BRIGHAM YOUNG UNIVERSITY

STAT 446 ADVANCED SHORT-TERM ACTUARIAL MODELING

ASSIGNMENT 7

OUTSTANDING CLAIMS RESERVES

1. You are given the following cumulative paid claims (in thousands) for a portfolio of short-term policies. Assume that claims are fully developed by the end of Development Year (DY) 3, that all payments are made in the middle of each year, and that the valuation date is December 31, 2025.

	Cumulative Paid Claims by Development Year (DY)			
Accident Year (AY)	0	1	2	3
2022	1,200	1,980	2,280	2,350
2023	1,350	2,180	2,520	
2024	1,500	2,400		
2025	1,620			

   (a) Calculate the chain ladder development factors $\hat{f}_j$ for $j = 0, 1, 2$. [1.6198; 1.1538; 1.0307]

   (b) Using the chain ladder method, calculate the estimated ultimate claims and outstanding claims for each accident year, and the total outstanding claims. [ultimate: 2350, 2597.4, 2854.3, 3120.6; outstanding: 0, 77.4, 454.3, 1500.6; total 2032.3]

   (c) Calculate the projected claim payments in each of calendar years 2026, 2027, and 2028. [1450.6; 488.7; 93.0]

   (d) Using an annual effective interest rate of 4%, calculate the discounted value of the total outstanding claims as of December 31, 2025. [1967.5]

   (e) The earned premium for AY 2025 is 3,300 and the expected loss ratio is 75%.

      (i) Calculate the Bornhuetter-Ferguson estimate of the outstanding claims for AY 2025. [1190.2]

      (ii) Show that the Bornhuetter-Ferguson estimate of the ultimate claims for AY 2025 is a credibility-weighted average of the chain ladder estimate and the prior estimate, and state the credibility factor. [$Z = 0.5191$]

      (iii) Calculate the expected loss ratio estimate of the outstanding claims for AY 2025. [855]

   (f) You learn that the AY 2025 payments in DY 0 include a single unusually large claim of 300 that has been fully settled.

      (i) Recalculate the chain ladder and Bornhuetter-Ferguson estimates of the outstanding claims for AY 2025 with this claim removed from the data. [CL: 1222.8; BF: 1190.2]

(ii) Comment on the sensitivity of the two methods to unusual claims in the most recent accident year.

2. You are given the following incremental claim payments by development year. Assume all incremental payments are made at the middle of each year, and claims are fully developed by the end of Development Year (DY) 4.

	Incremental Claims by Development Year (DY)				
Accident Year (AY)	0	1	2	3	4
0	81,345	55,315	12,299	2,085	755
1	98,020	70,574	20,231	1,699	
2	94,987	56,042	16,613		
3	101,760	66,144			
4	115,734				

You are also given the following inflation rates r_t, measured from mid-year t to mid-year $t+1$:

Year t	Inflation Rate (Past)	Year t	Inflation Rate (Assumed)
0	2.0%	4	5.8%
1	3.5%	5	4.4%
2	5.7%	6	3.6%
3	6.3%	7	3.0%

(a) Calculate the claims inflation index, Q_k, using the inflation rates, for $k = 0, 1, \ldots, 8$. Set $Q_0 = 100.0$. [100, 102, 105.57, 111.59, 118.62, 125.50, 131.02, 135.74, 139.81]

(b) Using the inflation-adjusted chain ladder method, calculate the estimated total ultimate claims from all five accident years of the table, expressed in terms of mid AY 4 monetary units. [985,892]

(c) Calculate the inflation-adjusted outstanding claims reserve. [127,091]

3. An insurer records the cumulative number of settled claims and the incremental claim payments for a portfolio. All claims are settled by the end of Development Year (DY) 3.

Cumulative Number of Settled Claims

AY	0	1	2	3
0	40	70	85	90
1	45	80	96	
2	50	88		
3	55			

Incremental Claim Payments

AY	0	1	2	3
0	8,000	24,000	22,500	12,000
1	9,450	27,300	26,400	
2	11,000	31,200		
3	12,100			

(a) Using the chain ladder method, project the number of claims settled in each future development year for AY 1, AY 2, and AY 3. [AY 1: 5.647; AY 2: 18.187, 6.246; AY 3: 41.963, 20.039, 6.882]

(b) Calculate the average incremental payment per settled claim for each development year. [213.42, 800.97, 1577.42, 2400]

(c) Estimate the outstanding claims for each accident year and in total using the frequency-severity method. [13,553; 43,679; 81,739; total 138,971]

(d) Estimate the total outstanding claims by applying the chain ladder method to the cumulative payments, and compare. [146,252]

(e) Explain why the average payment per settled claim increases with the development year.

4. An insurer records the following numbers of reported claims by accident year and development year. All claims are reported by the end of Development Year (DY) 3.

	Incremental Reported Claim Counts by Development Year (DY)			
Accident Year (AY)	0	1	2	3
0	120	60	25	10
1	130	70	30	
2	140	66		
3	150			

The insurer uses the Poisson model: the incremental counts $X_{i,j}$ are independent with

$$X_{i,j} \sim \text{Poi}\left(\mu_i \gamma_j\right), \qquad \mu_i, \gamma_j > 0, \qquad \sum_{j=0}^{3} \gamma_j = 1,$$

where μ_i and γ_j are estimated by maximum likelihood.

(a) Calculate the chain ladder development factors $\hat{f}_j$ for $j = 0, 1, 2$, based on the cumulative claim counts. [1.5026; 1.1447; 1.0488]

(b) Using the relationship between the Poisson model and the chain ladder method, determine $\hat{\gamma}_j$ for $j = 0, 1, 2, 3$ and $\hat{\mu}_i$ for $i = 0, 1, 2, 3$. [$\hat{\gamma}_j$: 0.5543, 0.2786, 0.1206, 0.0465; $\hat{\mu}_i$: 215, 241.22, 247.32, 270.59]

(c) The maximum likelihood estimates satisfy $\sum_{i=0}^{3-j} X_{i,j} = \hat{\gamma}_j \sum_{i=0}^{3-j} \hat{\mu}_i$ for each j. Verify that your estimates satisfy this equation for $j = 2$.

(d) Calculate the expected number of claims that will be reported

(i) in the next calendar year, and [116.42]

(ii) in total in all future years. [173.13]

(e) Treating the estimated parameters as known, calculate the standard deviation of the total number of claims still to be reported under the Poisson model. [13.16]

(f) Give two reasons why the standard deviation in (e) may understate the uncertainty in the number of claims still to be reported.

(g) The insurer instead uses the overdispersed Poisson model, with $\mathrm{Var}(X_{i,j}) = \phi\,\mu_i\gamma_j$.

(i) Estimate ϕ using Pearson residuals. [0.157]

(ii) Calculate the process standard deviation of the total number of claims still to be reported, and comment on your result. [5.21]

5. You are given the following cumulative claims triangle. Claims are fully developed by the end of Development Year (DY) 2.

AY	0	1	2
0	1000	1500	1600
1	1200	1650	
2	900		

You use the Bühlmann-Straub credibility model to estimate the outstanding claims.

(a) Calculate the chain ladder estimates of $\hat{\gamma}_j$ and of the ultimate claims for each accident year. [0.6548, 0.2827, 0.0625; 1600, 1760, 1374.55]

(b) Calculate $\hat{s}_0^2$, $\hat{s}_1^2$, and $\hat{v}$. [5741.6, 11,483.3; 8612.4]

(c) Calculate $\hat{a}$. [23,533]

(d) Calculate the credibility factors Z_0, Z_1, Z_2 and the credibility-weighted mean $\hat{\mu}$. [0.7321, 0.7192, 0.6415; 1585.88]

(e) Calculate the Bühlmann-Straub estimates of the outstanding claims for AY 1 and AY 2, and compare them with the chain ladder estimates. [61.11, 550.32; CL: 110.00, 474.55]

(f) Give one advantage and one disadvantage of the Bühlmann-Straub approach compared with the Bornhuetter-Ferguson method.

6. Your actuarial team is given the following cumulative paid claims run-off triangle for determining the indicated outstanding claims reserves.

	Cumulative Paid Claims by Development Year (DY)				
Accident Year (AY)	0	1	2	3	4
0	81,345	136,660	148,959	151,044	151,800
1	98,020	168,594	188,826	190,525	
2	94,987	151,029	167,642		
3	101,760	167,904			
4	115,734				

(a) Team member A determines the total outstanding claims reserves using the chain ladder (CL) method.

(i) Calculate $f_{i,j}$ for $i, j \leq 3$. [AY 0: 1.680, 1.090, 1.014, 1.005; AY 1: 1.720, 1.120, 1.009; AY 2: 1.590, 1.110; AY 3: 1.650]

(ii) Estimate $\hat{f}_j$ for $j \leq 3$. [1.6596, 1.1077, 1.0112, 1.0050]

(iii) Calculate the CL estimate of $C_{i,4}$ for $i = 1, 2, 3, 4$. [191,478.6; 170,368.5; 189,013.0; 216,216.6]

(b) Team member B determines the total outstanding claims reserves using the Bornhuetter-Ferguson (BF) method with prior estimates of the expected total losses for each accident year as:

AY	1	2	3	4
μ_i	200,000	180,000	220,000	240,000

(i) Calculate $\hat{\beta}_j$ for $j \leq 3$. [0.5353, 0.8883, 0.9840, 0.9950]

(ii) Calculate the BF estimate of outstanding claims reserves for Accident Year i, $i = 1, 2, 3, 4$. [996.0; 2880.6; 24,569.6; 111,535.5]

(c) Team member C assumes that incremental claim payments $X_{i,j}$ follow the Poisson model:

$$X_{i,j} \sim \text{Poi}\left(\mu_i \gamma_j\right), \qquad \mu_i, \gamma_j > 0, \qquad \sum_{j=0}^{4} \gamma_j = 1,$$

where μ_i and γ_j are estimated using maximum likelihood based on the given run-off triangle data.

Using the results in (a), estimate $\hat{\mu}_i$ and $\hat{\gamma}_j$ for $i, j = 0, 1, \ldots, 4$. [$\hat{\mu}_i$: 151,800; 191,478.6; 170,368.5; 189,013.0; 216,216.6. $\hat{\gamma}_j$: 0.5353, 0.3531, 0.0957, 0.0110, 0.0050]

(d) Team member D further examines the uncertainty of outstanding claims reserves using Mack's model.

(i) Calculate $\hat{\sigma}_j^2$ for $j \leq 3$. [286.99, 34.575, 2.0813, 0.1253]

(ii) Calculate the process standard deviation for outstanding claims for AY 3, using Mack's approximation. [2531.8]

(iii) Calculate the square root of the estimation error for AY 3, using Mack's approximation. [1565.5]