1. Lecture 7 (SOA #56) - A 10-year equity-linked policy is issued to (x). The policy offers a guaranteed minimum maturity benefit of 100% of the initial single premium. After an initial expense charge of 5% of the premium, the remainder is invested in separate fund. The insurer deducts a 0.2% management charge from the policyholder's fund at the end of each month, if the policyholder survives. You are given:

   - An investment at t = 0 of 1 in the underlying asset will have value S_t at t, where S_t follows a geometric Brownian motion, with volatility σ, and $S_0 = 1$
   - $BSP(K) = E_0^Q\left[e^{-10r}(K - S_{10})^+\right]$ is the Black-Scholes option price formula for a put option on an investment of 1 in the underlying stock at time 0, with strike price K.
   - There are no exits other than death.

   (a) The risk-neutral value of the GMMB option at issue is $\pi(0) = P({}_{10}p_x \xi BSP(K^*))$, define the parameters ξ and K^* in terms of the information given about this specific contract. [$\xi = 0.74712, K = \xi^{-1}$]

   (b) Given $\sigma = 0.25$, $r = 0.04$, $P = 100,000$, $x = 60$, and Standard Ultimate Life Table mortality, calculate $\pi(0)$. [17,091]

   (c) Assume that the insurer constructs a hedge portfolio at $t = 0$ based on the Black Scholes option valuation. The insurer does not rebalance the hedge until one month later. You are given that in the first month, the underlying asset values fell by 3%. Calculate the value of the hedge portfolio at the end of the first month, before rebalancing. [17,846]

2. Lecture 6 (SOA #50) - An insurer issues a Type A universal life policy with a face amount of 100,000 and an annual premium of 50,000. You are given:

Policy Year	Percent of Premium Charge	Annual Expense Charge	COI Rate per 1 of Insurance	Annual Discount Rate for COI	Annual Credited Interest Rate	Corridor Factor
1	20%	75	0.025	4.5%	6.50%	1.5
2	8%	75	0.030	4.5%	5.75%	1.4

(a) Calculate the account value at the end of two years. [90,838]

The policyholder surrenders the policy at the end of the second year, when she is age 60. The policy does not have a surrender charge. She uses the surrender value as a net single premium to purchase a special last-survivor life annuity-due with her husband, who is 10 years older than the policyholder.

This annuity provides the following payments at the beginning of each quarter:

- For the first ten years, a guaranteed payment of Q
- After the first ten years, a payment of Q if the policyholder is alive
- After the first ten years, a payment of $0.6Q$ if the policyholder is dead, but her husband is alive.

You are given that the net single premium and reserves for this annuity are calculated based on the following information and assumptions.

- The future lifetimes of the couple are independent.
- Mortality follows the Standard Ultimate Life Table.
- $i = 0.05$
- The two-term Woolhouse formula

(b) Calculate Q. [1629.6]

(c) You are given that the policyholder's husband dies during during the first 10 years of the annuity. Calculate the net premium reserve immediately prior to the payment of Q at the start of the eleventh year. [53,279]

(d) In the eleventh year, Person S decides that they no longer needs the annuity and asks the company to pay the reserve. The insurance company refuses to pay the full reserve. Explain why the insurance company would not agree to S's request.

3. Lecture 5 (SOA #46) - A person (J), who is 57, is a member of a final average salary defined benefit pension plan. The person has 35 years of service. You are given the following information.

   - The person's salary was 100,000 last year and this year.
   - The accrual rate is 1.8%
   - The pension is payable monthly to the plan member from age 65, with a 100% survivor pension paid to the member's partner after the member's death, provided this occurs after age 65.
   - There is no benefit on death after withdrawal before age 65.
   - The final average salary is the average of the final 2 year's salary before exit.
   - Salaries are expected to be frozen for the next year.
   - The person's partner (M) is also 57.

   You are also given the following valuation assumptions and information:

   - Decrements follow the Standard Service Table
   - $i = 0.05$
   - Mortality after exit follows the Standard Ultimate Life Table.
   - Withdrawals occur halfway through the year.
   - J and M have independent future lifetimes.
   - $\ddot{a}_{65}^{(12)} = 13.087$ and $\ddot{a}_{65:65}^{(12)} = 11.2158$
   - Uniform distribution of deaths between integer ages for all other fractional age calculations.
   - a_x^w represents the value on withdrawal at age x of an annuity of 1 per year paid monthly in advance from age 65, including the survivor's benefit.
   - $a_{58.5}^w = 10.5804$ and $a_{59.5}^w = 11.1456$
   - The valuation uses the traditional unit credit funding method.

   (a) i. Under the Standard Ultimate Life Table, calculate ${}_{7.5}E_{57.5:57.5}$. [0.65511]

   ii. Calculate $a_{57.5}^w$. [10.05]

   (b) Calculate the actuarial liability for the withdrawal benefit. [35,888.33]

   (c) Calculate the normal contribution for the withdrawal benefit. [850.94]

   (d) Person J withdraws at age 57.5 and immediately gets divorced. Under the divorce settlement, Person M will receive a pension of $X/3$ for life from age 65, and Person J will receive a pension of X for life from age 65. The value of the settlement is equal to the value of the joint and last survivor pension payable had the couple not divorced. Calculate X. [54,584.75]

4. Lecture 4 (SOA #26) - An insurer issues fully discrete whole life insurance policies to 10,000 lives, each age 45, with independent future lifetimes. The death benefit for each policy is 100,000. Gross premiums are determined using the equivalence principle. You are given the following information:

	Pricing and Policy Value Assumptions	Policy Year 1 Actual Experience	Policy Year 2 Actual Experience
Interest	5%	7%	Same as pricing
Expense at Start of the Year	75% of premium + 100 per policy in the first year; 10% of premium + 20 per policy thereafter	75% of premium + 105 per policy	Same as pricing
Settlement Expense	200 per policy	Same as pricing	220 per policy
Mortality	Standard Ultimate Life Table	Same as pricing	10 deaths

(a) Calculate the gross premium for each policy. [1015.80]

(b) Calculate the gross premium policy value for a policy inforce at the end of policy year 1. [84.30]

(c) For each of interest, expense and mortality, in that order, calculate the gain or loss by source in policy year 1 on this block of policies. [30,790; -53,500; 0]

(d) Explain the sources and direction of any gains or losses in policy year 2. Exact values are not necessary.

5. Lecture 3 (SOA #19) - Person P and Person R, each age 40, buy a fully discrete, last survivor insurance with a sum insured of 100,000 You are given:

- Premiums are payable while at least one life is alive, for a maximum of 20 years.
- Mortality of each follows the Standard Ultimate Life Table (SULT).
- $i = 0.05$
- With independent future lifetimes, $\ddot{a}_{40:40:\overline{20|}} = 12.9028$.

(a) Calculate the annual net premium assuming that the future lifetimes are independent. [623.42]

(b) State two reasons why couples may have dependent future lifetimes.

The insurer decides that premiums and policy values for this policy will be determined using a mortality model incorporating dependency. You are given the following information about this model:

- The future lifetimes for the first 20 years are not independent.
- If both lives survive 20 years, it is assumed that the future lifetimes from that time will be independent, and will follow the Standard Ultimate Life Table.
- The mortality of each of P and R, individually, follows the Standard Ultimate Life Table, whether the other is alive or dead.
- $\ddot{a}_{40:40:\overline{10|}} = 8.0703$; $\ddot{a}_{40:40:\overline{20|}} = 12.9254$; ${}_{20}E_{40:40} = 0.35912$; ${}_{100}E_{50:50}$.
- $A_{\overline{50:50}} = 0.13441$
- ${}_{10}p_{40:40} = 0.9866$; ${}_{10}p_{\overline{40:40}} = 0.9980$

Use the dependent mortality model for the rest of this question.

(c) i. Calculate $A_{40:40}$. [0.15792]
    ii. Calculate ${}_{10}E_{40:40}$. [0.60570]
    iii. Calculate $\ddot{a}_{50:50:\overline{10|}}$. [8.0157]

(d) Calculate the annual net premium. [644.62]

(e) Let ${}_kL$ denote the net future loss random variable at time k for the insurance.
    i. $E[{}_{10}L]$ given that only P is alive at time 10. [13,738.6]
    ii. $E[{}_{10}L]$ given that both P and R are alive at time 10. [8,223.3]
    iii. $E[{}_{10}L]$given that at least one of P and R is alive at time 10. [8,286.2]

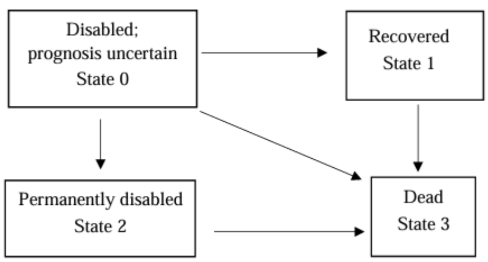
(f) Because the insurer is not informed of the first death for these policies, the actuary decides that they should hold the same reserve for all policies in force. Explain which, if any, of the policy values in part (e) would be a suitable value for the time 10 reserve for each in-force policy.

6. Lecture 2 (SOA #12) - Person G, who is age 45, has recently suffered a disabling injury on the job. G's prognosis is uncertain. His annual salary before the accident was 100,000 Person G receives a structured settlement from MRH Insurance. The settlement is a life annuity, starting immediately, and payable continuously at a rate of 90,000 per year while G is disabled. An additional annuity of 20,000 per year is payable continuously while G's prognosis is uncertain for up to two years, to offset medical expenses.

   (a) State two reasons why structured settlements often use an annuity format rather than a lump sum.

   (b) State two possible reasons why the long term annuity would replace less than 100% of G's pre-injury earnings.

   MRH uses the following multiple state model:



   You are given:

   - MRH calculates policy values equal to the expected present value of future benefits.
   - $i = 0.04$

   You have calculated the following table of annuity values and transition probabilities:

x	$\bar{a}_x^{00}$	$\bar{a}_x^{01}$	$\bar{a}_x^{02}$	$\bar{a}_x^{11}$	$\bar{a}_x^{22}$	${}_2p_x^{00}$	${}_2p_x^{01}$	${}_2p_x^{02}$
45	0.559	5.540	7.161	19.948	10.703	0.0301	0.2766	0.6164
47	0.559	5.420	7.104	19.528	10.623	0.0301	0.2766	0.6162

   (a) Calculate the expected present value of future benefits at $t = 0$, the start date for the annuity payments. [705,669]

   (b) i. Calculate ${}_2V^{(0)}$, the policy value at $t = 2$ if G is in State 0 at that time. [698,670]

   ii. Calculate ${}_2V^{(2)}$, the policy value at $t = 2$ if G is in State 2 at that time. [956,070]

   iii. Calculate the expected present value at $t = 0$ of the policy value at time $t = 2$. [564,054]

   iv. Calculate the expected present value at $t = 0$ of the payments during the first two years. [141,615]

   The chief actuary of MRH reviews your assumptions. The chief actuary asks you to redo your calculations, increasing μ_{45+t}^{01} by 0.01 for all t, with no other changes.

   (c) i. State with reasons whether the value of ${}_2V^{(2)}$ will increase, decrease or stay the same as a result of this change.

   ii. State with reasons whether the expected present value, at age 45, of the policy value at the end of two years will increase, decrease or stay the same as a result of this change.

7. Lecture 1 (SOA #28) - You are performing a profit test on a 50,000, fully discrete 10-year term life insurance policy issued to a healthy life age 60. The annual premium is waived when the insured is sick. You are given the following information regarding the policy value basis:

   - A Markov model with three states: Healthy (0), Sick (1), and Dead (2) is used to value the policy.
   - The annual probability transition matrix for an insured age $60 + k, k = 0, 1, \ldots, 9$ is:

$$\begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} \quad 0 \quad & \quad 1 \quad & \quad 2 \quad \end{array} \\ \begin{pmatrix} 0.90-0.01k & 0.05 & 0.05+0.01k \\ 0.70-0.01k & 0.20 & 0.10+0.01k \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

   - The annual gross premium is 5000 from the start of the third year.
   - Premiums in the first two years are lower.
   - Reserves are gross premium policy values.
   - Issue expenses are 300 per policy.
   - Maintenance expenses are 150 incurred at the start of each year, including the first, for all policies in force.
   - $i = 0.06$
   - The following actuarial present value functions, calculated at 6%:

k	$A^{02}_{60+k:\overline{10-k}\rvert}$	$A^{12}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{00}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{01}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{10}_{60+k:\overline{10-k}\rvert}$	$\ddot{a}^{11}_{60+k:\overline{10-k}\rvert}$
2	0.46667	0.49680	4.7328	0.2533	3.3340	1.4060

   (a) Calculate ${}_2p^{01}_{60}$. [0.055]

   (b) i. Calculate ${}_2V^{(0)}$. [417.415]

   ii. Calculate ${}_2V^{(1)}$. [8,881]

   iii. You are given that ${}_3V^{(0)} = 1788$. Calculate ${}_3V^{(1)}$. [10,200]

   You are given the following additional information regarding the profit test for this policy:

   - The earned rate is 5.7%.
   - The hurdle rate is 8%.
   - Pre-contract expenses are 200.
   - Maintenance expenses are 60 at the start of each year, including the first, for all policies in force.
   - Mortality and morbidity are the same as in the policy value basis.
   - Reserves are gross premium policy values.
   - There are no withdrawals.
   - The profit signature values for $t = 1, 2$ are $\Pi_1 = 84.74$ and $\Pi_2 = 80.35$

   (c) i. Calculate Π_3. [70.012]

   ii. Calculate the Discounted Payback Period. [3]