Stat 444 — Winter 2024

Homework Assignment 2

Due Date: Thursday, February 8th at 2:00 pm

General Notes:

- For Part I, you may submit your assignment to Learning Suite or in class.
- For Part II, you should use a spreadsheet.
  - Submit electronically the Excel spreadsheet you create to answer the questions in Part II to Learning Suite. Your spreadsheet should be neatly organized and labeled; each answer should be highlighted in some manner, and it should be very clear how each of your answers was obtained.

Part I

1. Consider a CCRC plan having three levels of care: ILU, ALU, and SNF. Assume that it is possible to move not only to the next greater level of care, but also to the next lesser level of care. The funding arrangement (pricing structure) for the plan is Type A, with a large entry fee ($90,000) and a flat monthly fee ($1,500) regardless of current level of care. The plan provides a refund of the entry fee upon death.

   The monthly costs of providing care (including expenses) are:

   - ILU: $1,700
   - ALU: $2,000
   - SNF: $4,000

   (a) Modify the state diagram for the CCRC model presented in the notes (Slide 2.16) in order to be appropriate for this plan.

   (b) Write the Kolmogorov Forward equations (with boundary conditions) for an individual currently in the ILU, i.e., $\frac{d}{dt}\,{}_tp_x^{0j}$ for $j = 0, 1, 2, 3$.

   You are given the following EPVs for an individual entering the CCRC at age 70:

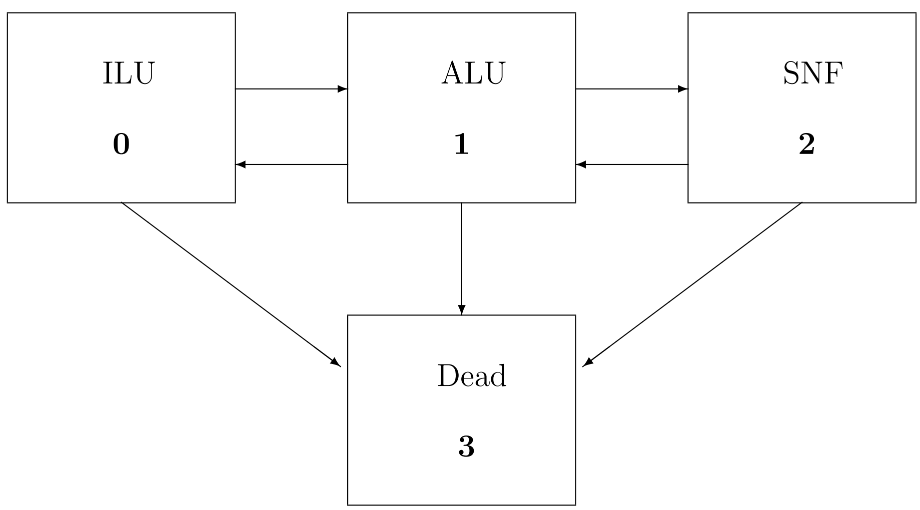
$$\ddot{a}_{70}^{00^{(12)}} = 4.52 \qquad \ddot{a}_{70}^{01^{(12)}} = 2.04 \qquad \ddot{a}_{70}^{02^{(12)}} = 1.25 \qquad \bar{A}_{70}^{03} = 0.31$$

(c) Show that $E[L_0] = -1512$ for this plan, assuming that all cash flows happen at the start of each month.

(d) The plan is considering changing their funding model to a Type B plan. Under this new Type B arrangement, the entry fee would be lower, and the monthly fee would be set at 90% of the actual monthly cost of care. In addition, the entry fee would no longer be refunded upon death. Determine the amount of the new entry fee, if we price the arrangement to keep the same overall level of profitability, i.e., we want to set the price so that $E[L_0] = -1512$ as it is currently. [21628.80]

Answer:

(a)



(b)

$$\frac{d}{dt}\,{}_tp_x^{00} = {}_tp_x^{01}\,\mu_{x+t}^{10} - {}_tp_x^{00}\,\mu_{x+t}^{01} - {}_tp_x^{00}\,\mu_{x+t}^{03}, \qquad {}_0p_x^{00} = 1$$
$$\frac{d}{dt}\,{}_tp_x^{01} = {}_tp_x^{00}\,\mu_{x+t}^{01} + {}_tp_x^{02}\,\mu_{x+t}^{21} - {}_tp_x^{01}\,\mu_{x+t}^{10} - {}_tp_x^{01}\,\mu_{x+t}^{12} - {}_tp_x^{01}\,\mu_{x+t}^{13}, \qquad {}_0p_x^{01} = 0$$
$$\frac{d}{dt}\,{}_tp_x^{02} = {}_tp_x^{01}\,\mu_{x+t}^{12} - {}_tp_x^{02}\,\mu_{x+t}^{21} - {}_tp_x^{02}\,\mu_{x+t}^{23}, \qquad {}_0p_x^{02} = 0$$
$$\frac{d}{dt}\,{}_tp_x^{03} = {}_tp_x^{00}\,\mu_{x+t}^{03} + {}_tp_x^{01}\,\mu_{x+t}^{13} + {}_tp_x^{02}\,\mu_{x+t}^{23}, \qquad {}_0p_x^{03} = 0$$

(c)

$$\begin{aligned} E[L_0] &= 1700(12)(4.52) + 2000(12)(2.04) + 4000(12)(1.25) + 90000(0.31) - 90000 \\ &\quad - 18000(4.52 + 2.04 + 1.25) \\ &= -1512 \end{aligned}$$

(d)

$$\begin{aligned} -1512 &= 1700(12)(4.52) + 2000(12)(2.04) + 4000(12)(1.25) - P \\ &\quad - 0.9[1700(12)(4.52) + 2000(12)(2.04) + 4000(12)(1.25)] \\ P &= 21628.8 \end{aligned}$$

2. Consider a single premium DI product issued to (35), which pays a continuous disability benefit of \$3,000 per month when the insured is disabled. There is a one month waiting period and no off period. The product expires at age 65, and we will assume that it is possible to recover from disability. You are given the following forces of transition (assuming the DI model given in the notes):

$$\mu_x^{01} = 0.02 \qquad \mu_x^{02} = 0.01 + 0.001x \qquad \mu_x^{10} = 0.01 \qquad \mu_x^{12} = 0.02 + 0.002x$$

(a) Give an expression for the EPV of benefits for this product.

(b) Calculate ${}_{10}p_{35}^{\overline{00}}$. [0.49659]

(c) Give an expression for the reserve at time 10 if the insured is Healthy at that time.

(d) Using Thiele's Differential Equation, give an expression for $\frac{d}{dt}\,{}_tV^{(0)}$.

Answer:

(a)

$$EPV = \int_0^{29\,11/12} (36000)\, e^{-\delta t}\, {}_tp_{35}^{00}\, \mu_{35+t}^{01} \left(\bar{a}^{\overline{11}}_{35+t:\overline{30-t}|} - \bar{a}^{\overline{11}}_{35+t:\overline{1/12}|} \right) dt$$

(b)

$$
\begin{aligned}
{}_{10}p_{35}^{\overline{00}} &= e^{-\int_0^{10} \mu_{35+t}^{01} + \mu_{35+t}^{02}\, dt} \\
&= e^{-\int_0^{10} 0.02+0.01+0.001(35+t)\, dt} \\
&= e^{-\int_0^{10} 0.065+0.001t\, dt} \\
&= e^{-\left[0.065t+0.0005t^2\right]_0^{10}} \\
&= e^{-[0.65+0.05]} \\
&= 0.49659
\end{aligned}
$$

(c)

$$
{}_{10}V^{(0)} = \int_0^{19\,11/12} (36000)\, e^{-\delta\, t}\, {}_tp_{45}^{00}\, \mu_{45+t}^{01} \left(\bar{a}_{45+t:\overline{20-t}|}^{\overline{11}} - \bar{a}_{45+t:\overline{1/12}|}^{\overline{11}} \right)\, dt
$$

(d)

$$
\frac{d}{dt}\, {}_tV^{(0)} = \delta\, {}_tV^{(0)} - \mu_{35+t}^{01} \left({}_tV^{(1)} - {}_tV^{(0)} \right) - \mu_{35+t}^{02} \left(0 - {}_tV^{(0)} \right)
$$

Part II

Consider a CCRC plan having three levels of care: ILU, ALU, and SNF. Assume that it is only possible to move to the next greater level of care, i.e., the CCRC model given in the notes. The forces of transition for an individual attained age y are:

$$\mu_y^{01} = 0.001y \qquad \mu_y^{03} = Be^{cy} \qquad \mu_y^{12} = 0.015 \qquad \mu_y^{13} = 2\mu_y^{03} \qquad \mu_y^{23} = 8\mu_y^{03},$$

$$\text{where } B = 10^{-5} \text{ and } c = 0.1$$

Assume that a person age x enters the plan in the ILU level, where x is the last two digits of your BYU ID number. If $x < 50$, then add 50. Assume that the plan ends at age 110. Also assume an interest rate of $i^{(12)} = 0.04$.

The yearly costs of providing care, including expenses, payable continuously, are:

- ILU: \$12,000
- ALU: \$25,000
- SNF: \$45,000

(a) Using Euler's Forward Method with step size $h = \frac{1}{12}$, calculate ${}_tp_y^{00}, {}_tp_y^{01}, {}_tp_y^{02}$, and ${}_tp_y^{03}$ for $t = \frac{1}{12}, \frac{2}{12}, \ldots$.

(b) Find the EPV at issue of this insurance, i.e., the single premium for this plan, using the probabilities calculated above.

Now suppose that we want to design a Type A funding arrangement for this plan. In particular, the insured will pay an entry fee of $10F$ and an annual fee, payable continuously, of F, regardless of the level of care the insured is in.

(c) Using the Equivalence Principle, calculate F, using the probabilities calculated above.