BRIGHAM YOUNG UNIVERSITY

STAT 446 ADVANCED SHORT-TERM ACTUARIAL MODELING

ASSIGNMENT 1

POLICY MODIFICATION

1. The ground-up loss X on an insurance policy has the following distribution:

x	$\Pr(X = x)$
200	0.4
800	0.3
2000	0.2
5000	0.1

   (a) Calculate $\mathbb{E}[X]$ and the loss elimination ratio for an ordinary deductible of 500. [1220; 0.3115]

   (b) For an ordinary deductible of 500, calculate the expected payment per loss and the expected payment per payment. [840; 1400]

   (c) For a franchise deductible of 500, calculate the expected payment per loss and the expected payment per payment. [1140; 1900]

   (d) Determine the ordinary deductible that would produce a loss elimination ratio of 40%. [680]

   (e) The policy is instead sold with a linearly disappearing deductible: no payment is made on a loss of 500 or less, full payment is made on a loss of 2500 or more, and the payment increases linearly between these two amounts. Calculate the expected payment per loss. [987.5]

   (f) Next year all losses are expected to increase by 10%, while the ordinary deductible remains 500.

      (i) Calculate the percentage increase in the expected payment per loss. [13.57%]

      (ii) Explain why your answer to (i) is larger than 10%.

2. Ground-up losses follow a Pareto distribution with $\alpha = 3$ and $\theta = 2000$. An insurance policy has an ordinary deductible of 250, a maximum covered loss of 5000, and a coinsurance factor of 80%.

   (a) Determine the maximum possible payment on a single loss. [3800]

   (b) Calculate the expected payment per loss. [566.79]

   (c) Calculate the expected payment per payment. [807.02]

   (d) Next year, all losses increase uniformly by 6%, and the deductible, maximum covered loss, and coinsurance factor do not change.

      (i) Calculate the expected payment per loss next year. [603.35]

      (ii) Calculate the percentage increase in the expected payment per loss, and explain why it is not exactly 6%. [6.45%]

(e) The insurer also sells policies with no deductible and no coinsurance, using a basic limit of 5000. Assuming that claim frequency does not depend on the policy limit, calculate the increased limit factor for a policy limit of 10,000. [1.0586]

3. An insurer has the following data on 1,000 ground-up losses:

Ground-Up Loss	Number of Losses	Total Ground-Up Losses
0–1000	500	210,000
1000–2500	300	510,000
2500–5000	120	420,000
5000–10,000	60	390,000
10,000+	20	380,000
Total	1,000	1,910,000

Assume that the number of ground-up losses does not depend on the deductible or the policy limit. Use the empirical distribution of these data to answer the following.

(a) Calculate the expected ground-up loss. [1910]

(b) For an ordinary deductible of 1000, calculate

(i) the expected payment per loss, [1200]

(ii) the expected payment per payment, and [2400]

(iii) the loss elimination ratio. [0.3717]

(c) The insurer sets its base rate using a deductible of 1000. Determine the indicated deductible relativity for

(i) a policy with no deductible, and [1.5917]

(ii) a policy with a deductible of 2500. [0.575]

(d) The insurer buys excess-of-loss reinsurance with a retention of 5000 per loss on policies with no deductible. Calculate the reinsurer's expected cost per loss. [370]

(e) Using a basic limit of 5000 for policies with no deductible, calculate the increased limit factor for a policy limit of 10,000. [1.1234]

(f) Give one reason why the deductible relativities in (c) might not be appropriate in practice.

4. The number of losses N^L on a policy with no deductible belongs to the $(a, b, 0)$ class with

$$\frac{\Pr(N^L = 1)}{\Pr(N^L = 0)} = 0.6 \qquad \text{and} \qquad \frac{\Pr(N^L = 2)}{\Pr(N^L = 1)} = 0.45.$$

Loss amounts follow an exponential distribution with mean 1000 and are independent of N^L.

(a) Determine a and b, and identify the distribution of N^L and its parameters. [$a = b = 0.3$; negative binomial, $r = 2$, $\beta = 3/7$]

(b) An ordinary deductible of 500 is imposed. Identify the distribution of the number of payments, N^P, and its parameters. [negative binomial, $r = 2$, $\beta^* = 0.2599$]

(c) Calculate $\Pr(N^P = 0)$, $\mathbb{E}[N^P]$, and $\mathrm{Var}(N^P)$. [0.6299; 0.5199; 0.6550]

(d) Suppose instead that N^L follows a zero-modified distribution, with the same a and b as in (a) and with $p_0^M = 0.6$. The deductible of 500 still applies.

(i) Calculate p_0^{M*}, the probability of no payments. [0.7098]

(ii) Calculate $\Pr(N^P = 1)$. [0.2039]

(iii) Calculate $\mathbb{E}[N^P]$. [0.4078]

5. For a collective risk model, you are given:

(i) The number of losses follows a zero-modified Poisson distribution with $\lambda = 0.5$, and $p_0^M = 0.3$.

(ii) Loss size follows a single-parameter Pareto distribution with $\theta = 200$ and $\alpha = 1.5$, so $\Pr(X > x) = (200/x)^{1.5}$ for $x \geq 200$.

(iii) An insurance coverage has a deductible of 500.

(a) Calculate the probability of no paid claim. [0.7886]

(b) Calculate the probability of one paid claim. [0.1983]

(c) Calculate the mean of the number of payments. [0.2250]

(d) Calculate the variance of the number of payments. [0.2029]