Stat 444 — Winter 2024
Homework Assignment 4
Due Date: Thursday, March 14th at 2:00 pm

General Notes:

- For Part I, you may submit your assignment on Learning Suite.

- For Part II, you should use a spreadsheet.

  - Submit electronically the Excel spreadsheet you create to answer the questions in Part II to Learning Suite. Your spreadsheet should be neatly organized and labeled; each answer should be highlighted in some manner, and it should be very clear how each of your answers was obtained.

Part I

1. XYZ Insurance issues fully discrete joint life 20-year endowment insurance policies to couples, each age 65, with independent future lifetimes. The sum insured is 100,000, payable at the end of the year of the first death, or at time 20 if both individuals survive to that time. Premiums are payable annually while both lives survive, for a maximum of 20 years. You are given the following assumptions for premiums and reserves:

   - Acquisition expenses are 500 plus 20% of the first year's premium.
   - Renewal expenses are 5% of each subsequent premium.
   - Mortality follows the Standard Ultimate Mortality Model (SULT).
   - Gross premiums are calculated using the equivalence principle.
   - $i = 0.05$
   - Reserves are gross premium policy values.

   (a) Calculate $_{20}E_{65:65}$ [0.15773]
   (b) Calculate the gross premium for the policy. [4753.04]
   (c) Calculate the gross premium policy value at time 10 [38,247.28]

   At time 10, XYZ Insurance had 100 identical and independent policies in force. During the 11th policy year, the actual experience was as follows: (i) The interest rate earned was 5.5%. (ii) Renewal expenses were 5% of the premium. (iii) There were 4 claims. (iv) Each claim had a termination expense of 100. The gross premium policy value at time 11 is 42,811.92.

(d) Calculate the total profit in the 11th policy year for the portfolio. [1,117.12]

(e) Calculate the profit by source in the 11th policy year, in the order of Interest, Mortality, and Expenses. [21,381.33; -19,864.11; -400]

At time 19, there were 40 policies remaining in force. During the 20th policy year, a pandemic occurred, such that the actual experience was: (i) The interest rate earned was 5.0% (ii) Renewal expenses were 5% of the premium. (iii) There were 35 claims. (iv) There were no termination expenses.

(f) Show that the total profit in the 20th policy year for the portfolio is zero.

(g) Explain why there is no mortality loss despite the pandemic.

2. For a special 3-year term life insurance issued to (50) with a premium refund feature, you are given:

- The death benefit is 100,000.

- The premium refund feature refunds the last premium payment, without interest, at the end of the 3-year term if the insured is still alive.

- The mortality rates are:

x	q_x
50	0.00592
51	0.00642
52	0.00697

- Pre-contract expenses are 155.

- Commissions are 5% of each premium.

- The hurdle rate is 14%.

- The reserves of this policy have been set to:

t	$_tV$
0	0
1	400
2	800

- The annual premium for this policy is 1100.

- The earned interest rates are:

Year 1	Year 2	Year 3
0.01	0.02	0.03

(a) Calculate the expected profit in policy year 2 for a policy in force at the start of year 2. [37.04]

(b) Calculate the profit vector of this policy. [-155; 65.82; 37.04; 111.02]

(c) Calculate the profit signature and Net Present Value (NPV) of this policy. [-155; 65.82; 36.82; 109.65; 5.08]

(d) Rank from low to high the Internal Rate of Return (IRR) of the following products, explaining your order. [B; A; C]

- Product A: The special 3-year term life insurance described above.
- Product B: A 3-year term life insurance policy with the following profit signature: [-155, 0, 0, 210]
- Product C: The same special 3-year term life insurance as Product A, except that the reserves of the product have been set to:

t	$_tV$
0	0
1	300
2	800

3. NED Life issues a fully discrete whole life insurance of 100,000 on (50) with a return of premium benefit subject to the following conditions:

- If death occurs in the first 10 years, no premiums are returned.
- If death occurs after 10 years, all premiums, including the first 10, are returned without interest at the end of the year of death.

You are given:

- The gross premium is 2,000.
- Premium expenses, payable at the beginning of the year, are 80% of the first year's premium and 5% of premium in subsequent years.
- Termination expenses of 1,000 are paid at the end of the year of death.
- Mortality follows the Standard Ultimate Life Table.
- $i = 0.05$
- L^g is the gross loss at issue random variable and $_kV^g$ denotes the gross premium policy value at the end of year k.
- $(IA)_{60} = 6.63303$

(a) If $T_{50} = 11.8$, calculate L_g. [53,422.49]

(b) Calculate $E[L_g]$. [-248.53]

(c) Calculate $_{10}V_g$ [20,072.15]

NED Life had 1000 such policies in force at the end of 15 years. You are given:

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- $_{15}V^g = 34,333.78$ and $_{16}V^g = 37,480.51$
- During year 16:
  - There were 7 deaths.
  - $i = 0.052$
  - Premium expenses were 4% of premium.
  - Termination expenses were 2,000 per death.

(d) Calculate the total profit in the 16th year. [-17,169.87]

(e) Calculate the gain by source in the following order: expenses (E), interest (I) and mortality (M). [15,085; 72,507.56; -104,723.65]

(f) An actuary at NED Life was asked to calculate the profit by source. He wasn't sure which order to use, so he calculated the profit based on all six possible orders. Referring to the sources of gain in order by their first letters, the order you did in part (e) was EIM. Without further calculation, state, using the three-letter ids, which, if any, of the other 5 orders had the same profit from interest as EIM. Justify your response. [MEI; EMI]

Part II

Assume that you issue 1,000 20-year term policies paying \$500,000 at the end of the year of death to people age x, where x is your BYU ID number (if $x < 40$ then add 40). Assume the following basis for premiums, policy values, and profit testing:

- Interest: 5% per year

- Pre-contract expenses: \$400 + 20% of the first premium

- Renewal expenses: 3.5% of premiums, including the first premium

- Mortality: SULT

- Gross Premium: Calculated using equivalence principle, paid at the beginning of each year

Experience in the first two years is

Year	Expenses	Interest Rate Earned	Deaths
1	\$242,000	5.1%	1
2	\$168,500	4.7%	2
3	\$165,000	4.6%	1

1. Calculate the gross premium.

2. Calculate the asset share at the end of year 2.

3. Calculate the total profit and the profit by source (MEI) for year 3.

4. (Note that because the gross premium is calculated using the equivalence principle, many of these values will seem odd) Calculate the

   (a) Internal Rate of Return
   (b) Net Present Value
   (c) Profit Margin
   (d) Discounted Payback Period