444 HW 7

Key

April 10, 2024

1a) i)
$\text{Pr}_0 = -e_0 - {}_0V = -200 - 300 = -500$
$\text{Fund}_1 = 10000(1 - .02)(1 + 0) = 9800$
$\text{EDB}_1 = 9800(.1)(.05) = 49$
$\text{ESB}_1 = -(1 - .05)(.1)(1 - .85)(9800) = -139.65$
$\text{E}_1V_{=}200(1 - .05)(1 - .1) = 171$
$\text{Pr}_1 = [{}_0V_{+}.02(10000)](1.03) - EDB_1 - ESB_1 - E$
$\text{Pr}_1 = [300 + 200](1.03) - 49 + 139.65 - 171$
$\text{Pr}_1 = 434.7$
$\text{Fund}_2 = 9800(1 - .01) = 9702$
$\text{EDB}_2 = 9702(.1)(.05) = 48.51$
$\text{ESB}_2 = 0$
$\text{E}_2V_{=}100(1 - .05)(1 - .1) = 85.5$
$\text{Pr}_2 = [{}_1V_{+}(.1 - .006)(9800)](1.03) - EDB_2 - ESB_2 - E$
$\text{Pr}_2 = [200 + .004(9800)](1.03) - 49.51 - 85.5$
$\text{Pr}_2 = 112.4$
$\text{Fund}_3 = 9702(1 - .01)(1.05) = 10085.23$
$\text{EDB}_2 = 10085.23(.1)(.05) = 50.43$
$\text{ESB}_3 = 0$
$\text{E}_3V_{=}0$
$\text{Pr}_3 = [{}_2V_{+}(.1 - .006)(9702)](1.03) - EDB_3 - ESB_3 - E$
$\text{Pr}_3 = [200 + .004(9800](1.03) - 50.43$
$\text{Pr}_3 = 92.5$
Pr = [-500, 434.7, 112.4, 92.5]
ii)
$\Pi_0 = -500$
$\Pi_1 = 434.7$
$\Pi_2 = Pr_2 * p_x^{(\tau)} = 112.4(1 - .05)(1 - .1) = 96.102$
$\Pi_3 = Pr_3 * {}_2p_x^{(\tau)} = 92.5 * (.95 * .9)^2 = 67.6198125$
NPV = -500 + (434.7/1.1) + $(96.102/1.1^2) + (67.6198125/1.1^3)$
NPV = 25.4

1b) i)

If the policy is at risk of being unprofitable due to GMMB risk, selling more policies will not help. Selling more policies with the same GMMB will increase the insurer's potential future liabilities due to GMMB's, which will just make the problem worse. Also, because most of the risk of a GMMB comes from poor investment returns, selling more of them will not mitigate the risk of not having the guaranteed amount in the fund for each policyholder. Instead, the insurer should modify the design of the policies to mitigate the GMMB risk.

ii)

Advantage: By buying an option instead of holding reserves, the insurer can take more profit each year since it does not have to hold reserves. Thus, the profitability of the policy will increase for each year after time 0.

Disadvantage: The option is expensive. It requires 3 times as much money as the initial reserve amount that would be required under the alternative option. Thus, it will increase the start-up costs associated with the policy, potentially making it less profitable overall.

2a) A GMMB is a guarantee that a VA policyholder's fund value will be at least the guarantee amount at the time of maturity. If the fund is below that amount, the insurer must make up the difference. A GMIB is a guarantee for the annuitization rate of the benefit base at maturity. If market annuitization rates at maturity are less favorable than the guaranteed rate, then the policyholder can annuitize at the guaranteed rate. Therefore, a GMMB sets a floor for the fund value at maturity, while a GMIB sets a floor for the annuitization rate at maturity.

2b) i)

$p(0) = 9000 * e^{-.04*10} * \Phi(-d_2(0)) - S_0 * (1-.06) * (1-.002)^{120} * \Phi(-d_1(0))$

$S_0^* = 10000(1-.06)((1-.002)^{120}) = 7392.5$

$d_1(0) = [ln(7392.5/9000) + (.04 + .25^2/2)(10)]/.25\sqrt{10}$

$d_1(0) = 0.6524$

$d_2(0) = d_1(0) - .25\sqrt{10} = -0.1382$

$\Phi(-d_2(0)) = \Phi(0.1382) = .55496$

$\Phi(-d_1(0)) = \Phi(-0.6524) = .25708$

$p(0) = 9000 * e^{-.04*10} * .55496 - 7392.5 * .25708$

p(0) = 3348 - 1900.5 = 1447.5

${}_{10}p_{50} = \ell_{60}/\ell_{50} = 96634.1/98576.4 = .9803$

$\Pi(0) = p(0) * {}_{10}p_{50} = .9803(3348 - 1900.5) = 3282 - 1863$

$\Pi(0) = 1419$

ii)

The bond portion is the first term in the hedge portfolio.

$\text{Bond}_0 = .9803(3348) = 3282$

2c) $p(1) = 9000 * e^{-.04*9} * \Phi(-d_2(1)) - S_0^* * 1.05 * \Phi(-d_1(1))$

$S_0^* * 1.05 = 7392.5 * 1.05 = 7762.2$

$d_1(0) = [ln(7762.2/9000) + (.04 + .25^2/2)(9)]/.25\sqrt{9}$
$d_1(0) = 0.65771$
$d_2(0) = d_1(0) - .25\sqrt{9} = -0.0923$
$\Phi(-d_2(1)) = \Phi(0.0923) = .53677$
$\Phi(-d_1(1)) = \Phi(-0.65771) = .25536$
${}_9p_{51} = \ell_{60}/\ell_{51} = 96634.1/98457.2 = .98148$
$p(1) = 9000 * e^{-.04*9} * .53677 - 7762.2 * .25536$
$p(1) = 1388.27$
$\Pi(1) = 1388.27{}_9p_{51} = .98148(1388.27) = 1362.5$
The hedge brought forward allowing for the survival bonus yields:
$[3282(e^{.04}) - 1863(1.05)] * \ell_{50}/\ell_{51} =$
$= [3282(e^{.04}) - 1863(1.05)] * [98576.4/98457.2] =$
$= 1461.5$
Thus, the rebalancing cost is
1362.5 - 1461.5 = -99
So, rebalancing yields a gain of 99.