BRIGHAM YOUNG UNIVERSITY

STAT 446 ADVANCED SHORT-TERM ACTUARIAL MODELING

ASSIGNMENT 5

CREDIBILITY

1. Two eight-sided dice, A and B, are used to determine the number of claims for an insured. The faces of each die are marked with either 0 or 1, representing the number of claims for that insured for the year.

Die	$\Pr(\text{Claims} = 0)$	$\Pr(\text{Claims} = 1)$
A	1/4	3/4
B	3/4	1/4

Two spinners, X and Y, are used to determine claim cost. Spinner X has two areas marked 12 and c. Spinner Y only has one area marked 12.

Spinner	$\Pr(\text{Cost} = 12)$	$\Pr(\text{Cost} = c)$
X	1/2	1/2
Y	1	0

To determine the losses of the year, a die is randomly selected from A and B and rolled. If a claim occurs, a spinner is randomly selected from X and Y and spun. For subsequent years, the same die and spinner are used to determine losses.

Losses for the first year are 12. Based upon the first year, you determine that the expected losses for the second year are 10.

Calculate c. [36]

2. You are given:

(i) An individual automobile insured has an annual claim frequency distribution that follows a Poisson distribution with mean λ.

(ii) λ is assumed to follow a gamma distribution with parameters α and θ.

(iii) The first actuary assumes that $\alpha = 1$ and $\theta = 1/6$.

(iv) The second actuary assumes the same mean for the gamma distribution, but only half the variance.

(v) A total of one claim is observed for the insured over a three year period.

(vi) Both actuaries determine the Bayesian premium for the expected number of claims in the next year using their model assumptions.

You may use the fact that if Λ is gamma with integer α, then $2\Lambda/\theta$ is chi-square with 2α degrees of freedom. The 2.5th, 50th, and 97.5th percentiles of the chi-square distribution with 4 degrees of freedom are 0.4844, 3.3567, and 11.1433.

(a) Determine the ratio of the Bayesian premium that the first actuary calculates to the Bayesian premium that the second actuary calculates. [$10/9 = 1.111$]

(b) Using the first actuary's model, calculate the Bayes estimate of λ under absolute loss and under zero-one loss. [0.1865; 0.1111]

(c) Using the first actuary's model, calculate a 95% equal-tailed credibility interval for λ. [(0.0269, 0.6191)]

3. You are given the following information about workers' compensation coverage:

(i) The number of claims for an employee during the year follows a Poisson distribution with mean

$$(100 - p)/100$$

where p is the salary (in thousands) for the employee.

(ii) The distribution of p is uniform on the interval $(0, 100]$.

An employee is selected at random. During the last 4 years, the employee has had a total of 5 claims.

Determine the Bühlmann credibility estimate for the expected number of claims the selected employee will have next year. [0.8]

4. For a portfolio of insurance risks, aggregate losses per year per exposure follow a normal distribution with mean θ and standard deviation 1000, with θ varying by class as follows:

Class	θ	Percent of Risks in Class
X	2000	60%
Y	3000	30%
Z	4000	10%

A randomly selected risk has the following experience over three years:

Year	Number of Exposures	Aggregate Losses
1	24	24,000
2	30	36,000
3	26	28,000

Calculate the Bühlmann-Straub estimate of the mean aggregate losses per year per exposure in Year 4 for this risk. [1137.84]

5. In one year, 500 policyholders had the following numbers of claims:

Number of Claims	Number of Policyholders
0	300
1	140
2	45
3	12
4	3

Each policyholder's annual number of claims is assumed to be Poisson, with a mean that varies by policyholder. Use semiparametric empirical Bayes estimation.

(a) Explain why the Poisson assumption allows you to estimate v from the same data used to estimate μ.

(b) Estimate μ, v, and a. [0.556; 0.556; 0.0882]

(c) Calculate the credibility factor for one year of data. [0.1369]

(d) Calculate the credibility estimate of next year's number of claims for a policyholder who had 2 claims, and for a policyholder who had no claims. [0.7536; 0.4799]

(e) A policyholder from the same population had 4 claims in the last 3 years. Calculate the credibility estimate of the number of claims next year. [0.8066]

6. You are given the following experience for two insured groups:

Group		Year 1	Year 2	Year 3	Total
1	Number of members	8	12	5	25
	Average loss per member	96	91	113	97
2	Number of members	25	30	20	75
	Average loss per member	113	111	116	113
Total	Number of members				100
	Average loss per member				109

Determine the nonparametric empirical Bayes credibility premium for group 1. [98.26]