BRIGHAM YOUNG UNIVERSITY

STAT 446 ADVANCED SHORT-TERM ACTUARIAL MODELING

ASSIGNMENT 2

AGGREGATE LOSS MODELS

1. A company sells three independent policies. For policy A, all claim payments are 10,000 or 20,000 with equal probabilities, and a single policy has a Poisson number of claims with mean 1. For policy B, all claim payments are 20,000 and a single policy has a Poisson number of claims with mean 0.5. For policy C, all claim payments are 40,000 and a single policy has a Poisson number of claims with mean 0.5.

   (a) The aggregate claim payments of the company follow a compound Poisson distribution. Specify its primary and secondary distributions. [Poisson with $\lambda = 2$; 10,000, 20,000, 40,000 with probabilities 0.25, 0.5, 0.25]

   (b) Calculate the expected value and standard deviation of the aggregate claim payments for the company. [45,000; 35,355]

   (c) Calculate the probability that the aggregate claim payments are no more than 30,000. [0.4257]

2. An insurer has a portfolio of 400 independent policies. For each policy:

   (i) The annual number of claims follows a Poisson distribution with mean 0.25.

   (ii) Claim amounts follow a lognormal distribution with $\mu = 7$ and $\sigma = 1$.

   (iii) Claim counts and claim amounts are independent.

   Let S denote the total annual claims for the portfolio.

   (a) Explain why S has a compound Poisson distribution, and state its Poisson parameter. [$\lambda = 100$]

   (b) Calculate $\mathbb{E}[S]$ and the standard deviation of S. [180,804; 29,810]

   (c) Using a normal approximation for S, calculate

      (i) the 95% VaR of S, and [229,837]

      (ii) the 95% Expected Shortfall (also called TVaR) of S. [242,293]

   (d) The insurer instead approximates S with a lognormal distribution with parameters μ^* and σ^*, chosen so that its mean and variance match those of S.

      (i) Show that $\sigma^* = 0.16$ to the nearest 0.01. You should calculate the value to the nearest 0.0001. [0.1638]

      (ii) Calculate the 95% VaR of S using the lognormal approximation. [233,546]

   (e) You are given that for a compound Poisson distribution with Poisson parameter λ,

$$\mathbb{E}\left[(S - \mathbb{E}[S])^3\right] = \lambda\,\mathbb{E}[X^3].$$

(i) Calculate the coefficient of skewness of S. [0.4482]

(ii) State with reasons which of the two approximations you would expect to give the more accurate estimate of the 95% VaR. [lognormal]

3. Let S denote the aggregate payments on a policy, in thousands. The number of losses is N, and the payment on each loss, after a per-loss deductible, is X (in thousands), so a loss can result in a payment of 0. You are given:

(i) N belongs to the $(a, b, 0)$ class with $\mathbb{E}[N] = 1.2$ and $\mathrm{Var}(N) = 0.96$.

(ii) X takes values in $\{0, 1, 2, \ldots\}$ and belongs to the $(a, b, 0)$ class with $\mathbb{E}[X] = 1.5$ and $\mathrm{Var}(X) = 3.75$.

(iii) N and the X's are independent.

(a) (i) Explain why N must have a binomial distribution, and determine its parameters. [$m = 6$, $q = 0.2$]

(ii) Identify the distribution of X and its parameters. [geometric, $\beta = 1.5$]

(b) Calculate $\mathbb{E}[S]$ and the standard deviation of S. [1.8; 2.5807]

(c) Determine the probability generating function of S. [$P_S(z) = \left(0.8 + \frac{0.2}{2.5 - 1.5z}\right)^6$]

(d) (i) Calculate $\Pr(S = 0)$. [0.4644]

(ii) Use the recursive formula to calculate $\Pr(S = 1)$ and $\Pr(S = 2)$. [0.1520; 0.1119]

(e) Calculate the net stop-loss premium for an aggregate deductible of

(i) 2 (that is, 2000), and [0.8808, or 880.8]

(ii) 1.5 (that is, 1500). [1.0726, or 1072.6]

4. For a collective risk model, you are given:

(i) The number of losses follows a binomial distribution with $m = 5$, and $q = 0.2$.

(ii) Loss size follows a lognormal distribution with $\mu = 5$ and $\sigma = 2$.

(a) With a span of $h = 100$, discretize the severity distribution of the losses using the method of rounding. You are required to calculate all values up to a discretized amount paid of 200. [$f_0 = 0.2932$, $f_1 = 0.2089$, $f_2 = 0.1007$]

(b) Use the recursive formula to calculate the probability that the discretized aggregate losses are no more than 200. [0.6461]

5. Loss amounts follow a Pareto distribution with $\alpha = 3$ and $\theta = 2000$, and the number of losses follows a Poisson distribution with mean 1.5. You discretize the loss amounts using a span of $h = 500$.

(a) Using local moment matching with one moment, calculate f_0, f_1, and f_2, the probabilities at 0, 500, and 1000. [0.2800, 0.3289, 0.1553]

(b) Using the method of rounding, calculate f_0, f_1, and f_2. [0.2977, 0.3177, 0.1516]

(c) Which of the two discretized distributions has the same mean as the loss amounts? Explain.

(d) Using the distribution from (a) and the recursive formula, calculate $\Pr(S = 0)$ and $\Pr(S = 500)$, where S is the discretized aggregate loss. [0.3396; 0.1675]

(e) Using your answer to (d), calculate the net stop-loss premium for an aggregate deductible of 1000. [923.36]

6. For a collective risk model:

(i) The number of losses has a negative binomial distribution with $\beta = 0.5$ and $r = 4$.

(ii) The common distribution of the individual losses is:

x	$f_X(x)$
200	0.4
400	0.4
600	0.2

There are two policies, A and B, available to insure these losses.

(a) Policy A has an ordinary deductible of 200 on each individual loss. Calculate the expected aggregate insurance payments from Policy A. [320]

(b) Policy B has an aggregate deductible of 400. Calculate the expected aggregate insurance payments from Policy B. [420.08]