BRIGHAM YOUNG UNIVERSITY

STAT 446 ADVANCED SHORT-TERM ACTUARIAL MODELING

ASSIGNMENT 3

PARAMETRIC MODELS: ESTIMATION & SELECTION

1. You are given:

   (i) Four independent lives are observed from time $t = 0$ until death.

   (ii) Deaths occur at times $t = 1, 2, 3$ and 4.

   (iii) The lives are assumed to be subject to the probability density function

   $$f(t) = \frac{te^{-t/c}}{c^2}, \qquad t > 0.$$

   (a) Calculate the maximum likelihood estimate for c. [1.25]

   (b) Calculate the variance of the maximum likelihood estimator for c. [0.1953]

   (c) Determine the maximum likelihood estimate for the probability that death occurs before $t = 5$ for a randomly selected life, i.e., $F(5)$. [0.9084]

   (d) Use the delta method to approximate the variance of the maximum likelihood estimator of $F(5)$. [0.01074]

2. A random sample of 6 claims is given:

   $$39 \quad 150 \quad 247 \quad 388 \quad 705 \quad 1000^+$$

   where the last claim is known to be greater than 1000.

   You fit the data to the following density function

   $$f(x) = \frac{\alpha\, 1000^\alpha}{(1000 + x)^{\alpha+1}}, \qquad x > 0.$$

   (a) Determine $\hat{\alpha}$, the maximum likelihood estimate of α. [2.5597]

   (b) Calculate the approximated variance of $\hat{\alpha}$. [1.3104]

   (c) Construct an approximate 95% normal confidence interval for α. [(0.316, 4.803)]

   (d) Construct an approximate 95% likelihood-based confidence interval for α, and compare it with the interval in (c). [(0.918, 5.502)]

3. Claim size X follows an inverse Pareto distribution, with $F(x) = \left(\frac{x}{x+\theta}\right)^\tau$ for $x > 0$. The maximum likelihood estimates of the two parameters τ and θ are $(2, 100)$, and the covariance matrix of these estimators is

   $$\begin{bmatrix} 0.01 & -0.1 \\ -0.1 & 5 \end{bmatrix}.$$

   Using the delta method, estimate the variance of $\Pr(X > 500)$. [0.0001285]

4. You are given the following random observations:

$$0.1 \quad 0.4 \quad 0.8 \quad 0.8 \quad 0.9$$

You test whether the sample comes from a distribution with probability density function:

$$f(x) = \frac{1+2x}{2}, \qquad 0 \le x \le 1.$$

Calculate the Kolmogorov-Smirnov test statistic. [0.320]

5. You are given the following:

- The observed number of claims for a group of 1000 risks has been recorded as follows:

Number of Claims	Number of Risks
0	729
1	242
2	29
3 or more	0

- The null hypothesis, H_0, is that the number of claims per risk follows a Poisson distribution.
- You use a chi-square test to measure the fit of a Poisson distribution. The minimum expected number of observations in any group should be 5. The maximum possible number of groups should be used.
- The estimated Poisson parameter is $\lambda = 0.3055$ from the sample.

Determine the chi-square statistic. [3.56]

6. You fit a Weibull distribution to a sample of 20 claim amounts $x_1, \ldots, x_{20}$ and use the likelihood ratio test to test the hypothesis

$$H_0 : \tau = 2 \quad \text{vs.} \quad H_1 : \tau \neq 2.$$

You are given that

$$\sum_{i=1}^{20} \ln x_i = 73.6177, \qquad \sum_{i=1}^{20} x_i^2 = 87{,}266,$$

and at the maximum likelihood estimate, the loglikelihood is -98.443.

Determine the result of the test. [test statistic 3.392 with 1 df: reject at 10%, do not reject at 5%]