

Stat 444 — Winter 2024

Homework Assignment 5

Due Date: Thursday, March 21st at 2:00 pm

General Notes:

- For Part I, you may submit your assignment on Learning Suite.
- For Part II, you should use a spreadsheet.
 - Submit electronically the Excel spreadsheet you create to answer the questions in Part II to Learning Suite. Your spreadsheet should be neatly organized and labeled; each answer should be highlighted in some manner, and it should be very clear how each of your answers was obtained.

Part I

1. You are the actuary for a defined benefit pension plan that offers a Final Average Salary (FAS) pension. The final average salary is the average of the earnings in the final two years, and the pension is paid as a whole life annuity due, payable monthly from retirement. The accrual rate is 2%. You are given the following valuation assumptions:
 - Decrements are assumed to follow the Standard Service Table up to age 61. All lives in force at age 61 retire immediately.
 - All retirements not occurring at exact ages 60 or 61 are assumed to occur at age 60.5.
 - $i = 0.05$
 - Under the post-retirement mortality assumptions:
 $\ddot{a}_{60}^{(12)} = 14.441$ $\ddot{a}_{60.5}^{(12)} = 14.315$ $\ddot{a}_{61}^{(12)} = 14.186$
 - The valuation date is 12/31/2022.
 - There were no salary increases in 2021 or 2022.
 - Salaries increased by 10% on 1/1/2023.
 - The valuation uses the Traditional Unit Credit funding method.

The plan has two members. Their information at the valuation date is given in the following table:

Employee	Age	Years of Service	FAS at Valuation
A	35	10	60,000
B	60	30	100,000

- (a) i. Calculate the Actuarial Liability for Employee A's retirement benefits. [20,772.35]
ii. You are given that Employee B did not retire at exact age 60. Calculate the Actuarial Liability for Employee B's retirement benefits. [809,859.22]
- (b) i. Calculate the Normal Contribution (normal cost) for Employee A's retirement benefits. [3,219.71]
ii. Calculate the Normal Contribution for Employee B's retirement benefits. [65,422.07]
iii. The Aggregate Normal Contribution Rate is the total Normal Contribution divided by the total annual salary rate as of 1/1/2023. Calculate the Aggregate Normal Contribution Rate for the plan. [39%]
- (c) A new entrant who is age 35 joins the plan on 12/31/2022.
i. State with reasons whether the plan's total Actuarial Liability for retirement benefits would increase, decrease, or stay the same.
ii. State with reasons whether the Aggregate Normal Contribution Rate for retirement benefits would increase, decrease, or stay the same.
2. A company has a defined contribution (DC) pension plan with the following features:
- The company matches the employee's contributions to their DC fund, up to 4% of the employee's salary. The employee may contribute more than this, but these additional contributions are not matched.
 - The employee's salary is paid at the end of each month; employee and employer contributions to the fund are made at this time.
 - The investment returns on the fund are $i^{(12)} = 0.06$.

You are given the following information regarding an employee.

- The employee entered the plan at age 25.
 - Their starting salary was 60,000 per year.
 - The employee is currently age 45.
 - The employee has received annual pay increases of 3% at the start of each subsequent year of employment.
 - The employee has contributed 7% of their salary to their DC fund since entering the plan.
- (a) Calculate the accumulated amount in the DC fund (including the employee and employer contributions which have just been paid). [322,137.46]

The employee is reviewing their retirement preparation, based on the following assumptions.

- They will continue to receive annual salary increases of 3%
 - They will retire at age 65.
 - They will receive a benefit of 18,000 at the start of each year from their government's retirement program, starting at age 65.
 - At age 65 they will use the accumulated amount in their DC fund to purchase a whole life annuity due, payable annually.
 - The individual is targeting a replacement ratio of 65%, including both the government and annuity benefits.
- (b) Calculate the yearly annuity payment required to meet the target replacement ratio. [105,514]
- (c) You are given that future investment returns will be $i^{(12)} = 0.06$ and that $\ddot{a}_{65} = 13.6$. Calculate the level contribution rate needed by the employee over the next 20 years to accumulate sufficient funds to purchase this annuity. [3.485%]
- (d) Suppose that the employee wishes to use the fund to purchase a life-and-10-year-guaranteed annuity due instead of a whole life annuity due. State with reasons whether their replacement ratio will increase, decrease, or stay the same.

Part II

You are the valuation actuary for a small company with a pension plan that pays 1% per year of service (capped at 30%) of the average of their 3 highest annual salaries. Employees are immediately vested in the pension. The current employee roster is available [here](#). Assume the following:

- Retirement can only happen at exact ages 60, 61, ..., 65.
- Salary increases 3.5% per year.
- Decrements follow the Standard Service Table (Excel version available [here](#))
- There are no death benefits.
- If the employee withdraws, they forfeit their pension benefit (this does not happen in real life, but it makes the calculations simpler)

Calculate each employee's actuarial liability (under TUC and PUC) and their expected replacement ratio.