BRIGHAM YOUNG UNIVERSITY

STAT 446 ADVANCED SHORT-TERM ACTUARIAL MODELING

ASSIGNMENT 6

RATEMAKING

1. An insurer has the following loss costs (pure premiums, including LAE) for a line of business:

Accident Year	Loss Cost
2020	210.5
2021	221.8
2022	238.0
2023	247.6
2024	266.1

   (a) Let $X =$ Accident Year $- 2020$. Fit the model $\ln(\text{Loss Cost}) = a + bX$ by least squares, and determine the indicated annual loss trend. [$a = 5.3481$, $b = 0.05788$; $+5.96\%$ per year]

   (b) New rates will be in effect for one year beginning January 1, 2026, and all policies are annual. Using the AY 2024 loss cost and the trend from (a),

      (i) determine the trend period, and [2.5 years]

      (ii) calculate the projected loss cost. [307.53]

   (c) Fixed expenses are 30 per exposure, variable expenses are 20% of premium, and the target underwriting profit is 4% of premium. Using the pure premium method, calculate the indicated average rate. [444.12]

   (d) The current average rate is 420. Calculate the indicated overall rate change. [$+5.74\%$]

   (e) Explain why the selected trend should not be based only on the fitted curve in (a), and give two factors that can drive loss trends.

2. An insurer is reviewing the rates for a line of business. You are given:

   (i) The new rates will be in effect for one year beginning January 1, 2027.

   (ii) The experience period is Accident Year (AY) 2025, with losses evaluated as of December 31, 2025.

   (iii) The selected annual loss trend is 4%.

   (iv) The following data for AY 2025:

Earned exposures	8,000
Earned premium at current rate level	4,000,000
Reported losses and LAE as of 12/31/2025	2,400,000
Loss development factor to ultimate	1.25
Projected fixed expenses	320,000

(v) Variable expenses are 15% of premium, and the target underwriting profit is 5% of premium.

(a) Determine the trend period and the trend factor if

(i) all policies are annual, [2.5 years; 1.1030]

(ii) all policies are six-month policies, and [2.25 years; 1.0923]

(iii) all policies are annual, but the experience period is Policy Year 2025 instead of AY 2025. [2 years; 1.0816]

(b) Assume all policies are annual. Calculate the projected ultimate losses and LAE for the AY 2025 experience. [3,309,060]

(c) Using the loss ratio method, calculate the indicated overall rate change. [$+13.41\%$]

(d) Using the pure premium method, calculate the indicated average rate, and show that it is consistent with your answer to (c). [567.04]

(e) Describe one situation in which the pure premium method is preferable and one situation in which the loss ratio method is preferable.

3. An insurer offers auto physical damage policies with a choice of three deductibles: 250 (the base level), 500, and 1000. You are given:

(i) Ground-up losses follow a Pareto distribution with $\alpha = 3$ and $\theta = 3000$.

(ii) The distribution of the number of ground-up losses is the same for every deductible option, and is independent of the loss amounts.

(a) (i) Calculate the expected payment per loss for each deductible option. [1278.11; 1102.04; 843.75]

(ii) Show that the indicated deductible relativities for the 500 and 1000 deductibles are 0.86 and 0.66 to the nearest 0.01. You should calculate the values to the nearest 0.0001. [0.8622; 0.6602]

The insurer currently uses the rounded relativities from (a)(ii), with a base rate of 150. The most recent experience, with losses developed and trended to the new rate period, is:

Deductible	Current Relativity	Earned Exposures	Losses
250	1.00	12,000	1,560,000
500	0.86	6,000	690,000
1000	0.66	2,000	160,000

(b) (i) Calculate the indicated relativities using the pure premium method. [1.00; 0.8846; 0.6154]

(ii) Calculate the indicated relativities using the loss ratio method. [1.00; 0.8846; 0.6154]

(iii) Explain why the two methods give the same answer here.

(c) Give one reason why the indicated relativity for the 1000 deductible in (b) is lower than the value from the Pareto model in (a).

(d) The insurer adopts the relativities from (b) and implements an overall rate increase of 6%.

(i) Calculate the off-balance factor. [1.0032]

(ii) Calculate the new base rate. [158.50]

(e) Calculate the percentage change in premium for each deductible option, and comment on the result for the 1000 deductible. [$+5.67\%$; $+8.69\%$; -1.48%]

4. You are given the following information and asked to perform a ratemaking analysis for an insurance product:

- The new rates are to become effective February 1, Calendar Year (CY) 6 for 12 months, and all policies are one-year policies.
- The loss development factors are estimated based on the following run-off triangle.

Table 1: Reported Losses (including LAE) as of

Accident Year (AY)	12/31/AY2	12/31/AY3	12/31/AY4
1	1,000,000	1,050,000	1,050,000
2	1,100,000	1,210,000	1,270,500
3		1,300,000	1,430,000

- The loss trend factor is 8% per annum effective.
- The permissible loss ratio is 0.80.
- The new rates are determined based on the AY 4 claims experience:

Territory	Class	AY 4 Earned Exposure	AY 4 Earned Premium	AY 4 Reported L&LAE
A	1	10,000	900,000	700,000
A	2	4,000	460,000	400,000
B	1	2,000	180,000	200,000
B	2	2,000	250,000	270,000

- The current base rate is 100 with differentials:

Differentials	Territory A	Territory B
Class 1	1.00	1.10
Class 2	1.20	1.32

(a) Determine the indicated overall rate change. [$+40.8\%$]

(b) Determine the new differentials. (Note: both territory and class differentials will change.) [Territory B: 1.4372; Class 2: 1.4649]

(c) Determine the new rate for each territory and class. [A1: 120.52; A2: 176.55; B1: 173.21; B2: 253.73]