Stat 344
Practice Final Exam

Name: ______________________________

This exam contains 10 pages (including this cover page) and 5 problems. Check to see if any pages are missing.

You may only use an SOA-approved calculator, the included FAM-L tables, and a pencil or pen on this exam.

You are required to show your work on each problem.

Problem	Points	Score
1	10	
2	17	
3	12	
4	18	
5	22	
Total:	79	

1. [10 pts] A 52 year-old is interested in a unique product which is purchased with an annual premium of π paid at the beginning of each year the policy is in force. The product pays \$5,000 at the end of the year of death if the policyholder dies in the first 10 years and \$10,000 at the end of the year of death if it occurs in the second 10 years. If they don't die in the 20 year term, they receive 10π at time 20. Mortality follows the SULT and $i = 0.05$. Calculate π using the equivalence principle.

$$E(\text{Out}) = 5000\, A^{1}_{52:\overline{10|}} + 10000\; {}_{10|}A^{1}_{52:\overline{10|}} + 10\pi\; {}_{20}E_{52}$$

$$E(\text{In}) = \pi\, \ddot{a}_{52:\overline{20|}}$$

$$\pi\, \ddot{a}_{52:\overline{20|}} = 5000\, A^{1}_{52:\overline{10|}} + 10000\; {}_{10}E_{52}\, A^{1}_{62:\overline{10|}} + 10\pi\; {}_{20}E_{52}$$

$$A^{1}_{52:\overline{10|}} = A_{52:\overline{10|}} - {}_{10}E_{52} = 0.617 - 0.59902 = 0.01798$$

$$A^{1}_{62:\overline{10|}} = 0.05292$$

$$\pi = \frac{5000\, A^{1}_{52:\overline{10|}} + 10000\; {}_{10}E_{52}\, A^{1}_{62:\overline{10|}}}{\ddot{a}_{52:\overline{20|}} - 10\; {}_{20}E_{52}}$$

$$= \frac{5000(0.01798) + 10000(0.59902)(0.05292)}{12.7862 - 10(0.34146)}$$

$$= 43.42$$

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2. For a 20 year term insurance policy, premiums are payable annually at the beginning of each year, and a benefit of \$100,000 is payable at the end of the year of death. Assume that the policyholder is currently 40, mortality follows the SULT, and $i = 0.05$. Expenses are:

- \$100 per policy at inception
- \$50 per policy at the beginning of years 2-20
- 10% of gross premium at inception
- 2% of gross premium at the beginning of years 2-20

a. [4 pts] Define the gross loss-at-issue random variable for this insurance (it should be a function of T_x and/or K_x).

$$L_0^g = \begin{cases} 100000v^{K_{40}+1} - (0.98\pi - 50)\ddot{a}_{\overline{K_{40}+1|}} + 50 + 0.08\pi & \text{if } K_{40} \le 19 \\ -(0.98\pi - 50)\ddot{a}_{\overline{20|}} + 50 + 0.08\pi & \text{otherwise} \end{cases}$$

b. [5 pts] Calculate the annual net premium using the equivalence principle.

$$P^n = \frac{100000 A^{1}_{40:\overline{20|}}}{\ddot{a}_{40:\overline{20|}}} = \frac{100000(0.38126 - 0.36663)}{12.9935}$$

$$= 112.59$$

c. [8 pts] Calculate the annual gross premium using the equivalence principle.

$$E(In) = \pi \ddot{a}_{40:\overline{20|}}$$

$$E(Out) = 100000 A^{\,\,1}_{40:\overline{20|}} + (0.02\pi + 50)\ddot{a}_{40:\overline{20|}} + 50 + 0.08\pi$$

$$\pi \ddot{a}_{40:\overline{20|}} = 100000 A^{\,\,1}_{40:\overline{20|}} + (0.02\pi + 50)\ddot{a}_{40:\overline{20|}} + 50 + 0.08\pi$$

$$\pi = \frac{100000 A^{\,\,1}_{40:\overline{20|}} + 50(\ddot{a}_{40:\overline{20|}} + 1)}{\ddot{a}_{40:\overline{20|}}(0.98) - 0.08}$$

$$= \frac{100000(0.38126 - 0.36663) + 50(12.9935 + 1)}{12.9935(0.98) - 0.08}$$

$$= 170.91$$

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3. [12 pts] For a special whole life insurance on (35), you are given:

- The annual benefit premium is payable at the beginning of each year.
- The death benefit is equal to 1000 plus the return of all benefit premiums paid in the past without interest.
- The death benefit is paid at the end of the year of death.
- $A_{35} = 0.42898$
- $(IA)_{35} = 6.16761$
- $i = 0.05$

Calculate the annual benefit premium for this insurance.

$$\ddot{a}_{35} = \frac{1 - A_{35}}{i/1+i} = 11.99142$$

$$E(In) = \pi \ddot{a}_{35}$$

$$E(Out) = 1000 A_{35} + \pi (IA)_{35}$$

$$\pi \ddot{a}_{35} = 1000 A_{35} + \pi (IA)_{35}$$

$$\pi = \frac{1000 A_{35}}{\ddot{a}_{35} - (IA)_{35}}$$

$$= \frac{1000 (0.42898)}{11.99142 - 6.16761}$$

$$= 73.66$$

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4. You are given the following information about a whole life policy purchased from a life insurance company by (x):

Gross Premium:	P paid at the beginning of each year that (x) is alive
Death Benefit:	\$500,000 plus the sum of all gross premiums paid by the insured over the course of the policy, all payable at the end of the year of death
Premium Tax Expenses:	2% of gross premiums paid (incurred at the time of premium payment)
Issue Expenses:	\$5,000 at the time of policy issue
Maintenance Expenses:	\$250 at the beginning of each year (including the first year)
Termination Expenses:	\$1,000 at the time of death benefit payment

The annual gross premium P is determined using the Equivalence Principle. You are given:

$$i = 6\%, \qquad A_x = 0.16079, \qquad (IA)_x = 4.17226$$

a. [2 pts] Calculate $\ddot{a}_x$.

b. [8 pts] Calculate P.

You are given the following additional information:

$$A_{x+10} = 0.24338, \qquad (IA)_{x+10} = 5.8923 \qquad \ddot{a}_{x+10} = 13.367$$

c. [8 pts] Calculate the net premium reserve at time 10.

Answer:

(a)

$$\begin{aligned} \ddot{a}_x &= \frac{1 - A_x}{0.06/1.06} \\ &= \boxed{14.82604} \end{aligned}$$

(b) First note that if (x) dies in the first year ($K_x = 0$), he will have paid one gross premium of amount P, so that the amount of the benefit will be 500,000 + P. If (x) dies in the second year ($K_x = 1$), he will have paid two gross premiums of amount P, so that the amount of the benefit will be 500,000 + $2P$. In general, if (x) dies in year k (so that $K_x = k + 1$), he will have paid $k + 1$ gross premiums of amount P, so that the amount of the benefit will be 500,000 + $(k + 1)\,P$. The EPV of a benefit that pays $k + 1$ if the insured dies in year k is denoted by $(IA)_x$; thus, the EPV of the total benefit is $500{,}000A_x + P\,(IA)_x$.

$$\begin{aligned}
\text{EPV of Premiums} &= [\text{EPV of Benefits}] + [\text{EPV of Expenses}] \\
P\,\ddot{a}_x &= [500,000A_x + P\,(IA)_x] + [(0.02)\,P\,\ddot{a}_x + 5,000 + (250)\,\ddot{a}_x + 1,000A_x] \\
P &= \frac{500,000A_x + 5,000 + (250)\,\ddot{a}_x + 1,000A_x}{\ddot{a}_x - (0.02)\,\ddot{a}_x - (IA)_x}
\end{aligned}$$

$$\begin{aligned}
P &= \frac{500,000A_x + 5,000 + (250)\,\ddot{a}_x + 1,000A_x}{(0.98)\,\ddot{a}_x - (IA)_x} \\
&= \frac{500,000(0.16079) + 5,000 + (250)\,(14.82604) + (1,000)(0.16079)}{(0.98)\,(14.82604) - (4.17226)} \\
&= \boxed{8{,}618.33}
\end{aligned}$$

(c) First we have to calculate the net premium:

$$\begin{aligned}
\text{EPV of Premiums} &= \text{EPV of Benefits} \\
P\,\ddot{a}_x &= 500,000A_x + P\,(IA)_x \\
P &= \frac{500,000A_x}{\ddot{a}_x - (IA)_x} \\
P &= \frac{500,000(0.16079)}{14.82604 - 4.17226} \\
P &= \boxed{7{,}546.15}
\end{aligned}$$

Then,

$$\begin{aligned}
{}_{10}V^n &= 500,000A_{x+10} + 10PA_{x+10} + P(IA)_{x+10} - P\ddot{a}_{x+10} \\
&= 500,000(0.24338) + 10(7,546.15)(0.24338) + (7,546.15)(5.8923) - (7,546.15)13.367 \\
&= \boxed{83{,}650.61}
\end{aligned}$$

5. You perform a study measuring the number of months between when a claim is first reported to when it is finally closed. Some of the claims were still open when you finished the study (denoted with a $^+$). Here are the 15 claims from your study:

$$2, 2, 3^+, 4^+, 6, 6, 6, 8^+, 8^+, 8, 8, 10^+, 12, 15^+, 21$$

a. [4 pts] Calculate the Kaplan-Meier estimate of $S(9)$.

b. [6 pts] Calculate the variance of the estimate in (a).

c. [4 pts] Calculate the Nelson-Åalen estimate of $S(8)$.

d. [8 pts] Calculate the 95% linear confidence interval of the estimate in (c).

t	r	d
2	15	2
6	11	3
8	8	2
9	4	0
12	3	1
21	1	1

(a) $S(9) = \left(\frac{13}{15}\right)\left(\frac{8}{11}\right)\left(\frac{6}{8}\right) = 0.4727$

(b) $Var = 0.4727^2\left[\left(\frac{2}{15(13)}\right)+\left(\frac{3}{11(8)}\right)+\left(\frac{2}{8(6)}\right)\right]$

$= 0.01922$

(c) $H(8) = \frac{2}{15} + \frac{3}{11} + \frac{2}{8} = 0.65606$

$S(8) = e^{-H(8)} = 0.51889$

(d) $Var = 0.51889^2 \times$

$\left[\frac{2(13)}{15^3} + \frac{3(8)}{11^3} + \frac{2(6)}{8^3}\right] = 0.01324$

$CI = 0.51889 \pm 1.96\sqrt{0.01324}$

$= [0.2934, 0.7444]$