\ln(k/100,000)}{\delta}$, we can find the desired value of k as follows:

$$\begin{aligned} \int_0^q f_{50}(t)\,dt &= 0.975 \\ \int_0^q \frac{1}{70}\,dt &= 0.975 \\ \left.\frac{t}{70}\right|_0^q &= 0.975 \\ \frac{q}{70} &= 0.975 \\ q &= 68.25 \end{aligned}$$

so that $k = 523.39$. Similarly, the 97.5^{th} percentile is 87,399.40.

(d) Now assume that, instead of paying the benefit at the moment of death, the insurance pays at the end of the month of death. Find the value of the random variable $K_{50}^{(12)}$ and the PV of the benefit if the person dies at the following ages:

i. 50.02 [99360.71]

Answer: $T_{50} = 0.02 \Rightarrow K_{50}^{(12)} = \frac{0}{12} = 0$. Then the PV is $100,000\,v^{0+\frac{1}{12}} = 99,360.71$

ii. 61.97 [39711.38]

Answer: $T_{50} = 11.97 \Rightarrow K_{50}^{(12)} = 11\frac{11}{12}$. Then the PV is $100,000\,v^{11+\frac{12}{12}} = v^{12} = 39,711.38$

iii. 91.48 [4101.23]

Answer: $T_{50} = 41.48 \Rightarrow K_{50}^{(12)} = 41\frac{5}{12}$. Then the PV is $100,000\, v^{41+\frac{6}{12}} = 4,101.23$.

2. A person currently age 65 wants to purchase a policy that will make a payment of \$500,000 on her 85^{th} birthday if she is alive on her 85^{th} birthday, and will also make a payment of \$1,000,000 on her 100^{th} birthday if she is still alive on her 100^{th} birthday. Use $i = 8\%$. Assume that for this person[1],

$$ {}_tp_{65} = \begin{cases} 1 & \text{for } 0 \le t \le 1 \\ \sqrt{\frac{1}{t}} & \text{for } 1 < t < 50 \\ 0 & \text{for } t \ge 50 \end{cases} $$

(a) Calculate ${}_{10|5}q_{65}$ for this person. [0.058]

Answer: ${}_{10|5}q_{65} = {}_{10}p_{65} - {}_{15}p_{65} = \sqrt{\frac{1}{10}} - \sqrt{\frac{1}{15}} = 0.058$

(b) Find the EPV of this policy. [35419.54]

Answer: We have ${}_{20}p_{65} = \sqrt{\dfrac{1}{20}} = 0.22361$ and ${}_{35}p_{65} = \sqrt{\dfrac{1}{35}} = 0.16903$. Then the EPV for this policy is

$$ \begin{aligned} \text{EPV} &= 500,000 \cdot {}_{20}p_{65} \cdot v^{20} + 1,000,000 \cdot {}_{35}p_{65} \cdot v^{35} \\ &= 500,000 \cdot 0.22361 \cdot \left(\frac{1}{1.08}\right)^{20} + 1,000,000 \cdot 0.16903 \cdot \left(\frac{1}{1.08}\right)^{35} \\ &= 23,987.22 + 11,432.32 \\ &= \boxed{35{,}419.54} \end{aligned} $$

(c) Write the EPV in terms of actuarial symbols for pure endowment EPVs.

Answer: We can write this EPV in actuarial symbols as

$$ \boxed{500,000\, A_{65:\overline{20}|}^{\;\;\,1} + 1,000,000\, A_{65:\overline{35}|}^{\;\;\,1}} \quad \text{or} \quad \boxed{500,000\, {}_{20}E_{65} + 1,000,000\, {}_{35}E_{65}} $$

(d) Sketch the CDF (cumulative distribution function) for the present value of benefits for this policy.

Answer: Graph not shown here.

[1]Note that while this is a perfectly valid survival function — it satisfies the three required conditions — it does *not* meet all of the usual assumptions we normally impose. These assumptions are strictly for mathematical convenience and the failure of this function to meet them won't cause problems for us here.

3. Suppose that the force of mortality for a particular person is given by $\mu_x = 0.04 \quad \forall x$. Assume that $i = 10\%$.

(a) Show that ${}_tp_x = e^{-0.04t}$

Answer:

$$
\begin{aligned}
{}_tp_x &= \exp\left\{-\int_0^t \mu_{x+s}\, ds\right\} \\
&= \exp\left\{-\int_0^t 0.04\, ds\right\} \\
&= \exp\left\{-[\,0.04\, s\,]_0^t\right\} \\
&= \exp\left\{-0.04\, t\right\}
\end{aligned}
$$

(b) Calculate ${}_{20|10}q_{30}$ for this person. [0.14813]

Answer:

$$
\begin{aligned}
{}_{20|10}q_{30} &= {}_{20}p_{30} - {}_{30}p_{30} \\
&= \exp\{-0.04(20)\} - \exp\{-0.04(30)\} \\
&= \boxed{0.14813}
\end{aligned}
$$

(c) Suppose that this person (who is currently age 30) wants to buy a 5-year term insurance, with a benefit of \$500,000 payable at the moment of death.

i. Find the EPV of this insurance (including the correct actuarial symbol for this EPV). [72669.14]

Answer:

$$
\begin{aligned}
500,000\bar{A}^{\,1}_{30:\overline{5}|} &= \int_0^5 \exp(-\delta\, t)\; {}_tp_{30}\; \mu_{30+t}\, dt \\
&= (500,000) \times \left\{ \int_0^5 \exp(-0.0953\, t)\; \exp\{-0.04\, t\}\; (0.04)\, dt \right\} \\
&= (500,000) \times \left\{ (0.04) \int_0^5 \exp(-0.1353\, t)\, dt \right\} \\
&= (500,000) \times \left\{ \frac{-0.04}{0.1353} \left[\exp(-0.1353\, t)\right]_0^5 \right\} \\
&= (500,000) \times \left\{ \frac{0.04}{0.1353} \left[1 - \exp(-0.1353 \times 5)\right] \right\} \\
&= (500,000) \times 0.14534 \\
&= \boxed{72{,}669.14}
\end{aligned}
$$

ii. Calculate the probability that the PV of the benefit paid is greater than $\$400,000$. [0.089389]

Answer: Letting Z denote the PV of the benefit:

$$
\begin{aligned}
P(Z > 400,000) &= P\left(500,000\, v^{min(T_{30},5)} > 400,000\right) \\
&= P\left(500,000\, v^{T_{30}} > 400,000\right) \\
&= P\left(e^{-\delta\, T_{30}} > 0.8\right) \\
&= P\left(-\delta\, T_{30} > \ln(0.8)\right) \\
&= P\left(T_{30} < -\frac{\ln(0.8)}{\delta}\right) \\
&= P\left(T_{30} < 2.341\right) \\
&= 1 - {}_{2.341}p_{30} \\
&= \boxed{0.089389}
\end{aligned}
$$

(d) Now suppose that the interest rate is changed to $i = 5\%$. State with reasons (but without calculations) how this change would impact the results in the previous part.

Answer: Present values are inversely related to the interest rate. Thus, as the interest rate decreases, we would expect that the EPV in part (ci) would increase. We would expect that for part (cii) that since the entire distribution of PVs will shift up, the probability that the PV of DB will exceed 400,000 will also increase.

4. You are given the following life table excerpt (Table 3.1 from Dickson et al.) and an interest assumption of $i = 0.10$:

x	ℓ_x	d_x
30	10,000.00	34.78
31	9,965.22	38.10
32	9,927.12	41.76
33	9,885.35	45.81
34	9,839.55	50.26
⋮	⋮	⋮

(a) Consider a 3-year term insurance that will pay a benefit of \$500,000 at the end of the year of death. Find the EPV of this insurance for someone who is currently age 30. [4724.41]

Answer:

$$\begin{aligned} 500,000\, A^{\,1}_{30:\overline{3}|} &= q_{30}\, v + p_{30}\, q_{31}\, v^2 + p_{30}\, p_{31}\, q_{32}\, v^3 \\ &= 500,000\, [0.003161818 + 0.00314876 + 0.003138242] \\ &= 500,000\, [0.00944882] \\ &= \boxed{4{,}724.41} \end{aligned}$$

(b) Under the UDD assumption, calculate $500,000\bar{A}^{\,1}_{30:\overline{3}|}$ [4956.88]
Answer:

$$\begin{aligned} 500,000\bar{A}^{\,1}_{30:\overline{3}|} &\overset{UDD}{=} 500,000 \left(\frac{i}{\delta}\, A^{\,1}_{30:\overline{3}|} \right) \\ &\overset{UDD}{=} \frac{i}{\delta}\, 500,000\, A^{\,1}_{30:\overline{3}|} \\ &\overset{UDD}{=} (1.049206)(4,724.41) \\ &\overset{UDD}{=} \boxed{4{,}956.88} \end{aligned}$$

(c) Under the UDD assumption, calculate $500,000A^{(4)}{}^{\,1}_{30:\overline{3}|}$ [4898.06]

Answer:

$$
\begin{aligned}
500,000 A^{(4)}{}^{\,1}_{30:\overline{3}|} &\overset{UDD}{=} 500,000\left(\frac{i}{i^{(4)}}\,A^{\,1}_{30:\overline{3}|}\right)\\
&\overset{UDD}{=} \frac{i}{i^{(4)}}\,500,000\,A^{\,1}_{30:\overline{3}|}\\
&\overset{UDD}{=} (1.036756)(4,724.41)\\
&\overset{UDD}{=} \boxed{4,898.06}
\end{aligned}
$$

(d) Under the CFM assumption, calculate $500,000A^{(2)}{}^{\,1}_{30:\overline{2}|}$ [3232.36]
Answer:

$$
\begin{aligned}
A^{(2)}{}^{\,1}_{30:\overline{2}|} &= 500,000\left[\frac{{}_{0.5}q_{30}}{(1.1)^{0.5}}+\frac{{}_{0.5|0.5}q_{30}}{(1.1)^{1}}+\frac{{}_{1|0.5}q_{30}}{(1.1)^{1.5}}+\frac{{}_{1.5|0.5}q_{30}}{(1.1)^{2}}\right]\\
&= 500,000\left[\frac{1-{}_{0.5}p_{30}}{(1.1)^{0.5}}+\frac{({}_{0.5}p_{30})({}_{0.5}q_{30.5})}{(1.1)^{1}}+\frac{({}_{1}p_{30})({}_{0.5}q_{31})}{(1.1)^{1.5}}+\frac{({}_{1.5}p_{30})({}_{0.5}q_{31.5})}{(1.1)^{2}}\right]\\
&\overset{CF}{=} 500,000\left[\frac{1-(p_{30})^{0.5}}{(1.1)^{0.5}}+\frac{((p_{30})^{0.5})(1-(p_{30})^{0.5})}{(1.1)^{1}}\right.\\
&\qquad\left.+\frac{(p_{30})(1-(p_{31})^{0.5})}{(1.1)^{1.5}}+\frac{(p_{30})(p_{31})^{0.5}(1-(p_{31})^{0.5})}{(1.1)^{2}}\right]\\
&\overset{CF}{=} 500,000\,[0.0016595+0.0015795+0.0016528+0.0015729]=\boxed{3232.36}
\end{aligned}
$$

(e) If $A_{33}=0.6$, find A_{30}. [0.455069]

Answer:

$$
\begin{aligned}
A_{32} &= v\,q_{32}+v\,p_{32}\,A_{33}\\
&= \left(\frac{1}{1.1}\right)\left(\frac{41.76}{9,927.12}\right)+\left(\frac{1}{1.1}\right)\left(\frac{9,885.35}{9,927.12}\right)(0.6)\\
&= 0.546984606
\end{aligned}
$$

Similarly, $A_{31}=0.498833289$ and $A_{30}=\boxed{0.455069406}$.

(f) Calculate the EPV of a 2-year endowment insurance with a benefit amount of \$50,000 payable at the end of the year of death for a person age 32. [41339.70]

Answer:

$$
\begin{aligned}
EPV &= 50,000\left[vq_{32}+v^2p_{32}q_{33}+v^2p_{32}p_{33}\right]\\
&= 50,000\left[vq_{32}+v^2p_{32}\right]\\
&= \boxed{41,339.70}
\end{aligned}
$$

(g) Under the UDD assumption, calculate the EPV of 2-year endowment insurance with a benefit amount of $\$50,000$ payable at the end of the quarter of death for a person age 32. [41353.73]

Answer:

$$\begin{aligned} EPV &= 50,000\left\{\frac{i}{i^{(4)}}\left[vq_{32}+v^2p_{32}q_{33}\right]+v^2p_{32}p_{33}\right\} \\ &= \boxed{41{,}353.73} \end{aligned}$$

(h) Under the UDD assumption, calculate the EPV of 2-year endowment insurance with a benefit amount of $\$50,000$ payable at the moment of death for a person age 32. [41358.49]

Answer:

$$\begin{aligned} EPV &= 50,000\left\{\frac{i}{\delta}\left[vq_{32}+v^2p_{32}q_{33}\right]+v^2p_{32}p_{33}\right\} \\ &= \boxed{41{,}358.49} \end{aligned}$$

5. Consider a type of insurance contract issued to (x) that is in effect for 30 years total and pays \$10 at the end of year 10 if the policyholder dies in the first 10 years of the policy, pays nothing if the policyholder dies in the next 10 years of the policy, and pays \$100 at the end of year 30 if the policyholder dies in the final 10 years of the policy.

$${}_5p_x = 0.98 \qquad {}_5p_{x+5} = 0.97 \qquad {}_{15}p_{x+5} = 0.91 \qquad {}_{10|10}q_{x+10} = 0.12 \qquad i = 0.07$$

(a) Find the EPV of this contract. [1.7497]

Answer: The EPV is

$$10\,v^{10}\,{}_{10}q_x + 100\,v^{30}\,{}_{20|10}q_x.$$

Note that

$${}_{10}q_x = 1 - {}_{10}p_x = 1 - {}_5p_x\,{}_5p_{x+5} = 1-(0.98)(0.97) = 0.0494.$$

Also,

$${}_{20|10}q_x = {}_{10}p_x\,{}_{10|10}q_{x+10} = (0.98)(0.97)(0.12) = 0.114072.$$

Then the EPV is

$$10\,(1/1.07)^{10}\,(0.0494) + 100\,(1/1.07)^{30}\,(0.114072) = 1.7497.$$

(b) Calculate the probability that this contract pays a benefit. [0.163472]

Answer: The contract pays a benefit if the insured dies in the first 10 years or the third 10 years, which happens with probability ${}_{10}q_x + {}_{20|10}q_x = 0.0494 + 0.114072 = \boxed{0.163472}$

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	$\ell_{[x]+2}$	ℓ_{x+3}	$x+3$
21	944,710	943,435	941,916	940,202	24
22	942,944	941,652	940,108	938,359	25
23	941,143	939,835	938,265	936,482	26
24	939,279	937,964	936,379	934,572	27
⋮	⋮	⋮	⋮	⋮	⋮

6. Below is an excerpt of a select and ultimate mortality table with a 3 year select period (Table 3 from Jordan's Life Contingencies). Assume $i = 8\%$.

   (a) Calculate $A^{\ 1}_{[22]:\overline{2}|}$ [0.002673]

   (b) Calculate ${}^2A^{\ 1}_{[22]:\overline{2}|}$ [0.002378]

   Answer: First, we find that $v = \frac{1}{1.08} = 0.925926$ and $v^2 = 0.857339$. Also,

$$q_{[22]} = \frac{\ell_{[22]} - \ell_{[22]+1}}{\ell_{[22]}} = \frac{942,944 - 941,652}{942,944} = 0.001370, \quad p_{[22]} = 1 - q_{[22]} = 0.998630.$$

$$q_{[22]+1} = \frac{\ell_{[22]+1} - \ell_{[22]+2}}{\ell_{[22]+1}} = \frac{941,652 - 940,108}{941,652} = 0.001640. \text{ Then we have}$$

$$\begin{aligned} A^{\ 1}_{[22]:\overline{2}|} &= v\, q_{[22]} + v^2\, p_{[22]}\, q_{[22]+1} \\ &= (0.925926)\,(0.001370) + (0.857339)\;(0.998630)\;(0.001640) \\ &= \boxed{0.002673} \end{aligned}$$

and

$$\begin{aligned} {}^2A^{\ 1}_{[22]:\overline{2}|} &= (v)^2\, q_{[22]} + \left(v^2\right)^2\; p_{[22]}\, q_{[22]+1} \\ &= (0.857339)\,(0.001370) + (0.857339)^2\;(0.998630)\;(0.001640) \\ &= \boxed{0.002378} \end{aligned}$$

7. You are given the following life table excerpt (Table 3.1 from Dickson et al.). Assuming $i = 8\%$, calculate the EPV of a 3-year arithmetically increasing term insurance for (30), payable at the end of the year of death, with a benefit of 1 in the first year and benefit increases occurring annually. Also find the variance of the PV of this benefit. [0.019698, 0.03748]

x	ℓ_x	d_x
30	$10,000.00$	34.78
31	$9,965.22$	38.10
32	$9,927.12$	41.76
33	$9,885.35$	45.81
34	$9,839.55$	50.26
$\vdots$	$\vdots$	$\vdots$

Answer:

$$\begin{aligned} E[Z] &= vq_{30} + 2v^2 p_{30} q_{31} + 3v^3 p_{30} p_{31} q_{32} \\ &= \boxed{0.019698} \end{aligned}$$

$$\begin{aligned} E[Z^2] &= v^2 q_{30} + 4v^4 p_{30} q_{31} + 9v^6 p_{30} p_{31} q_{32} \\ &= 0.037868 \end{aligned}$$

so that $Var(Z) = E\left[Z^2\right] - E[Z]^2 = \boxed{0.03748}$

8. Assuming that $x, n,$ and m are all positive integers, with $n > m$, **prove** that

$$A_{x:\overline{n}|} = A^1_{x:\overline{m}|} + {}_mE_x \, A_{x+m:\overline{n-m}|}$$

Be sure to explicitly show each step of your proof.

Answer:

$$
\begin{aligned}
A_{x:\overline{n}|} &= A^{1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}} \\
&= A^{1}_{x:\overline{m}|} + {}_{m|}A^{1}_{x:\overline{n-m}|} + A_{x:\overset{1}{\overline{n}|}} \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x\, A^{\;1}_{x+m:\overline{n-m}|} + A_{x:\overset{1}{\overline{n}|}} \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x\, A^{\;1}_{x+m:\overline{n-m}|} + v^n\, {}_{n}p_x \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x\, A^{\;1}_{x+m:\overline{n-m}|} + v^m\, v^{n-m}\, {}_{m}p_x\, {}_{n-m}p_{x+m} \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x\, A^{\;1}_{x+m:\overline{n-m}|} + v^m\, {}_{m}p_x\, v^{n-m}\, {}_{n-m}p_{x+m} \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x\, A^{\;1}_{x+m:\overline{n-m}|} + {}_{m}E_x\, v^{n-m}\, {}_{n-m}p_{x+m} \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x \left(A^{\;1}_{x+m:\overline{n-m}|} + v^{n-m}\, {}_{n-m}p_{x+m}\right) \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x \left(A^{\;1}_{x+m:\overline{n-m}|} + A_{x+m:\overset{1}{\overline{n-m}|}}\right) \\
&= A^{1}_{x:\overline{m}|} + {}_{m}E_x\, A_{x+m:\overline{n-m}|}
\end{aligned}
$$

Part II

Deterministic Analysis

Subpart A: Level Benefit Term Insurance

Assume that (y) wishes to purchase a 30-year level benefit term insurance. This insurance policy will pay a benefit of $\$500,000$ at the end of the year of death, if death occurs within 30 years of purchase. As was the case for the first assignment y is the last two digits of your BYU ID number; if your BYU ID number is between $00 - 19$, add 20. Assume $i = 6\%$ and that mortality is given by the Standard Select Mortality Model.

1. Find the EPV of the death benefit for this insurance.

2. Find the standard deviation of the present value of the death benefit for this insurance.

3. Find the 90^{th} percentile of the present value of the death benefit for this insurance.

4. What is the probability that this insurance policy will pay the death benefit?

5. Create a graph in Excel showing the PMF (probability mass function) of the present value of death benefit.

   - You can use a "Column" plot or "Scatter" plot or any type of plot that you think adequately shows the shape of the pmf. Make sure that both axes are properly labelled and the scales of the axes (numbers next to the tick marks on the axes) are correct.

Subpart B: Increasing Benefit Term Insurance

Now consider a 30-year term insurance (again sold to a y-year-old) in which the death benefit is $\$300,000$ if the insured dies within the first year, $\$310,000$ if the insured dies in the second year, $\$320,000$ if the insured dies in the third year, etc. No death benefit is paid if the insured lives at least 30 years.

1. Redo Question 1 (from Subpart A) for this increasing benefit term insurance policy.

2. Redo Question 2 (from Subpart A) for this increasing benefit term insurance policy.

3. Redo Question 3 (from Subpart A) for this increasing benefit term insurance policy.

4. Redo Question 4 (from Subpart A) for this increasing benefit term insurance policy.

5. Redo Question 5 (from Subpart A) for this increasing benefit term insurance policy.

6. Suppose that this person buys both the level benefit policy (described in Subpart A) as well as the increasing benefit policy (described in Subpart B). What is the probability that the increasing benefit policy pays a greater benefit than the level benefit policy?

Optional Bonus Section – worth up to 10% bonus

Notes:

- This part is perhaps easier to complete in **R** than Excel. You can use **R** for this part, even if you used Excel for the deterministic analysis.
- If you use **R**, submit the code you use, as well as your output, in a PDF, text, or Word document. Your document should be neatly organized and labelled; it should be very clear how each of your answers was obtained.

Stochastic Analysis

Now you will consider an analysis based on simulations. Specifically, you will simulate the future lifetime of the same y-year old 1,000 times and base your answers on the results of these simulations. Use the same mortality and interest assumptions as above. (Note: You can use 10,000 or 100,000 simulations if you prefer and if it doesn't cause a severe processing burden for your computer.)

Subpart A: Level Benefit Term Insurance

Redo Questions 1-5 in Part I — Subpart A based on your simulated values.

Subpart B: Increasing Benefit Term Insurance

Redo Questions 1-5 in Part I — Subpart B based on your simulated values.