444 HW 4

Key

April 9, 2024

1a) ${}_{20}E_{65:65} = {}_{20}p_{65} * {}_{20}p_{65} * 1.05^{-20}$
${}_{20}E_{65:65} = (\ell_{85}/\ell_{65})^2 * 1.05^{-20}$
${}_{20}E_{65:65} = (\frac{61184.9}{94579.7})^2 * 1.05^{-20}$
${}_{20}E_{65:65} = 0.15772$

1b) E(benefits) $= 100000(A^{\ 1}_{65:65:\overline{20|}} + {}_{20}E_{65:65}) + 500 + .15G + .05G\ddot{a}_{65:65:\overline{20|}}$
E(premiums) $= G\ddot{a}_{65:65:\overline{20|}}$
$A^{\ 1}_{65:65:\overline{20|}} = A_{65:65} - ({}_{20}E_{65:65} * A_{85:85})$
$A^{\ 1}_{65:65:\overline{20|}} = .44336 - (.15772 * .76652) = .32276$
$\ddot{a}_{65:65:\overline{20|}} = \ddot{a}_{65:65} - ({}_{20}E_{65:65} * \ddot{a}_{85:85})$
$\ddot{a}_{65:65:\overline{20|}} = 11.6831 - (.15772 * 4.9030) = 10.9098$
$100000(A^{\ 1}_{65:65:\overline{20|}} + {}_{20}E_{65:65}) + 500 + .15G + .05G\ddot{a}_{65:65:\overline{20|}} = G\ddot{a}_{65:65:\overline{20|}}$
100000(.32276 + .15772) + 500 + .15G + .05(10.9098)G = 10.9098G
48548 = 10.21409G
G = 4753.04

1c) ${}_{10}V = 100000A_{75:75:\overline{10|}} - .95(4753.04)(\ddot{a}_{75:75:\overline{10|}})$
${}_{10}E_{75:75} = {}_{10}E_{75} * {}_{10}E_{75} * 1.05^{10} = .31657$
$A_{75:75:\overline{10|}} = A_{75:75} - ({}_{10}E_{75:75} * A_{85:85}) + {}_{10}E_{75:75}$
$A_{75:75:\overline{10|}} = .60912 - (.31657 * .76657) + .31657 = .68303$
$\ddot{a}_{75:75:\overline{10|}} = 6.6563$
${}_{10}V = 100000(.68303) - .95(4753.04)(6.6563) = 38247.22$
${}_{10}V = 38247.22$

1d) $\Pr_{11} = 100(38247.22)(1.055) + 100(4753.04)(1 - .05)(1.055) - 4(100100) - 96({}_{11}V)$
$\Pr_{11} = 100(40350.8171 + 4763.73434 - 400400 - 96(42811.92)$
$\Pr_{11} = 1117.12$

1e) I:
$I_{11} = 100(38247.22 + .95(4753.04)) * (.055 - .05) = 100(42762.668) * .005$
$I_{11} = 21381.33$
M:
$q_{75:75} = 1 - (1 - q_{75})^2 = (1 - .018433)^2 = .036526$
$M_{11} = [100(q_{75:75}) - 4] * (100000 - {}_{11}V)$
$M_{11} = [100(.036526) - 4] * (100000 - 42811.92)$
$M_{11} = -19864.11$
E:
$E_{11} = 100[(.05 * 4753.04) - (.05 * 4753.04)] * 1.055 - 400$
$E_{11} = 0 - 400 = -400$

1f) $Pr_{20} = 40({}_{19}V * 1.05 + .95G * 1.05) - (35 + 5) * 100000$
Under the equivalence principle, ${}_{19}V * 1.05 + .95G * 1.05 = 100000$
since the payment is guaranteed to happen at time 20 regardless of mortality experience.
Thus, $Pr_{20} = 40({}_{19}V * 1.05 + .95G * 1.05) - (35 + 5) * 100000$
$Pr_{20} = 40(100000) - (35 + 5) * 100000$
$Pr_{20} = 0$

1g) There is no mortality loss due to the pandemic because every policy in force at time 19 is guaranteed a payment at time 20, regardless of mortality experience. Premiums and policy values were set such that the insurer will hold exactly enough in reserve to be able to make this payment at time 20 for every policy still in force at time 19. Thus, the reserves exactly cover the payments, and there is no loss due to mortality, as mortality experience does not affect the probability that a payment is made at time 20.

2a) $Pr_2 = 1100(.95)(1.02) + 400(1.02) - .00642(100000) - (1 - .00642) * 800$
$Pr_2 = 37.04$

2b) $Pr_0 = -155$ (Pre-contract expenses)
$Pr_1 = 1100(.95)(1.01) + 0(1.01) - .00592(100000) - (1 - .00592) * 400$
$Pr_1 = 65.82$
$Pr_2 = 37.04$ from part a
$Pr_3 = 1100(.95)(1.03) + 800(1.03) - .00697(100000) - (1 - .00697) * 1100$
$Pr_3 = 111.02$
Thus, Pr = [-155, 65.82, 37.04, 111.02]

2c) $\Pi_0 = -155$
$\Pi_1 = 65.82$
$\Pi_2 = 37.04 * p_{50} = 37.04(1 - .00592) = 36.82$
$\Pi_3 = 111.02 * {}_2p_{50} = 111.02(1 - .00592)(1 - .00642) = 109.65$
$\Pi = [-155, 65.82, 36.82, 109.65]$
NPV = -155 + 65.82*1.14^{-1} + $36.82 * 1.14^{-2} + 109.65 * 1.14^{-3}$
NPV = 5.08

2d) Lowest: Product B. All profits are pushed back to year 3, leading to a lower IRR.

Middle: Product A. Profits happen earlier than in Product B, but reserves are higher than product C.

Highest: Product C. Reducing reserves will raise IRR, and reserves are smaller for this policy than they are for Product A.

3a) $\mathrm{T}_{50} = 11.8$

$\mathrm{L}_g = -2000(\ddot{a}_{\overline{12}|})(.95) + .75(2000) + 10100v^{12} + 12(200)v^{12}$

$\ddot{a}_{\overline{12}|} = (1 - v^{12})/(.05/1.05) = 9.30641$

$\mathrm{L}_g = -2000(9.30641)(.95) + .75(2000) + 10100v^{12} + 12(200)v^{12}$

$\mathrm{L}_g = -17682.179 + 1500 + 10100(1.05^{-12}) + 24000(1.05^{-12})$

$\mathrm{L}_g = 53422.49$

3b) $\mathrm{E}[\mathrm{L}_g] = 10100A_{50} + {}_{10}E_{50}^{(10)(2000)}A_{60} + 2000(IA)_{60} + .75(2000) - .95(2000)(\ddot{a}_{50})$

$\mathrm{E}[\mathrm{L}_g] = 10100(.18931) + .60182[(20000)(.29028) + 2000(6.63303)] + 1500 - 1900(17.0245)$

$\mathrm{E}[\mathrm{L}_g] = 19120.31 + 11477.71 + 1500 - 32346.55 = -248.53$

3c) ${}_{10}V = 10100A_{60} + [(10)(2000)A_{60} + 2000(IA)_{60}] - .95(2000)(\ddot{a}_{60})$

${}_{10}V = 10100(.29028) + 20000(.29028) + 2000(6.63303) - 1900(14.9041)$

${}_{10}V = 20072.15$

3d) $\mathrm{Pr}_{15} = {}_{15}V(1.052)(1000) + 2000(.96)(1.052)(1000) - 7(100000 + 2000 + 2000(16)) - 993({}_{16}V)$

$\mathrm{Pr}_{15} = 34333.78(1052) + 2019.84(1000) - 7(134000) - 993(37480.51)$

$\mathrm{Pr}_{15} = -17169.87$

3e) E: [2000(1000)(.05)-2000(1000)(.04)](1.05)-(2000-1000)(1000)(q_{65}) =

= .01(2000000)(1.05)-1000000(.005915) =

= 15085

I: [2000(.96)(1000) + 34333.78(1000)](.052-.05) =

= (1920000 + 34333780) * (.002) = 72507.56

M: [1000(q_{65}) − 7](100000 + 2000 + 2000(16) − ${}_{16}V$) =

= [1000(.005915)-7] * (134000 - 37480.51) =

= (5.915-7) * (134000 - 37480.51) = -104723.65

3f) The size of the profit due to interest depends on whether profit is calculated for interest or expenses first. So, the profit will be the same for MEI and EMI as it was for EIM. This is because E was calculated before I in each of these three methods. Thus, MEI and EMI will use actual expenses in the interest calculation, just like we did for EIM in part e. The other three possible methods will use expected expenses to calculate profit due to interest.