444 HW 5

Key

April 10, 2024

1a) i) FAS = 60000 YoS = 10 accrual rate = .02 i = .05

$\ell_{35} = 218833.9 \ell_{60-} = 93085.4 \ell_{60+} = 65159.8 \ell_{61} = 58699.9$

r$_{60-} = 27925.6 r_{60+} = 6187.6$

Pr("retire at exact age 60") = 27925.6/218833.9 = .12761

Pr("retire at age 60.5") = 6187.6/218833.9 = .028275

Pr("retire at age 61") = 58699.9/218833.9 = .26824

AL$_{35} = 60000 * 10 * .02 * [.12761 * v^{60-35} * \ddot{a}_{60}^{(12)} + .028275 * v^{60.5-35} * \ddot{a}_{60.5}^{(12)} + .26824 * v^{61-35} * \ddot{a}_{61}^{(12)}]$

AL$_{35} = 12000 * [.12761 * v^{60-35} * 14.441 + .028275 * v^{60.5-35} * 14.315 + .26824 * v^{61-35} * 14.186]$

AL$_{35} = 12000 * 1.73103 = 20772.35$

ii)

AL$_{60} = 100000 * 30 * .02 * [(6187.6/65159.8) * v^{0.5} * \ddot{a}_{60.5}^{(12)} + (58699.9/65159.8) * v * \ddot{a}_{61}^{(12)}]$

AL$_{60} = 60000 * [(6187.6/65159.8) * v^{0.5} * 14.315 + (58699.9/65159.8) * v * 14.186]$

AL$_{60} = 60000 * [13.49765] = 809859.22$

1b) i)

AL$_{35} + NC_{35E} PVB_{35} = v * p_{35}^{(\tau)} * AL_{36}$

AL$_{36} = (60000 + 60000(1.1))/2 * 11 * .02 * [\frac{r_{60-}}{\ell_{36}} * v^{60-36} * 14.441 + \frac{r_{60+}}{\ell_{36}} * v^{60.5-36} * 14.315 + \frac{\ell_{61}}{\ell_{36}} * v^{61-36} * 14.186]$

AL$_{36} = (60000 + 60000(1.1))/2 * 11 * .02 * [\frac{27925.6}{207821.8} * v^{60-36} * 14.441 + \frac{65159.8}{207821.8} * v^{60.5-36} * 14.315 + \frac{58699.9}{2017821.8} * v^{61-36} * 14.186]$

AL$_{36} = 13860 * [1.91343] = 26520.15$

NC$_{35} = v * p_{35}^{(\tau)} * AL_{36} - AL_{35} + EPVB_{35}$

EPVB$_{35} = 0$

AL$_{35} = 20772.35$

NC$_{35} = 1.05^{-1} * (207821.8/218833.9) * 26520.15 - 20772.35$

NC$_{35} = 3219.79$

ii)

AL$_{60} + NC_{60} - EPVB_{60} = v * p_{60}^{(\tau)} * AL_{61}$

AL$_{61} = (100000 + 100000(1.1))/2 * 31 * .02 * \ddot{a}_{61}^{(12)}$

$AL_{61} = 65100 * 14.186 = 923508.6$
$NC_{60} = v * p_{35}^{(\tau)} * AL_{61} - AL_{60} + EPVB_{60}$
$EPVB_{60} = (6187.6/65159.8)*(1.05^{-0.5})*14.315*[.75(100000)+.25(110000)]* 30.5 * .02$
$EPVB_{60} = 82945.51$
$AL_{60} = 809859.22$
$NC_{60} = 1.05^{-1} * (58699.9/65159.8) * 923508.6 - 809859.22 + 82945.51$
$NC_{60} = 65422.07$
iii)
ANCR = Sum(NC_{60})/$Sum(SalaryRate)$
ANCR = (3219.79 + 65422.07) / [(100000 + 600000)(1.1)]
ANCR = 68641.78 / 176000
ANCR = 0.39

1c) i)

The total actuarial liability would STAY THE SAME. Actuarial liability depends only on past service, so a new employee will have an actuarial liability of 0 and have no impact on the total actuarial liability.

ii)

Since younger, less experienced employees have a lower normal cost relative to their salary, adding a new employee who is age 35 will DECREASE the ANCR. For example, employee A's normal cost is about .05 times their salary, while employee B's is about .65 times their salary.

2a) $i^{(12)}/12 = .06/12 = .005$
$i = (1.005)^{12} - 1 = .06168$
$a_{\overline{12}|.005} = \frac{1-1.005^{-12}}{.005} = 11.61893$
P = 60000(.11)(1/12)($a_{\overline{12}|.005}$) = 6390.41
j = (i-g)/(1+g) = .03076
PV at age 24 = P$\ddot{a}_{\overline{20}|_j}/(1+i)$ =
= [6390.41 * 1-(1+j)$^{-12}$/$(j/1+j)$]/(1.06168) =
= 6390.41(15.22851) / 1.06168 = 91662.88
FV at age 45 = 91662.88($1.005^{12(21)}$) = 322137.48

2b) Let P = annual annuity payment
Final salary = 60000$(1.03)^{39}$ = 190021.61
Gov't benefit = 18000
.65 = (18000 + P) / 190021.61
P = .65(190021.61) - 18000
P = 105514

2c) Starting amount = 322137.48 (age 45)
Age 45 salary = $60000(1.03^{20}) = 108366.67$
P = $108366.67(1/12)(r)(a_{\overline{12}|.005}) = 104925.4187r$
Goal: $105514a_{65} = 105514(13.6) = 1434990.4$
FV of starting amount = $322137.48(1.005^{240}) = 1066340.86$
Difference = 1434990.4 - 1066340.86 = 368349.54
$P\ddot{a}_{\overline{20}|_j}/(1+i) * 1.005^{240} = 368349.54$
P = 368349.54 / ($15.22851 * 1.005^{240}$)
P = 7313.09758
P = 104925.4187r
r = 7313.09758 / 104925.4187 = 0.069698
Due to 1-1 matching, the employee must contribute r / 2
r / 2 = .069698 / 2 = 0.03485

2d) Their replacement ratio will decrease. The guaranteed payments in the life and 10-year certain annuity due make it slightly more expensive than a whole life annuity due. So, the same amount of money would buy a lower annual payment for the life and 10-year certain annuity due than for the whole life annuity due. Since the annual benefit amount will decrease, the replacement ratio will also decrease.