Stat 344 — Fall 2023

Homework Assignment 6

Due Date: Thursday, December 14th in class

1. A person age x purchases a fully discrete 3-year term insurance. The death benefit for the policy is $\$100,000$ plus the sum of all premiums paid. Use the following basis for all calculations:

$$q_x = 0.1, \qquad q_{x+1} = 0.2, \qquad q_{x+2} = 0.3, \qquad i = 10\%$$

   (a) Calculate the net annual premium. [26135.78]

   (b) Write a prospective formula for ${}_1V^n$ in terms of insurance and annuity symbols. Use this equation to calculate ${}_1V^n$. [17928.7]

   (c) Write an equation for ${}_2V^n$ in terms of ${}_1V^n$. Use this equation to solve for ${}_2V^n$.

   (d) Write an equation for ${}_2V^n$ in terms of ${}_3V^n$. Use this equation to solve for ${}_2V^n$.

2. A single premium continuous whole life annuity is issued to (65). The annuity pays at a rate of $5,000 per year. The premium and policy value bases are:

   - Mortality is modeled as a constant force of mortality, $\mu = 0.07$.
   - Interest is given by $\delta = 0.03$.
   - The only expenses are an amount of $1,000 incurred at policy issue and amount of $500 incurred at the policy termination.

   (a) Calculate the gross single premium for this policy. [51,350]

   (b) Calculate the gross premium policy value at time 5 for this policy. [50,350]

3. A fully discrete whole life policy is issued to (45) whose death benefit is $100,000 until the insured reaches age 65, and $50,000 thereafter. Premiums are level and paid anually. Use the following basis for all calculations:

   - Mortality follows the SULT.
   - $i = 5\%$
   - Expenses are:
     - 5% of all gross premiums
     - $1,000 at the start of year 1; $100 at the start of each subsequent year the policy is in force

   (a) Calculate the net and gross premiums for this policy. [492.60, 676.96]

(b) Calculate ${}_{10}V^n$ and ${}_{10}V^g$ using prospective formulas. [5086.51, 4275.30]

(c) Calculate ${}_{11}V^n$ and ${}_{11}V^g$ using recursion formulas. [5670.07, 4869.74]

4. A single premium whole life annuity-due is issued to (65) that pays a monthly benefit of \$1,000. The first 10 years of payments are guaranteed. The only expenses are \$700 at issue and \$45 per year in renewal years. (Assume that the renewal expenses are incurred only while the policyholder is alive.) Assume that mortality is given by the SULT; use the UDD fractional age assumption where necessary. Also assume an annual effective interest rate of 5%.

   (a) Calculate the net single premium for this policy. [160,546.62]

   (b) Calculate the gross single premium for this policy. [161,811.40]

   (c) Calculate ${}_8V^n$, ${}_8V^g$, and ${}_8V^e$ for this policy, assuming the insured is alive at that time. [126,850.76; 127,346.10; 495.36]

   (d) Calculate ${}_{12}V^n$ for this policy. [109,798.95]

   (e) Calculate ${}_{0.05}p_{76.95}$ for this person. [0.998946]

   (f) Calculate ${}_{11.95}V^n$ for this policy. [109,415.96]

5. Jill purchases a \$250,000 fully discrete 20-year term life insurance policy at age 35. Mortality is given by the SULT. Assume an annual effective interest rate of 5%. Expenses are as follows:

   - Issue expenses of \$1,000.
   - Maintenance expenses of \$50 per year in renewal years.
   - Premium taxes of 2% of all gross premiums.
   - Claims expenses of \$100 at the end of the year of death.

   (a) Calculate ${}_5V^n$, ${}_5V^g$, and ${}_5V^e$ for this policy. [444.02, -347.06, -791.08]

   (b) Calculate the FPT modified premiums for this policy. [93.10, 187.63]

   (c) Calculate ${}_5V^{FPT}$ for this policy. [366.02]

6. Time to death from the onset of a disease in a study of 10 lives finds the following sample:

   1, 2, 4, 4, 5, 6, 6, 7, 10, 15

   (a) Find $S(5.5)$

      i. Using Kaplan-Meier [.5]

      ii. Using Nelson-Åalen [.534]

Now assume that 6 other people left the study for causes other than death at times:

3, 3, 4, 6, 8, 10.

(b) Find $S(5.5)$

  i. Using Kaplan-Meier [.6482]

  ii. Using Nelson-Åalen [.6657]

(c) Calculate the variance of the two estimates in (b). [KM = 0.0166, NA = 0.0135]

(d) Calculate the 95% linear confidence interval of the NA survival estimate in (b). [(0.4383, 0.8930)]

(e) Calculate the 95% log-confidence interval for the KM survival estimate in (b). [(0.3449, 0.8381)]