## Name:

This exam contains 13 pages (including this cover page) and 6 problems. Check to see if any pages are missing.

You may only use an SOA-approved calculator and a pencil or pen on this exam.

You are required to show your work on each problem on this exam.

Grade calculation errors: If I made an arithmetic mistake (I miscounted your total points) please come and see me and I will fix it.

Regrade requests: I make every effort to grade your test (and those of your classmates) fairly. If you feel I graded a portion of your test too harshly, please write an ex-

| Problem | Points | Score |
| :---: | :---: | :---: |
| 1 | 8 |  |
| 2 | 14 |  |
| 3 | 14 |  |
| 4 | 5 |  |
| 5 | 5 |  |
| 6 | 6 |  |
| Total: | 52 |  | planation on the back of the test and turn it into me by Wednesday March 15th in class. Please note that to maintain fairness your entire test will be regraded, potentially resulting in a lower overall grade.

1. Use the following life table excerpt

| $x$ | $\ell_{x}$ |
| :---: | :---: |
| 40 | 1000 |
| 41 | 995 |
| 42 | 989 |
| 43 | 983 |
| 44 | 976 |

(a) (1 point) Calculate $d_{43}$
(b) (1 point) Calculate ${ }_{2} p_{41}$
(c) (1 point) Calculate ${ }_{1 \mid} q_{40}$
(d) (2 points) Calculate $e_{41: 31}$
(e) (3 points) Under UDD, calculate ${ }_{1.5} p_{41.6}$

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2. You are the actuary at your life insurance company. You are given the following extract from a 2-year select-and-ultimate life table, where selection corresponds to being underwritten for life insurance:

| $[x]$ | $\ell_{[x]}$ | $\ell_{[x]+1}$ | $\ell_{x+2}$ | $x+2$ |
| :---: | :---: | :---: | :---: | :---: |
| 40 | 33,519 | 33,485 | 33,440 | 42 |
| 41 | 33,467 | 33,428 | 33,378 | 43 |
| 42 | 33,407 | 33,365 | 33,309 | 44 |
| 43 | 33,340 | 33,294 | 33,231 | 45 |
| 44 | 33,265 | 33,213 | 33,143 | 46 |

Note: For all parts of this problem, carry your calculations to at least 6 decimal places. Make whatever assumptions you deem appropriate, and be sure to note any assumptions you make in your calculations.
(a) (3 points) Calculate ${ }_{3.7} p_{[41]}$
(b) (3 points) Calculate ${ }_{3.7} p_{[40]+1}$

Joe and John are both exact age 41. Joe has just purchased a $\$ 100,000$ whole life policy from your company (and hence has just been underwritten), whereas John purchased a $\$ 100,000$ whole life policy from your company one year ago (and hence was underwritten one year ago).
(c) (1 point) Who is more likely to reach age 44.7 ?
(d) (3 points) Calculate the probability that both Joe and John reach age 44.7.

John requests that the death benefit of his policy be increased from $\$ 100,000$ to $\$ 1,000,000$. A sales manager at your company suggests that because John has already been underwritten a year ago, he does not need to be underwritten again.
(e) (2 points) In a sentence or two, give a general definition, in your own words, of the term adverse selection.
(f) (2 points) In a few sentences, explain how the principle of adverse selection applies to John's request, noting the potential financial consequences to your company.

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3. Suppose that the survival function for a newborn is given by

$$
S_{0}(t)=e^{-k t^{2}}, \quad t \geq 0
$$

for some constant $k>0$.
(a) (4 points) Show that $S_{0}(t)$ meets the requirements to be a valid survival function.
(b) (2 points) Show that ${ }_{t} p_{x}=e^{-k t(2 x+t)}$ for this model.
(c) (2 points) Derive an expression for $\mu_{x}$ for this model, simplifying as far as possible. Now suppose that $k=0.0002$.
(d) (3 points) Calculate the median of $T_{30}$, i.e., the median future lifetime for (30).
(e) (3 points) Calculate $P\left[K_{30}^{(4)}=12.75\right]$

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4. (a) (4 points) Prove that (be sure to show all steps)

$$
A_{x: \bar{n} \mid}=A_{x: \overline{n-1}}^{1}+v_{n-1}^{n} p_{x}
$$

(b) (1 point) Explain in words why the equation above is true.

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5. A person currently age 65 wants to purchase a policy that will make a payment of $\$ 500,000$ on her $85^{\text {th }}$ birthday if she is alive on her $85^{\text {th }}$ birthday, and will also make a payment of $\$ 1,000,000$ on her $100^{\text {th }}$ birthday if she is still alive on her $100^{\text {th }}$ birthday. Assume that mortality follows the SULT and $i=0.05$.
(a) (1 point) Calculate $\left.{ }_{10}\right|_{5} q_{65}$ for this person.
(b) (2 points) Find the EPV of this benefit.
(c) (2 points) Find the variance of this benefit.

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6. Assume that the survival function for a newborn is

$$
S_{0}(t)=\frac{100-t}{100}
$$

Assume also that $i=0.06$.
(a) (1 point) Calculate the probability that a newborn dies between the ages of 1 and 2 .
(b) (2 points) Find the survival function for the future lifetime of someone currently age 30 .
(c) (3 points) Calculate $\bar{A}_{30: \overline{10}}$.

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