Stat 344 **Name:** ____________________

Practice Exam 1

This exam contains 9 pages (including this cover page) and 6 problems. Check to see if any pages are missing.

You may only use an SOA-approved calculator and a pencil or pen on this exam.

You are required to show your work on each problem on this exam.

Grade calculation errors: If I made an arithmetic mistake (I miscounted your total points) please come and see me and I will fix it.

Regrade requests: I make every effort to grade your test (and those of your classmates) fairly. If you feel I graded a portion of your test too harshly, please write an explanation on the back of the test and turn it into me by Wednesday March 15th in class. Please note that to maintain fairness your entire test will be regraded, potentially resulting in a lower overall grade.

Problem	Points	Score
1	8	
2	14	
3	14	
4	5	
5	5	
6	6	
Total:	52	

1. Use the following life table excerpt

x	ℓ_x
40	1000
41	995
42	989
43	983
44	976

(a) (1 point) Calculate d_{43}

(b) (1 point) Calculate ${}_2p_{41}$

(c) (1 point) Calculate ${}_{1|}q_{40}$

(d) (2 points) Calculate $e_{41:\overline{3}|}$

(e) (3 points) Under UDD, calculate ${}_{1.5}p_{41.6}$

Solution:

(a) $d_{43} = \ell_{43} - \ell_{44} = 983 - 976 = 7$

(b) ${}_2p_{41} = \frac{\ell_{43}}{\ell_{41}} = \frac{983}{995} = 0.9879$

(c) ${}_{1|}q_{40} = \frac{\ell_{41}-\ell_{42}}{\ell_{40}} = 0.006$

(d)

$$\begin{aligned}
e_{41:\overline{3}|} &= \sum_{k=0}^{2} k \cdot \Pr[K_{41} = k] + 3 \cdot \Pr[K_{41} \geq 3] \\
&= 0 \cdot q_{41} + 1 \cdot {}_{1|}q_{41} + 2 \cdot {}_{2|}q_{41} + 3 \cdot {}_3p_{41} \\
&= 0 + 1 \cdot \frac{989 - 983}{995} + 2 \cdot \frac{983 - 976}{995} + 3 \cdot \frac{976}{995} \\
&= 2.963
\end{aligned}$$

(e)

$$\begin{aligned}
{}_{1.5}p_{41.6} &= \frac{\ell_{43.1}}{\ell_{41.6}} \\
&= \frac{\ell_{43} - 0.1 \cdot d_{43}}{\ell_{41} - 0.6 \cdot d_{41}} \\
&= \frac{983 - 0.1 \cdot 7}{995 - 0.6 \cdot 6} \\
&= 0.9908
\end{aligned}$$

2. You are the actuary at your life insurance company. You are given the following extract from a 2-year select-and-ultimate life table, where selection corresponds to being underwritten for life insurance:

$[x]$	$\ell_{[x]}$	$\ell_{[x]+1}$	ℓ_{x+2}	$x+2$
40	33,519	33,485	33,440	42
41	33,467	33,428	33,378	43
42	33,407	33,365	33,309	44
43	33,340	33,294	33,231	45
44	33,265	33,213	33,143	46

Note: For all parts of this problem, carry your calculations to at least 6 decimal places. **Make whatever assumptions you deem appropriate, and be sure to note any assumptions you make in your calculations.**

(a) (3 points) Calculate ${}_{3.7}p_{[41]}$

(b) (3 points) Calculate ${}_{3.7}p_{[40]+1}$

Joe and John are both exact age 41. Joe has just purchased a \$100,000 whole life policy from your company (and hence has just been underwritten), whereas John purchased a \$100,000 whole life policy from your company one year ago (and hence was underwritten one year ago).

(c) (1 point) Who is more likely to reach age 44.7?

(d) (3 points) Calculate the probability that both Joe and John reach age 44.7.

John requests that the death benefit of his policy be increased from \$100,000 to \$1,000,000. A sales manager at your company suggests that because John has already been underwritten a year ago, he does not need to be underwritten again.

(e) (2 points) In a sentence or two, give a general definition, in your own words, of the term **adverse selection**.

(f) (2 points) In a few sentences, explain how the principle of adverse selection applies to John's request, noting the potential financial consequences to your company.

Solution:

(a) Under UDD: ${}_{3.7}p_{[41]} = \dfrac{\ell_{[41]+3.7}}{\ell_{[41]}} \overset{UDD}{=} \dfrac{0.3\ell_{44} + 0.7\ell_{45}}{\ell_{[41]}} \overset{UDD}{=} \dfrac{(0.3)(33,309) + (0.7)(33,231)}{33,467}$
$\overset{UDD}{=} 0.9936475$

Under CF: ${}_{3.7}p_{[41]} = {}_{3}p_{[41]}\,{}_{0.7}p_{44} \overset{CF}{=} \dfrac{\ell_{44}}{\ell_{[41]}}\left(\dfrac{\ell_{45}}{\ell_{44}}\right)^{0.7} \overset{CF}{=} \dfrac{33,309}{33,467}\left(\dfrac{33,231}{33,309}\right)^{0.7} \overset{CF}{=} 0.9936469$

(b) Under UDD: ${}_{3.7}p_{[40]+1} = \dfrac{\ell_{[40]+4.7}}{\ell_{[40]+1}} \overset{UDD}{=} \dfrac{0.3\ell_{44} + 0.7\ell_{45}}{\ell_{[40]+1}} \overset{UDD}{=} \dfrac{(0.3)(33,309) + (0.7)(33,231)}{33,485}$
$\overset{UDD}{=} 0.9931133$

Under CF: ${}_{3.7}p_{[40]+1} = {}_{3}p_{[40]+1}\,{}_{0.7}p_{44} \overset{CF}{=} \dfrac{\ell_{44}}{\ell_{[40]+1}}\left(\dfrac{\ell_{45}}{\ell_{44}}\right)^{0.7} \overset{CF}{=} \dfrac{33,309}{33,485}\left(\dfrac{33,231}{33,309}\right)^{0.7} \overset{CF}{=}$
0.9931128

(c) Joe is more likely to reach 44.7; since he has been more recently underwritten, he will have a higher survival probability, as is evidenced by the probabilities calculated above.

(d) Assuming independence of Joe and John's future lifetimes, the probability that both reach age 44.7 is ${}_{3.7}p_{[41]}\,{}_{3.7}p_{[40]+1}$. Under UDD for both John and Joe, this is equal to $(0.9936475)(0.9931133) = 0.9868045$; under CF for both John and Joe, this is equal to $(0.9936469)(0.9931128) = 0.9868035$. (We could also use different fractional age assumptions for each person if we desired.)

(e) Adverse selection is the tendency of potential insureds to use their "inside" knowledge of their particular situation to their advantage when purchasing insurance, and hence to the disadvantage of the insurer; people who are more likely to collect an insurance benefit are more likely than average to purchase insurance coverage. This could be manifested in their decision to purchase insurance, or in the timing or amount of their insurance purchase.

(f) Additional underwriting is needed at the time of a benefit increase in order to prevent **adverse selection**; if no underwriting were done, policyholders could adversely select against the insurance company by means of the timing of their policy benefit increases. For example, an insured such as John who needs $\$1,000,000$ of coverage might buy a policy with a small death benefit (say, $\$100,000$) and wait until they were very ill to increase their benefit amount to $\$1,000,000$. Then they would be effectively getting coverage in the amount of $\$1,000,000$ while only paying premiums for a $\$100,000$ policy. Adverse selection makes this type of scenario — which would hurt the company financially — more likely to happen than would naturally occur by chance.

3. Suppose that the survival function for a newborn is given by

$$S_0(t) = e^{-kt^2}, \qquad t \geq 0$$

for some constant $k > 0$.

(a) (4 points) Show that $S_0(t)$ meets the requirements to be a valid survival function.

(b) (2 points) Show that ${}_tp_x = e^{-kt(2x+t)}$ for this model.

(c) (2 points) Derive an expression for μ_x for this model, simplifying as far as possible.

Now suppose that $k = 0.0002$.

(d) (3 points) Calculate the median of T_{30}, i.e., the median future lifetime for (30).

(e) (3 points) Calculate $P\left[K_{30}^{(4)} = 12.75\right]$

Solution:

(a) (i) $\lim_{t\to\infty} S_0(t) = \lim_{t\to\infty} e^{-kt^2} = 0$

(ii) $S_0(0) = e^{-k(0)} = e^{-0} = 1$

(iii) $\frac{d}{dt}S_0(t) = e^{-kt^2} \cdot (-2tk)$. The first term is negative and the second term is positive, so the product is negative for all t, meaning that $S_0(t)$ is a non-increasing function of t.

Then we have shown that $S_0(t)$ is a valid survival function.

(b)

$$\begin{aligned}
{}_tp_x &= \frac{S_0(x+t)}{S_0(x)} \\
&= \frac{e^{-k(x+t)^2}}{e^{-kx^2}} \\
&= \frac{e^{-k(x^2+2tx+t^2)}}{e^{-kx^2}} \\
&= e^{-k(2xt+t^2)} \\
&= e^{-kt(2x+t)}
\end{aligned}$$

(c)

$$\begin{aligned}
\mu_x &= \frac{-\frac{d}{dx}S_0(x)}{S_0(x)} \\
&= \frac{-\left[-2xke^{-kx^2}\right]}{e^{-kx^2}} \\
&= 2xk
\end{aligned}$$

(d) To calculate the median, we set ${}_tp_{30}$ equal to 0.5 and then solve for t. Applying the quadratic formula gives an answer of 36.07392.

(e)

$$
\begin{aligned}
{}_{u|t}q_x &= {}_u p_x \, {}_t q_{x+u} \\
&= e^{-ku(2x+u)} \cdot (1 - {}_t p_{x+u}) \\
&= e^{-ku(2x+u)} \cdot \left(1 - e^{-kt(2(x+u)+t)}\right)
\end{aligned}
$$

$P\left[K_{30}^{(4)} = 12.75\right] = {}_{12.75|0.25}q_{30} = 0.00355$ from the formula above.

4. (a) (4 points) Prove that (be sure to show all steps)

$$A_{x:\overline{n}|} = A^{1}_{x:\overline{n-1}|} + v^n {}_{n-1}p_x$$

(b) (1 point) Explain in words why the equation above is true.

Solution:

(a)

$$\begin{aligned}
A_{x:\overline{n}|} &= \sum_{k=0}^{n-1} v^{k+1} {}_{k|}q_x + {}_nE_x \\
&= \sum_{k=0}^{n-2} v^{k+1} {}_{k|}q_x + v^n {}_{n-1|}q_x + v^n {}_n p_x \\
&= \sum_{k=0}^{n-2} v^{k+1} {}_{k|}q_x + v^n \left[{}_{n-1|}q_x + {}_n p_x \right] \\
&= \sum_{k=0}^{n-2} v^{k+1} {}_{k|}q_x + v^n \left[{}_{n-1}p_x q_{n+x-1} + {}_{n-1}p_x p_{n+x-1} \right] \\
&= \sum_{k=0}^{n-2} v^{k+1} {}_{k|}q_x + v^n \left[{}_{n-1}p_x \left(q_{n+x-1} + p_{n+x-1} \right) \right] \\
&= \sum_{k=0}^{n-2} v^{k+1} {}_{k|}q_x + v^n {}_{n-1}p_x \\
&= A^{1}_{x:\overline{n-1}|} + v^n {}_{n-1}p_x
\end{aligned}$$

(b) With an endowment insurance, if you live to the start of the last year (time $n-1$) you will be paid at the end of the year, by the insurance if you die and by the endowment if you live.

5. A person currently age 65 wants to purchase a policy that will make a payment of \$500,000 on her 85^{th} birthday if she is alive on her 85^{th} birthday, and will also make a payment of \$1,000,000 on her 100^{th} birthday if she is still alive on her 100^{th} birthday. Assume that mortality follows the SULT and $i = 0.05$.

   (a) (1 point) Calculate ${}_{10|5}q_{65}$ for this person.

   (b) (2 points) Find the EPV of this benefit.

   (c) (2 points) Find the variance of this benefit.

   Solution:

   (a) ${}_{10|5}q_{65} = \frac{\ell_{75}-\ell_{80}}{\ell_{65}} = \frac{85{,}203.5-75{,}657.2}{94{,}579.7} = \boxed{0.10093}$

   (b)

$$\begin{aligned} EPV &= 500,000{}_{20}E_{65} + 1,000,000{}_{35}E_{65} \\ &= 500,000(0.24381) + 1,000,000\frac{\ell_{100}}{\ell_{65}}(1.05)^{-35} \\ &= 500,000(0.24381) + 1,000,000\frac{6,248.2}{94,579.7}(1.05)^{-35} \\ &= \boxed{133{,}881.54} \end{aligned}$$

   (c)

$$\begin{aligned} E[X^2] &= (0){}_{20}q_{65} + \left[500,000v^{20}\right]^2{}_{20|15}q_{65} + \left[500,000v^{20} + 1,000,000v^{35}\right]^2{}_{35}p_{65} \\ &= 0 + \left[500,000v^{20}\right]^2\frac{\ell_{85}-\ell_{100}}{\ell_{65}} + \left[500,000v^{20} + 1,000,000v^{35}\right]^2\frac{\ell_{100}}{\ell_{65}} \\ &= 29,657,887,780 \\ Var[X] &= 29,657,887,780 - 133,881.54^2 \\ &= \boxed{11{,}733{,}621{,}030} \end{aligned}$$

6. Assume that the survival function for a newborn is

$$S_0(t) = \frac{100 - t}{100}$$

Assume also that $i = 0.06$.

(a) (1 point) Calculate the probability that a newborn dies between the ages of 1 and 2.

(b) (2 points) Find the survival function for the future lifetime of someone currently age 30.

(c) (3 points) Calculate $\bar{A}_{30:\overline{10|}}$.

Solution:

(a) ${}_{1|1}q_0 = \frac{S_0(1)-S_0(2)}{S_0(0)} = \boxed{0.01}$

(b) $S_{30}(t) = \frac{S_0(30+t)}{S_0(30)} = \frac{70-t}{70}$

(c)

$$\begin{aligned}
\bar{A}_{30:\overline{10|}} &= \int_0^{10} e^{-\delta t} f(t)dt + v^{10}{}_{10}p_{30} \\
&= \int_0^{10} e^{-\delta t}\frac{1}{70}dt + v^{10}\frac{60}{70} \\
&= \frac{1}{70}\left[\frac{e^{-\delta t}}{\delta}\right]_0^{10} + v^{10}\frac{60}{70} \\
&= \frac{1}{70}\left[\frac{1-e^{-10\delta}}{\delta}\right]_0^{10} + v^{10}\frac{60}{70} \\
&= \boxed{0.58687}
\end{aligned}$$