Stat 444
Advanced Long-term Actuarial Math

Lecture 5: Pension Mathematics

Pension Plans

Many companies sponsor pension plans for their employees. There are a variety of reasons why a company might choose to have a pension plan:

- To compete for good employees (marketplace competition)
  - Obtain new employees
  - Retain current employees
- There are often tax advantages to pensions
- To encourage older employees to retire (to facilitate turnover)

The relative importance of these factors will influence the design of the plan.

There are two main categories of pension plans: Defined Contribution (DC) and Defined Benefit (DB).

DC Plans

In a DC plan, the employer sets up a fund for each employee:

- The employee (and maybe the employer as well) contribute to the fund.
- The fund is invested at the employee's discretion among various choices.
- Employee takes most or all of the investment risk.
- The fund is often paid to the employee's estate upon death.

DB Plans

In a DB plan, the employer promises a certain benefit for the employee upon retirement:

- The benefit is often based on:
  - salary (e.g., final salary, career avg. salary, final avg. salary)
  - length of service (e.g., number of years worked with company)
- Employer takes most or all of the investment risk.
  - And maybe other risks as well?
- May offer other benefits as well (e.g., death and/or disability benefits)
- Sometimes retirement benefits include a COLA (Cost of Living Adjustment) to help offset inflation.

Replacement Ratio

Companies will often set up a retirement plan to target a particular **replacement ratio** for employees:

Replacement Ratio

$$R = \frac{\text{pension income in the year after retirement}}{\text{salary in the year before retirement}}$$

Might target a replacement ratio of 50% - 80%. Usually assume that less than 100% is needed:

- May have lower taxes
- Don't need to save for retirement
- Don't have work-related expenses

DB Example

A pension has a monthly benefit of 0.3% of final salary for each year worked. Retirement happens on the employee's 65^{th} birthday. Jim joined the company on July 1, 2013 at exact age 42 with a salary of $50,000.

Assuming that annual pay raises of 3% happen each January 1:

(a) Find the amount of Jim's monthly benefit if he lives to retirement. [6,709.72]

(b) Calculate Jim's replacement ratio. [82.8%]

(c) Repeat parts (a) and (b) if benefits are based on a final 3-year average salary. [6,516.18, 80.4%]

DC Example

Jim's identical twin John starts work at a different company on the same day as Jim with the same salary. (Unlike his brother, John does not receive any raises.) John's company offers a DC plan with a 1-1 matching contribution. John makes a contribution on each birthday from age 43 to age 65, inclusive, that is a constant percentage of his salary.

At age 65, he wants to take the accumulated value and buy a life annuity due paying $60,000 per year. (This annuity is valued using the SULT and 5% interest.) Assume that John's account earns an 8% annual rate of return.

What percentage of his salary does John need to contribute each year to meet his goal? [22.25%]

Salary rate and salary scale

The **salary rate** and **salary scale** functions are used to model the progression of salaries over time.

The salary rate function $\bar{s}_y$ describes the annual salary rate being earned at exact age y.

We set $\bar{s}_{x_0}$ to some arbitrary value. Then the ratio $\bar{s}_y/\bar{s}_x$ is the ratio of annual salary rate at age y to annual salary rate at age x, for $x_0 \leq x < y$.

Example: For a particular person, you are given:

- Their annual salary rate at age 30 is $50,000.
- $x_0 = 20$
- $\bar{s}_y = 1.03^{y-20}$ for $y \geq 20$

(a) Calculate this person's annual salary rate at age 40. [67,195.82]

(b) Calculate the total amount of salary they earn between the ages of 40 and 43. [210,795.64]

Salary rate and salary scale

The salary scale s_y serves a similar function to the salary rate function. We again set s_{x_0} to some arbitrary value. Then

$$\frac{s_y}{s_x} = \frac{\text{salary earned in the year between ages } y \text{ and } y+1}{\text{salary earned in the year between ages } x \text{ and } x+1}$$

If we're given the salary rate function, we can use it to derive the salary scale:

$$\frac{s_y}{s_x} = \frac{\int_0^1 \bar{s}_{y+t}\, dt}{\int_0^1 \bar{s}_{x+t}\, dt}$$

Or alternatively, the salary scale is often presented as a table:

x	s_x	x	s_x	x	s_x	x	s_x
30	1.000	40	2.005	50	2.970	60	3.484
31	1.082	41	2.115	51	3.035	61	3.536
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
39	1.894	49	2.897	59	3.432		

Modeling Benefits Using Multiple Decrement Models

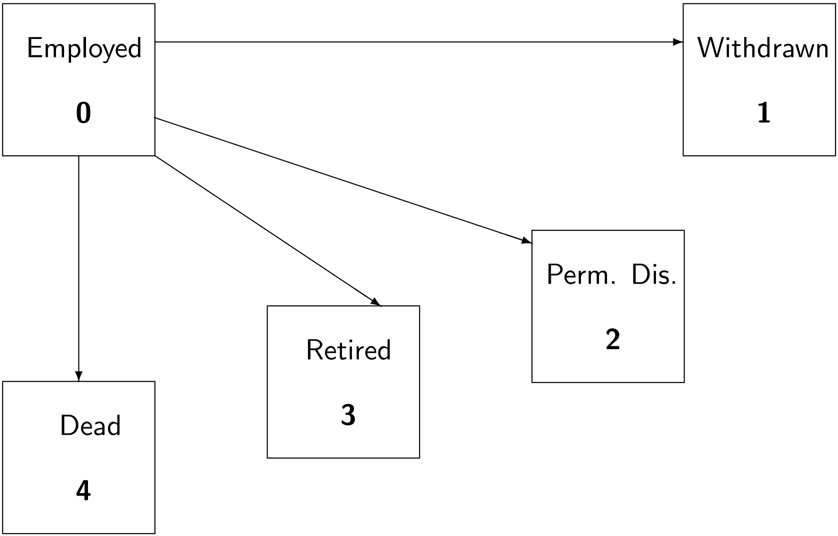
Often, pension plans will offer not only benefits for retiring from active service, but also other types of benefits as well:

- Benefits for permanent disability
- Lump-sum death benefit for death in service
- Deferred pension for withdrawals or terminations for employment

These different reasons for leaving active employment result in different financial obligations for the company sponsoring the pension plan.

Thus, it's often convenient to model the pension plans using a multiple decrement model.

Modeling Benefits Using Multiple Decrement Models



Modeling Benefits Using Multiple Decrement Models

We can specify the various forces of decrement; there may also be positive probabilities of decrements at exact ages.

Example: For employees of a company, decrements are given by:

$$\mu_x^{01} = \mu_x^{w} = \begin{cases} 0.1 & x < 60 \\ 0 & x \geq 60 \end{cases}$$

$$\mu_x^{02} = \mu_x^{i} = 0.002$$

$$\mu_x^{03} = \mu_x^{r} = \begin{cases} 0 & x < 60 \\ 0.2 & x \geq 60 \end{cases}$$

$$\mu_x^{04} = \mu_x^{d} = 0.03$$

In addition, 20% of employees active at age 60 retire at this age; everyone still active at 70 retires then.

Build a multiple decrement table (often called a **service table** in this context) for this situation.

Service Table for Example

x	ℓ_x	$d_x^{(w)}$	$d_x^{(i)}$	$d_x^{(r)}$	$d_x^{(d)}$
20	10000	936.81	18.74	0	281.04
21	8763.41	820.97	16.42	0	246.29
$\vdots$					
59	58.11	5.44	0.11	0	1.63
60^-	50.92	0	0	10.18	0
60^+	40.74	0	0.07	7.27	1.09
61	32.30	0	0.06	5.77	0.86
$\vdots$					
69	5.05	0	0.01	0.90	0.14
70^-	4.00	0	0	4.00	0

If the service table is given (rather than constructed from forces of decrement), we may need fractional age assumptions to do various calculations.

Example

Suppose a person joins a company with a pension plan at exact age 50 with a salary of \$80,000. His company's pension plan has the following benefits:

- The monthly benefit for retirement in service is 2% of the final year's salary for each year of service. The benefit is paid monthly for life.
- Retirement is allowed at exact ages 60, 61, . . . , 70. Assume that retirement happens at the beginning of the year.
- The benefit for death in service is two times the current annual salary, payable at the end of the year of death.
- There is no benefit for withdrawal or disability.

Assume annual raises of 3% and use the Standard Service Table to write an expression for the EPV of this employee's benefit package at the time of hire.

Accrued Benefits and Actuarial Liability

We're often interested in valuing the **accrued benefits** of a current member of a pension plan, that is, the benefits attributable to the past service of the plan member.

- The EPV at time of valuation of all of the benefits accrued up to that time is called the **actuarial liability**.

The **traditional unit credit (TUC)** approach values the accrued benefits without projecting future salary increases.

The **projected unit credit (PUC)** method projects salary to the exit date when valuing the accrued benefits.

Actuarial Liability — Example

Donna, currently age 50, earned 100,000 in the past year and has been a plan member for 20 years. Her pension plan offers the following retirement benefits:

- An annual benefit of 5% of final year's salary for each year of service.
- Retirement can occur at exact ages 60, 61, 62, 63, 64, or 65.
- Regardless of retirement age, payments begin at age 65 (if the retiree is alive then) and continue for the life of the retiree. If the retiree dies prior to age 65, no benefit is paid.
- There are no benefits payable for withdrawals, disabilities or death in service.

Actuarial Liability — Example (continued)

Pre-retirement decrements are given by the Standard Service Table, and post-retirement mortality is given by the SULT.

Assume that the employee receives a 3% raise on each birthday. Using $i = 5\%$, calculate the actuarial liability for this person (at age 50), using:

(a) TUC [55,352.30]

(b) PUC [80,918.67]

Normal Cost

The EPV of the current year's benefits is known as the **normal cost** for the plan for the year.

We can solve for the normal cost in year t (NC_t) for a plan member currently age x in a recursive manner:

$$AL_t + NC_t - EPVB_t = v\, p_x^{(\tau)}\, AL_{t+1}$$

where $EPVB_t$ is the EPV at time t (beginning of the year) of the benefits resulting from exits from the plan during the year.

- For simplicity, it's sometimes assumed that all exits happen mid-year.