Calculations for Kolmogorov Forward Equations Example

From the information given in the problem, we start with the following:

t	μ_{20+t}^{01}	μ_{20+t}^{02}	μ_{20+t}^{12}	μ_{20+t}^{10}	μ_{20+t}^{20}	μ_{20+t}^{21}	${}_tp_{20}^{00}$	${}_tp_{20}^{01}$	${}_tp_{20}^{02}$
0	0.03	0.06	0.1	0	0	0	1	0	0
0.25	0.03	0.0605	0.101	0	0	0	− − −	− − −	− − −
0.5	0.03	0.061	0.102	0	0	0	− − −	− − −	− − −
0.75	0.03	0.0615	0.103	0	0	0	− − −	− − −	− − −
1	0.03	0.062	0.104	0	0	0	− − −	− − −	− − −

Using the Kolmogorov Forward Equations with step size $h = 0.25$, we can calculate ${}_{0.25}p_{20}^{00}$ as follows, with $x = 20, t = 0, h = 0.25, i = 0,$ and $j = 0$:

$$\begin{aligned}
{}_{0+0.25}p_{20}^{00} &= {}_0p_{20}^{00} + (0.25)\sum_{k=1,2}\left({}_0p_{20}^{0k}\,\mu_{20}^{k0} - {}_0p_{20}^{00}\,\mu_{20}^{0k}\right) \\
&= {}_0p_{20}^{00} + (0.25)\left[\left({}_0p_{20}^{01}\,\mu_{20}^{10} - {}_0p_{20}^{00}\,\mu_{20}^{01}\right) + \left({}_0p_{20}^{02}\,\mu_{20}^{20} - {}_0p_{20}^{00}\,\mu_{20}^{02}\right)\right] \\
&= 1 + (0.25)\left[(0\cdot 0 - 1\cdot 0.03) + (0\cdot 0 - 1\cdot 0.06)\right] \\
&= 0.9775
\end{aligned}$$

Similarly, ${}_{0+0.25}p_{20}^{01} = 0.0075$ and ${}_{0+0.25}p_{20}^{02} = 0.015$, giving:

t	μ_{20+t}^{01}	μ_{20+t}^{02}	μ_{20+t}^{12}	μ_{20+t}^{10}	μ_{20+t}^{20}	μ_{20+t}^{21}	${}_tp_{20}^{00}$	${}_tp_{20}^{01}$	${}_tp_{20}^{02}$
0	0.03	0.06	0.1	0	0	0	1	0	0
0.25	0.03	0.0605	0.101	0	0	0	0.9775	0.0075	0.015
0.5	0.03	0.061	0.102	0	0	0	− − −	− − −	− − −
0.75	0.03	0.0615	0.103	0	0	0	− − −	− − −	− − −
1	0.03	0.062	0.104	0	0	0	− − −	− − −	− − −

Then we can use these recently computed values in order to get ${}_{0.5}p_{20}^{00}$ as follows, with $x = 20, t = 0.25, h = 0.25, i = 0,$ and $j = 0$:

$$\begin{aligned}
{}_{0.25+0.25}p_{20}^{00} &= {}_{0.25}p_{20}^{00} + (0.25) \sum_{k=1,2} \left({}_{0.25}p_{20}^{0k}\, \mu_{20.25}^{k0} - {}_{0.25}p_{20}^{00}\, \mu_{20.25}^{0k} \right) \\
&= {}_{0.25}p_{20}^{00} + (0.25) \left[\left({}_{0.25}p_{20}^{01}\, \mu_{20.25}^{10} - {}_{0.25}p_{20}^{00}\, \mu_{20.25}^{01} \right) + \left({}_{0.25}p_{20}^{02}\, \mu_{20.25}^{20} - {}_{0.25}p_{20}^{00}\, \mu_{20.25}^{02} \right) \right] \\
&= 0.9775 + (0.25) \left[(0.0075 \cdot 0 - 0.9775 \cdot 0.03) + (0.015 \cdot 0 - 0.9775 \cdot 0.0605) \right] \\
&= 0.955384063
\end{aligned}$$

Similarly, ${}_{0.25+0.25}p_{20}^{01} = 0.014641875$ and ${}_{0.25+0.25}p_{20}^{02} = 0.029974063$, giving:

t	μ_{20+t}^{01}	μ_{20+t}^{02}	μ_{20+t}^{12}	μ_{20+t}^{10}	μ_{20+t}^{20}	μ_{20+t}^{21}	${}_tp_{20}^{00}$	${}_tp_{20}^{01}$	${}_tp_{20}^{02}$
0	0.03	0.06	0.1	0	0	0	1	0	0
0.25	0.03	0.0605	0.101	0	0	0	0.9775	0.0075	0.015
0.5	0.03	0.061	0.102	0	0	0	0.95538	0.01464	0.02997
0.75	0.03	0.0615	0.103	0	0	0	− − −	− − −	− − −
1	0.03	0.062	0.104	0	0	0	− − −	− − −	− − −