Stat 444 - Hartman
Practice Final Exam
Time Limit: 180 Minutes

Name: _____

This exam contains 14 pages (including this cover page) and 6 problems. Check to see if any pages are missing.

You may only use SOA-approved calculators and a pencil or pen on this exam.

You are required to show your work on each problem on this exam.

Problem	Points	Score
1	37	
2	22	
3	30	
4	20	
5	20	
6	16	
Total:	145	

1. Linda, age 40 and Healthy, purchases a 20-year disability income (DI) insurance policy paying at a continuous rate of \$50,000 per year while she is Disabled. Gross premiums of \$10,000 per year are payable continuously while she is Healthy. Expenses are incurred at a continuous rate of \$100 per year in the Healthy state and \$300 per year in the Disabled state.

   To model this policy, you use the following 3-state Markov model and forces of transition, along with a force of interest of $\delta = 0.06$:
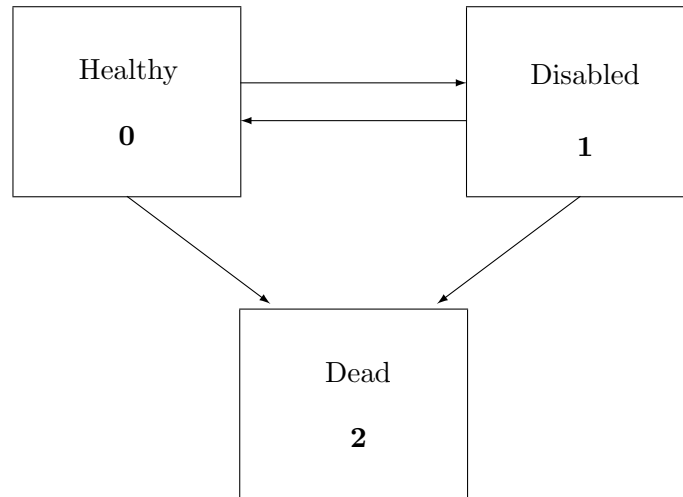
$$\mu_x^{01} = 0.03 \qquad \mu_x^{10} = 0.02 \qquad \mu_x^{02} = 0.005 + 0.001x \qquad \mu_x^{12} = 0.02 + 0.002x$$

   You are also given the following EPVs and probabilities, calculated at $\delta = 0.06$:

$$\bar{a}_{50}^{00} = 7.2847 \qquad \bar{a}_{50}^{01} = 1.0765 \qquad \bar{a}_{50}^{10} = 0.7399 \qquad \bar{a}_{50}^{11} = 3.9147$$

$$\bar{a}_{60}^{00} = 6.3865 \qquad \bar{a}_{60}^{01} = 0.7979 \qquad \bar{a}_{60}^{10} = 0.5446 \qquad \bar{a}_{60}^{11} = 3.2073$$

$$\bar{a}_{50:\overline{10|}}^{11} = 3.4896$$

$$_{10}p_{50}^{00} = 0.41321 \qquad _{10}p_{50}^{01} = 0.09309 \qquad _{10}p_{50}^{10} = 0.06313 \qquad _{10}p_{50}^{11} = 0.22581$$

   a. [8 pts] Calculate the probability that Linda remains Healthy for the entirety of the 20 years, and give the symbol for this probability.

   b. [9 pts] Write the Kolmogorov Forward Equations, with boundary conditions, for $\frac{d}{dt}\,_tp_x^{00}$, $\frac{d}{dt}\,_tp_x^{01}$, and $\frac{d}{dt}\,_tp_x^{02}$.

   c. [8 pts] Use Euler's Forward Method with a step size of 0.25 to estimate the probability that Linda is Disabled at time 0.5.

d. [6 pts] Calculate $\bar{a}^{10}_{50:\overline{10}|}$.

e. [6 pts] Calculate the gross premium reserve at time 10, if Linda is in state 1 (Disabled) at that time.

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2. Consider a \$200,000 fully-discrete, first-to-die, 11-year term life insurance policy issued to (55) and (45). Assume that mortality for these individuals is given by the SULT, they have independent future lifetimes, and that $i = 5\%$. The gross annual premium for this policy is \$2,700.

   Expenses for this policy are:

   - \$1,000 at issue

   - \$50 maintenance expense every year (including the first year)

   - 3% of gross premiums

   a. [2 pts] Calculate $_{11}q_{55:45}$.

   b. [2 pts] Calculate the probability that a benefit is paid for this policy.

   c. [4 pts] Calculate $A^{\hspace{0.2em}1}_{\overline{56:46}:\overline{10|}}$.

   d. [6 pts] Calculate the gross premium reserve at duration 1 for this policy, i.e., $_1V^g$.

   e. [8 pts] Calculate the FPT reserves at durations 1 and 2 for this policy, i.e., $_1V^{FPT}$ and $_2V^{FPT}$.

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3. [30 pts] You are doing a profit analysis for a block of 1000 fully discrete 20-year term insurance policies, issued to independent lives age 55. Each policy pays 250,000 at the end of the year of death. The reserve basis for the policies is:

   - $q_{55} = 0.003 \qquad q_{56} = 0.004$
   - $_0V = 0 \qquad _1V = 1292.26 \qquad _2V = 2949.98$
   - $i = 0.06$
   - The gross annual premium is 2,523 per policy
   - Issue expenses are 500 per policy, incurred at the start of the first year
   - Maintenance expenses are 100 per policy per year (at the start of the year), including the first year

   The actual experience for the first two years is given below:

Year	Interest Earned	Expenses	Deaths
1	4%	430,000	5
2	6%	110,000	5

   For each year, calculate the profit or loss due to mortality, interest, expenses, and overall.

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4. You sell a Type B Equity-Indexed Universal Life insurance product with the following features:

- Premiums are paid at the start of the year; there is a 2% charge on each premium paid.
- Maintenance expense and COI deductions are done at the start of each year.
- The specified amount is 200,000.
- Assume for now that corridor factors do not apply.
- The credited rate is determined according to the following:
  - Participation rate: 70%
  - Index: DJIA
  - Cap: 9%
  - Floor: 1%

You are also given the following information:

Policy Year (k)	Premium	% of Premium Charge	COI Charge	Maint. Expense Charge	DJIA Return	AV_k	Surrender Charge
9	3000	60	920	50	6%	78,000.00	5%
10	3000	60	950	50	-2%	80,739.40	4%
11	3000	60	990	50	15%	–	3%

(AV_k represents the account value at the end of policy year k.)

a. [2 pts] Determine the credited interest rate for policy year 9.

b. [2 pts] Determine the credited interest rate for policy year 10.

c. [2 pts] Determine the credited interest rate for policy year 11.

d. [4 pts] Calculate the cash surrender value at the end of policy year 10.

e. [4 pts] Calculate the account value at the end of policy year 11.

f. [2 pts] Calculate the death benefit payable if the insured dies in policy year 11.

g. [4 pts] If the corridor factor is $\gamma = 2.5$, verify that the DB (or NAR) is sufficiently large so that the death benefit in policy year 11 does not need to be increased. Show your calculations.

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5. An insurer is designing a 20-year single premium variable annuity policy with a guaranteed maturity benefit of 80% of the single premium. The basis and policy information is:

- Age at issue: 60
- Front end expense loading: 2%
- Annual management charge: 2% at each year end, including the first
- Survival model: Standard Ultimate Survival Model
- Lapses: 5% at each year end, except the final year
- Risk-free rate: 4% per year, continuously compounded
- Volatility: 20% per year

Answer the following questions:

a. [6 pts] Calculate the value of the GMMB at the issue date for a single premium of 100.

b. [14 pts] Calculate the value of the GMMB two years after issue, assuming the policy is still in force and that the underlying stock prices have decreased by 5% since inception.

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6. You act as the valuation actuary for a corporation which sponsors a final average salary defined benefit pension plan for its employees. The age retirement benefit provisions and valuation assumptions for the corporation's plan are described below.

   • The accrual rate is 2% per year of service.

   • The final average salary is defined as the salary over the final year of employment.

   • The Normal Form of pension is a life annuity with no guarantee, paid monthly in advance.

   • The normal retirement age is 65.

   • Salaries increase each year on 1 January at a rate of 2.5% per year.

   • $i = 0.05$

   • Mortality of active members and retirees follows the Standard Ultimate Life Table.

   • There are no exits prior to retirement at age 65, other than death.

   • The two-term Woolhouse formula is used for annuities paid more frequently than annually.

   • The plan is funded using the Traditional Unit Credit method.

   You are also given the following summary membership data, as of the valuation date, 1 January 2020.

Age	Number of members	Status	Pension in payment	Salary per member in 2019	Years of service per member
35	20	Active	-	45,000	8
60	5	Active	-	62,000	25
70	1	Retired	32,000	-	30

   a. [4 pts] Calculate the total actuarial liability for the corporation's pension plan as of the valuation date.

   b. [6 pts] Calculate the normal cost for 2020, expressed as a percentage of the total payroll at the valuation date.

   c. [2 pts] The corporation terminated the employment of all the 35-year-old members on the valuation date. Calculate the revised normal contribution rate for the corporation, expressed as a percentage of the total payroll of the remaining plan members.

   d. [4 pts] Without further calculation, state whether the change in the normal contribution rate would be greater or smaller under Projected Unit Credit funding. Justify your answer.

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