

Stat 444 — Winter 2024

Homework Assignment 7

Due Date: Tuesday, April 16th at 2:00 pm

General Notes:

- For Part I, you may submit your assignment on Learning Suite.
- For Part II, you may also submit your assignment on Learning Suite.

Part I

1. An insurer sells a 3-year single premium Variable Annuity contract, with the following provisions:
 - The policyholder deposits the 10,000 premium at the start of the contract.
 - The insurer deducts a 2% front-end charge from the premium before allocating the remainder to the policyholder fund.
 - At the start of each year after the first, the insurer deducts a management charge of 1% of the fund value.
 - The death benefit under the contract is 110% of the fund value at the end of the year of death, payable at that time.
 - If the policyholder surrenders during the first year, they receive 85% of their fund value at the end of the year. In subsequent years, the policyholder receives the full fund value at the end of the year of surrender.
 - The contract has a Guaranteed Minimum Maturity Benefit (GMMB) rider, which is 100% of the premium.

An actuary conducts a stress test of this contract using the following assumptions:

- Initial expenses are 200 at contract inception.
- Renewal expenses are 0.6% of the policyholder's fund value (before management charge deductions) incurred at the start of the second and third policy years.
- Mortality rates are 0.05 each year.
- At the end of the first and second years, 10% of the surviving policyholders surrender their policies.

- The policyholder's fund earns 0% in the first year, 0% in the second year, and 5% in the third year.
 - The insurer's funds earn 3% in each year.
 - The insurer holds a reserve of 300, 200, and 100 at the start of years 1, 2, and 3 respectively, for each contract in force at the start of the year.
- (a) Calculate the policyholder fund value at the end of year 3, assuming the policy remains in force.
- i. Determine the profit vector of the contract using the stress test assumptions. [-500,434.7,112.4,92.5]
 - ii. Calculate the net present value of the profit per policy issued, using a hurdle rate of 10% per year. [25.4]
- (b) The actuary is concerned about the GMMB risk.
- i. One colleague suggests selling more policies to mitigate the risk. Explain why this strategy would be ineffective.
 - ii. Another colleague suggests a hedging strategy for the GMMB. The price at issue of the required put option is 900. Describe one advantage and one disadvantage of hedging the guarantee, compared with holding reserves.
2. An insurer issues a 10-year Variable Annuity with a single premium of 10,000 to (50). You are given:
- The GMMB is 90% of the single premium. There are no other guarantees.
 - The initial expense charge is 6% of the premium.
 - The management charge is 0.2% of the policyholder's fund, deducted at the end of each month, including at time 10.
 - The underlying fund asset value follows a geometric Brownian motion, with volatility $\sigma = 0.25$
 - The risk-free rate is $r = 4\%$ per year, compounded continuously.
 - There are no exits other than death.
 - Mortality is assumed to follow the Standard Ultimate Life Table.
- (a) In the context of a Variable Annuity policy, describe the difference between a Guaranteed Minimum Maturity Benefit (GMMB) and a Guaranteed Minimum Income Benefit (GMIB).
- (b) The insurer uses the Black-Scholes formula to construct a hedge portfolio of 10-year zero-coupon bonds and the underlying fund assets.
- i. Calculate the value of the hedge portfolio at time 0. [1419]

- ii. Write down the value of the bond portion of the hedge portfolio at time 0. [3282]
- (c) The insurer does not rebalance the hedge portfolio until time 1. At that time the policy is still in force and the price of the underlying fund assets has increased by 5%. Calculate the rebalancing cost for the hedge portfolio. [-99]

Part II

Because it is the last assignment of the semester, let's make Part II a little more casual. Using the example [spreadsheet](#) from class, what happens to the value of the GMDB (and why?) as you increase and decrease the:

1. mortality rate (the mortality rates are positively related to all of the A, B and c parameters)?
2. t (current time)?
3. S_t (the value of the underlying stock)?
4. management charge?
5. risk free rate?
6. volatility?