444 Practice Final

Key

April 16, 2024

1a) Symbol: ${}_{20}p_{40}^{\bar{0}\bar{0}}$

${}_{20}p_{40}^{\bar{0}\bar{0}} = exp(-\int_0^{20} \mu_{40+t}^{01} + \mu_{40+t}^{02}\, dt$

${}_{20}p_{40}^{\bar{0}\bar{0}} = exp(-\int_0^{20} .03 + .005 + .001(40+t)\, dt$

${}_{20}p_{40}^{\bar{0}\bar{0}} = exp(-\int_0^{20} .075 + .001t\, dt$

${}_{20}p_{40}^{\bar{0}\bar{0}} = exp(-(.075t + .0005t^2))$ evaluated from 0 to 20

${}_{20}p_{40}^{\bar{0}\bar{0}} = exp(-1.7) = .182683524$

1b) d/dt ${}_tp_x^{00} = -{}_tp_x^{00}[\mu_x^{01} + \mu_x^{02}] + {}_tp_x^{01}\mu_x^{10}, {}_0p_x^{00} = 1$

d/dt ${}_tp_x^{01} = -{}_tp_x^{01}[\mu_x^{10} + \mu_x^{12}] + {}_tp_x^{00}\mu_x^{01}, {}_0p_x^{01} = 0$

d/dt ${}_tp_x^{02} = {}_tp_x^{01}\mu_x^{12} + {}_tp_x^{00}\mu_x^{02}, {}_0p_x^{02} = 0$

1c) ${}_0p_{40}^{00} = 1$

at t = .25, d/dt ${}_tp_{40}^{00} = -[.03 + .005 + .001(40)] = -0.075$

${}_{.25}p_{40}^{00} = 1 + .25(-.075) = .98125$

at t = .25, d/dt ${}_tp_{40}^{01} = 0 - 0 + 1(.03) = .03$

${}_{.25}p_{40}^{01} = 0 + .25(.03) = .0075$

at t = .5, d/dt ${}_tp_{40}^{01} = -.0075(.02) - .0075(.02 + .002(40.25)) + .98125(.03) =$ $.02853375$

${}_{.5}p_{40}^{01} = .0075 + .25(.02853375) = 0.014633438$

1d) $\bar{a}_{50:\overline{10|}}^{10} = \bar{a}_{50}^{10} - [{}_{10}p_{50}^{11} * e^{-.06(10)} * \bar{a}_{60}^{10}] - [{}_{10}p_{50}^{10} * e^{-.06(10)} * \bar{a}_{60}^{00}]$

$\bar{a}_{50:\overline{10|}}^{10} = .7399 - [*e^{-.6} * .22581 * .5446] - [e^{-.6} * .06313 * 6.3865]$

$\bar{a}_{50:\overline{10|}}^{10} = 0.451139536$

1e) ${}_{10}Vg = E(benefits) - E(prems)$

E(benefits) - E(prems) = $[50000\bar{a}_{50:\overline{10|}}^{11} + 300\bar{a}_{50:\overline{10|}}^{11} + 100\bar{a}_{50:\overline{10|}}^{10}] - 10000\bar{a}_{50:\overline{10|}}^{10}$

${}_{10}Vg = 50300(3.4896) - 9900(.451139536)$

${}_{10}Vg = 171060.5986$

2a) $_{11}q_{55:45} = 1 - [_{11}p_{55} * _{11}p_{45}]$
$_{11}q_{55:45} = 1 - [\frac{94020.3}{97846.2} * \frac{97651.2}{98957.6}]$
$_{11}q_{55:45} = 0.051786576$

2b) Pr("benefit paid") = $_{11}q_{55:45} = 0.051786576$

2c) $A_{\overset{1}{56}:46:\overline{10|}} = A_{56:46} - {}_{10}E_{56:46}A_{66:56}$
$A_{56:46} = .27244, A_{66:56} = .40365$
$_{10}E_{56:46} = {}_{10}E_{46} * {}_{10}E_{56} * 1.05^{10} = .60581(.59109)(1.05^{10}) = .583288$
$A_{\overset{1}{56}:46:\overline{10|}} = .27244 = .583288(.40365) = .036995799$

2d) $_{1}Vg = 200000A_{\overset{1}{56}:46:\overline{10|}} - (2700(.97) - 50) * \ddot{a}_{46:56:\overline{10|}}$
$_{1}Vg = 200000(.036995799) - 2569(7.9740) = -13086.0462$

2e) By definition, $_{1}V^{FPT} = 0$
$(_{1}V_{+}P_{1})(1+i) = q_{46:56}(200000) + p_{46:56} * {}_{2}V^{FPT}$
Renewal premiums (under FPT) P = $[200000A_{\overset{1}{56}:46:\overline{10|}}]/7.974 = 927.91$
$p_{46:56} = p_{46} * p_{56} = .99695$
(0 + 927.91)(1.05) = (1 - $p_{46:56}$)(200000) + $p_{46:56} * {}_{2}V^{FPT}$
974.306219 = .00305(200000) + .99695 * $_{2}V^{FPT}$
$_{2}V^{FPT} = [974.306219 - .00305(200000)]/.99695$
$_{2}V^{FPT} = 365.42$

3) Year 1:

Total: [1000(2523) - 430000](1.04) - 250000(5) - 1292.26(995) = -359078.7

M: (250000 - 1292.26)(.003(1000) - 5) = -497415.5

I: [1000(2523) - 600(1000)](.04 - .06) = -38460

E: [600(1000) - 430000](1.04) = 176800

M + I + E = -497415.5 - 38460 + 176800 = -359078

Year 2:

Total: [995(2523 + 1292.26) - 110000](1.06) - 250000(5) - 2949.98(990) = -263125.48

M: (250000 - 2949.98)(.004(995) - 5) = -251991.02

I: [995(2523 + 1292.26) - 100(100)](.06 - .06) = 0

E: [100(995) - 110000](1.06) = -11130

M + I + E = -251991.02 + 0 - 251991.02 = -263121.02

4a) $\text{CreditedRate}_9 = max(min(.7{*}.06, .09), .01) = max(min(.042, .09), .01) = max(.042, .01) = .042$ or 4.2 percent

4b) $\text{CreditedRate}_{10} = max(min(.7{*}{-}.02, .09), .01) = max(min(-.014, .09), .01) = max(-0.014, .01) = .01$ or 1 percent

4c) $\text{CreditedRate}_{11} = max(min(.7{*}15, .09), .01) = max(min(.105, .09), .01) = max(.09, .01) = .09$ or 9 percent

4d) $\text{CSV}_{10} = AV_{10} * (1 - SC_{10}) = 80739.4(1 - .04) = 77509.824$

4e) $\text{AV}_{11} = [80739.4 + 3000 - 60 - 990 - 50](1.09) = 90076.946$

4f) $\text{DB}_{11} = 200000 + AV_{11} = 200000 + 90076.496 = 290076.496$

4g) $\text{AV}_{11} * \gamma = 2.51(90076.496) = 225192.365$
Since $\text{DB}_{11} > AV_{11} * \gamma$, (290,076 is greater than 225,192.365),
the DB is sufficiently large, so the corridor factor does not apply.

5a) GMMB = .8 * P

$\xi = (1-.02)(1-.02)^{20} = .654255812$

$\text{p}(0) = 80\text{e}^{-.04(20)} * \Phi - d_2(0) - .654255812 * 100 * \Phi - d_1(0)$

$\text{d}_1(0) = [ln(65.4255812/80) + (.04 + \frac{.2^2}{2})(20)]/(.2 * \sqrt{20}) = 1.1168$

$\text{d}_2(0) = 1.1168 - .2\sqrt{20} = -.222$

$\Phi - d_2(0) = \Phi - .222 = .4121569$

$\Phi - d_1(0) = \Phi - 1.1168 = .1320399$

$\text{p}(0) = [80\text{e}^{-.04(20)} * .4121569] - [.654255812 * 100 * .1320399]$

p(0) = 14.81552263 - 8.597502 = 6.218

$\Pi(0) = 6.218 * {}_{20}p_{60}^{(\tau)}$

$\Pi(0) = 6.218 * .95^{20} * 75,657.2/96,634.1$

$\Pi(0) = 1.745$

5b) GMMB = .8 * P

$\xi = (1-.02)(1-.02)^{20} = .654255812$

$\text{p}(2) = 80\text{e}^{-.04(18)} * \Phi - d_2(2) - .654255812 * 95 * \Phi - d_1(2)$

$\text{d}_1(2) = [ln(62.15430217/80) + (.04 + \frac{.2^2}{2})(18)]/(.2 * \sqrt{18}) = .975328179$

$\text{d}_2(2) = .975328179 - .2\sqrt{18} = .1268$

$\Phi - d_2(2) = \Phi - .1268 = 0.4495493$

$\Phi - d_1(2) = \Phi - .975328179 = 0.1646987$

$\text{p}(2) = [80\text{e}^{-.04(18)} * 0.4495493] - [.654255812 * 95 * 0.1646987]$

p(2) = 17.50553088 - 10.23673276 = 7.26879812

$\Pi(2) = 7.26879812 * {}_{18}p_{62}^{(\tau)}$

$\Pi(2) = 7.26879812 * .95^{18} * 75,657.2/95,940.6$

$\Pi(2) = 2.28$

6a) $\ddot{a}_{65}^{12} = \ddot{a}_{65} - (11/24) = 13.09147$
$\ddot{a}_{70}^{12} = \ddot{a}_{70} - (11/24) = 11.54997$
$\text{AL}_{35} = 8 * 45000 * .02 * [\ell_{65}/\ell_{35}] * \ddot{a}_{65}^{(12)} * v^{30}$
$\text{AL}_{35} = 8 * 45000 * .02 * [94579.7/99556.7] * (13.09147) * 1.05^{-30}$
$\text{AL}_{35} = 20719$
$\text{AL}_{60} = 25 * 62000 * .02 * [\ell_{65}/\ell_{60}] * \ddot{a}_{65}^{(12)} * 1.05^{-5}$
$\text{AL}_{60} = 25 * 62000 * .02 * [94579.7/96634.1] * (13.09147) * 1.05^{-5}$
$\text{AL}_{60} = 311223$
$\text{AL}_{70} = 32000\ddot{a}_{70}^{(12)} = 32000(11.54997) = 369599$
Total AL = 20(20719) + 5(311223) + 369599 = 2340094

6b) Total payroll before raises = 20(45000) + 5(62000) = 1210000
After raises: 121000(1.025) = 1240250
$\text{AL}_{36} = 9 * 1.025(45000) * .02 * [\ell_{65}/\ell_{36}] * \ddot{a}_{65}^{(12)} * v^{29}$
$\text{AL}_{36} = 9 * 46125 * .02 * [94579.7/99517.8] * (13.09147) * 1.05^{-29}$
$\text{AL}_{36} = 25096$
$\text{NC}_{35} = v * p_{35} * AL_{36} - AL_{35}$
$\text{NC}_{35} = 1.05^{-1} * [\ell_{36}/\ell_{35}] * 25096 - 20719$
$\text{NC}_{35} = 3173$
$\text{AL}_{61} = 1.025(62000) * .02 * 26 * [\ell_{65}/\ell_{61}] * \ddot{a}_{65}^{(12)} * v^4$
$\text{AL}_{61} = 63550 * .02 * 26 * [94,579.7/96,305.8] * 13.09147 * v^4$
$\text{AL}_{61} = 359103$
$\text{NC}_{60} = 1.05^{-1} * [\ell_{61}/\ell_{60}] * 359103 - 311223$
$\text{NC}_{60} = 20541$
Total NC = 20(3173) + 5(20541) = 166165
As a percent of payroll, 166165 / 1240250 = 0.133977 or 13.3977 percent

6c) Removing the 35 year old members, and using values calculated in part b,
Total NC = 5(20541) = 102705
Total Salary = 5(63550) = 317750
102705 / 317750 = 0.3232 or 32.32 percent

6d) The change would be smaller under PUC. Under PUC the curve of contribution rates by age is less steep, so the NC at age 60 will be closer to the NC at age 35, implying that the change will be smaller. The reason is that the PUC pre-pays for future pay increases on accrued benefits, while the TUC does not. That means that TUC NC rates at older ages must pay for the new accrued benefit, and in addition must pay to upgrade all past accruals for the most recent pay rise. This creates a very steep curve of contribution rates at older ages.