Stat 446
Advanced Short-Term Actuarial Modeling

Chapter 6: Ratemaking

Outline

1. Ratemaking Review
   - Projecting Premium
   - Loss Development and Trend
   - Expenses and Profit
2. Ratemaking Steps
   - Overall Rating Indications
   - Risk Classification Differentials
   - Balance Back

Introduction

- In this chapter, we will first briefly review how to properly determine the rates for short-term insurance coverages, including projecting premium, losses, expenses, and expected profit for the effective period.
- We will then discuss the major steps for a complete ratemaking process, i.e., determine the overall rate changes, changes required for different rate classes, and the new base rate.

Fundamental Insurance Equation

- The fundamental insurance equation:

$$\text{Premium} = \text{Losses} + \text{LAE} + \text{UW Expenses} + \text{UW Profit}$$

where

  - **Premium**: aggregate amount the insured pays for insurance coverage; written/earned/unearned/in-force premium.
  - **Exposure**: basic unit of risk that underlies the insurance premium; written/earned/unearned/in-force exposure.
  - **Losses & LAE**: see prior coursework on loss reserving.
  - **UW expenses**: commissions and brokerage, other acquisition, general taxes, licenses, and fees
  - **UW profit**: operating income (*not including* investment income)

Prospective Ratemaking

- Ratemaking is *prospective*:
  - Make adjustments to the historical information considering multiple factors, such as changes in rate, operation, inflationary pressures, mix of business written, and law.
  - Project each component to the level expected during the **expected effective period**.
- The **experience period** is usually the most recent one to five years, not all past data: older years reflect conditions (coverage, mix of business, law) that no longer apply, while using several years adds stability. The latest months are immature, so their losses must be developed.

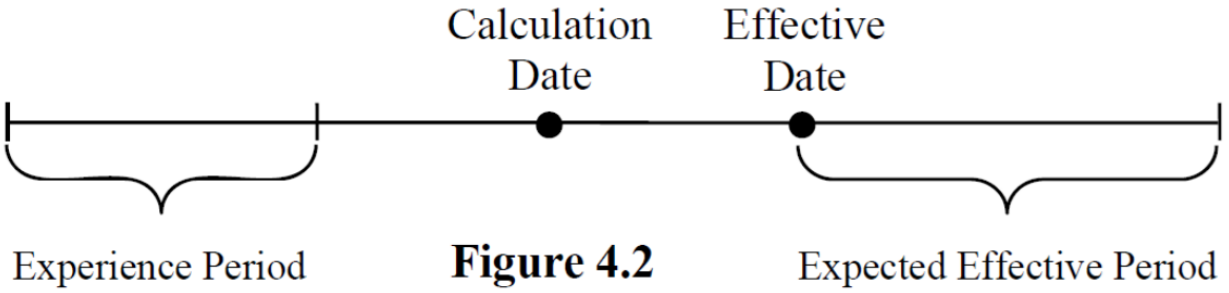


Figure 4.2

Adjustments to Premium

- To project future premium, we must first bring historical premium to the **current rate level**, or put it **"on-level"**.
  - Rate changes may have occurred since the experience period.
  - Extension of Exposures
  - Parallelogram Method
- **Premium trend**: the average premium level also changes due to
  - Inflationary pressure (e.g., inflation-sensitive exposures such as payroll)
  - Characteristics of the policies written (i.e., distributional changes; e.g., acquiring new business, moving to higher deductibles)
  - *Premium trend* will not be covered; see WM Ch. 5 for details.

Adjustments to Losses

To project future losses, we generally need to:

- Remove extraordinary events (e.g., individual shock losses and catastrophe losses) from historical losses and replace them with a provision more in line with long-term expectations.
- Develop immature losses to their ultimate settlement level.
  - Loss development triangle method.
- **Loss trend factors**:
  - Driving factors: monetary inflation, increasing medical costs, advancements in safety technology, social influences (e.g., court practices), and distributional changes.
  - Determine the 1) **annual trend** and 2) **trend period**.

Annual Loss Trend

- Measure trend in *loss costs*:
  - **Least squares** curve fitting: linear vs. nonlinear (e.g., exponential).
  - **Regression analysis** (includes the above as a special case).
  - May directly apply to loss costs or severity/frequency separately.
- Trend selection:
  - The trend factor should not be determined purely from a "black box" mathematical tool.
  - Professional actuarial judgment is necessary.

Example: Exponential Curve Fitting

Accident Year	X **= Year − 2008**	**Loss Cost (Pure Premium)**	ln**(Loss Cost)**
2008	0	119.39	4.782
2009	1	133.97	4.897
2010	2	129.89	4.867
2011	3	158.57	5.066
2012	4	188.72	5.240

- Fit a straight line to ln(Loss Cost) (as Y)

$$Y = 4.7534 + 0.1085X$$

- 0.1085 may be chosen as the *exponential growth* factor δ.
- The projected loss cost may be calculated as

$$\text{Projected Loss Cost} = (\text{Experience Loss Cost}) \cdot e^{\delta t}$$

Experience Periods and LAE

- **Calendar year (CY)**: transactions (payments and reserve changes) made during the year, whenever the accident happened or the policy was written.
- **Accident year (AY)**: losses from accidents that occur during the year.
- **Policy year (PY)**: losses on policies written during the year. With annual policies, a policy year's accidents span two calendar years.
- **Loss adjustment expenses (LAE)**: costs of investigating and settling claims, both claim-specific (allocated, e.g., legal fees) and general (unallocated, e.g., claims department overhead). Losses are often stated including LAE (L&LAE).

Trend Period

- Loss **trend period**: time between the **average accident/loss occurrence date** of the experience period and the forecasting period.
- It depends on
  - whether the experience period is accident year or policy year.
  - both the term of the policy and the expected duration for the new rates

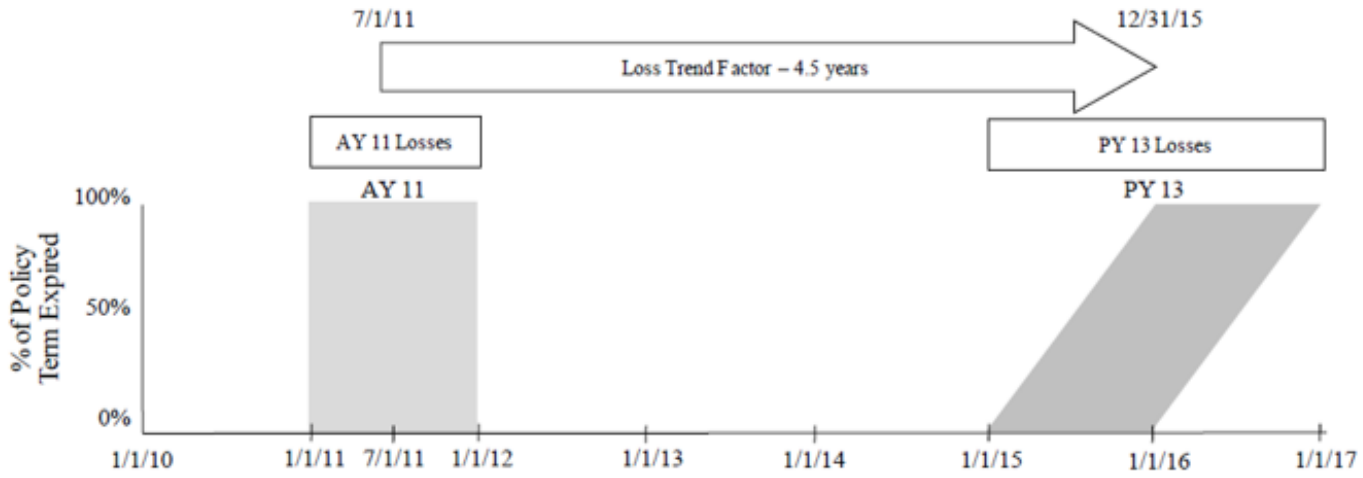
Example

Determine the trend factor for Accident Year and Policy Year 2011 losses, respectively, if the proposed new rate will be in effect for one year starting from January 1, 2015; the selected *annual* pure premium trend is $+1\%$; and the term of the policy is 1) annual and 2) 6-month. [AY: $1.01^{4.5}$, $1.01^{4.25}$; PY: 1.01^{4} for both]

AY Experience Period: Annual Policy

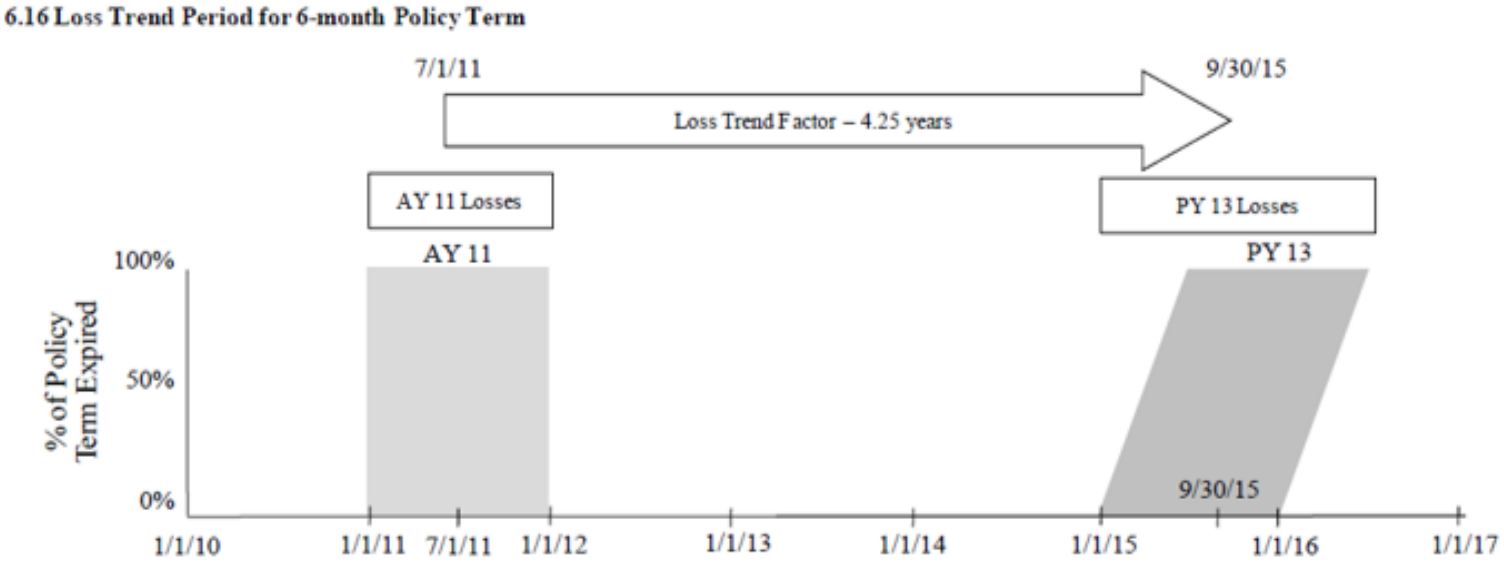
- Trend from 7/1/2011 to 12/31/2015.
- Trend period is 4.5 years.
- The trend factor is $(1 + 1\%)^{4.5}$.

6.15 Loss Trend Period for 12-month Policy Term



AY Experience Period: 6-Month Policy

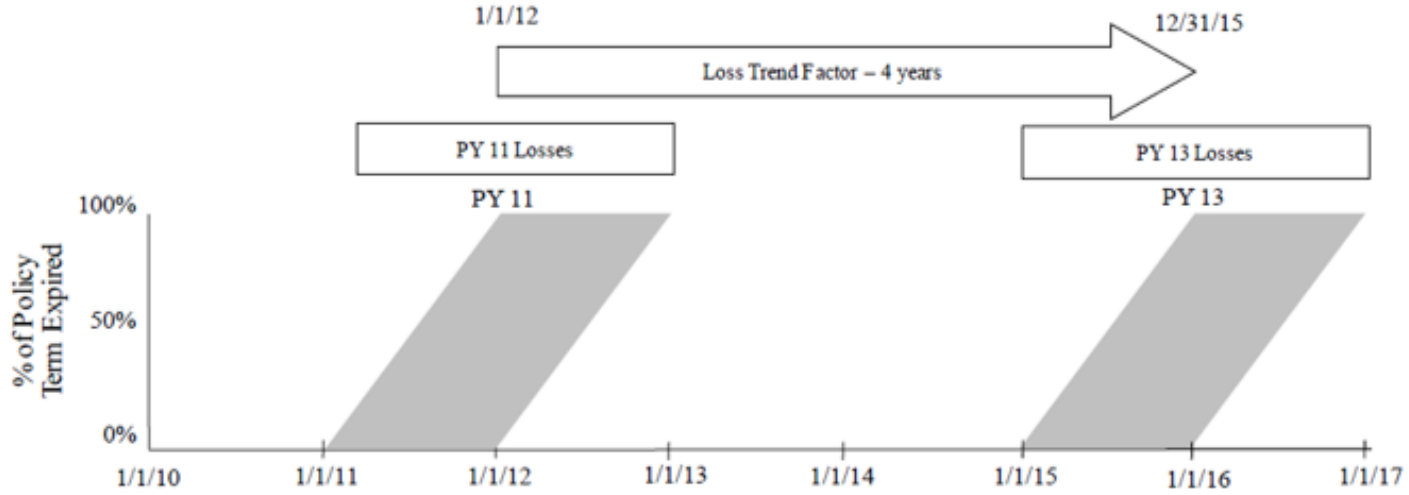
- Trend from 7/1/2011 to 9/30/2015.
- Trend period is 4.25 years.
- The trend factor is $(1 + 1\%)^{4.25}$.



PY Experience Period: Annual Policy

- Trend from 1/1/2012 to 12/31/2015.
- Trend period is 4 years.
- The trend factor is $(1 + 1\%)^4$.

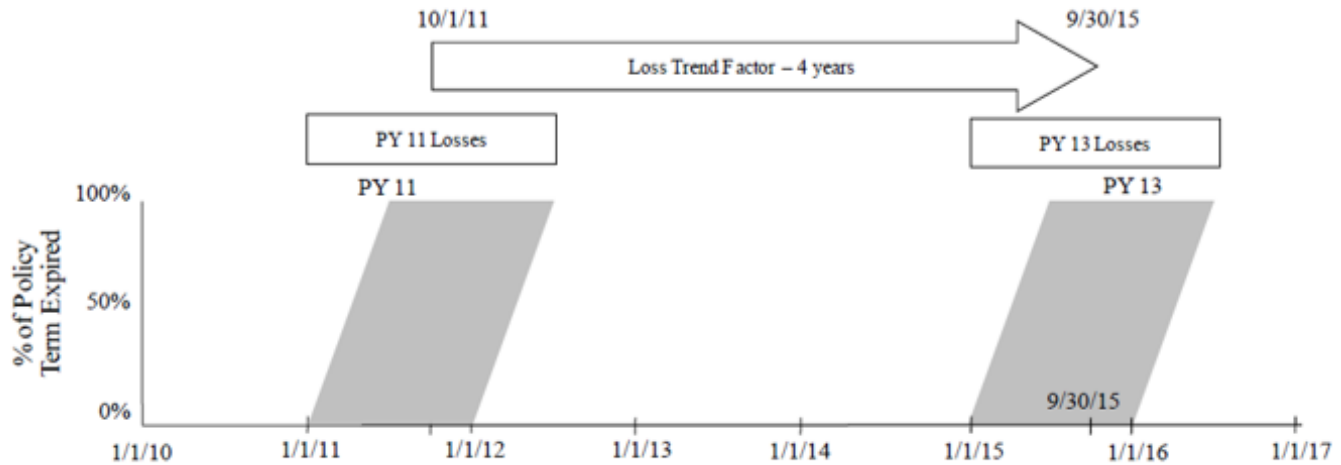
6.17 Loss Trend Period for 12-month Policy Term and PY experience period



PY Experience Period: 6-Month Policy

- Trend from 10/1/2011 to 9/30/2015.
- Trend period is 4 years.
- The trend factor is $(1 + 1\%)^4$.

6.18 Loss Trend Period for 6-month Policy Term and PY experience period



Expenses and Profit Provisions

- LAE projection: ALAE is often included with the loss projection, while ULAE needs special treatment.
- Underwriting expenses: variable V vs. fixed F
- Underwriting profit Q_T: achieve the target rate of return after considering investment income.
- **Permissible Loss Ratio**: percentage of each premium dollar intended to pay for the projected loss and LAE
  - **Variable Permissible Loss Ratio** (VPLR): $1 - V - Q_T$.
  - **(Total) Permissible Loss Ratio** (PLR): $1 - (F + V) - Q_T$.
  - VPLR and PLR are the same if all expenses are considered as variable.

Rate Changes

- Three major steps for ratemaking:
  - Determine the average, or overall, rate change required.
  - Decide on changes required in the differentials that apply to the rate classification parameters (e.g., class or territory).
  - Determine the new base rate/balance back (i.e., adjust the results so that the overall change in premium income is actually the one needed).
- We focus on the overall rating indications:
  - Pure Premium/Loss Cost Method
  - Loss Ratio Method

Pure Premium Method

- Recall the Fundamental Insurance Equation

$$\overline{P} = \left(\overline{L} + \overline{E}_L\right) + \left(\overline{E}_F + V \cdot \overline{P}\right) + Q_T \cdot \overline{P}$$

where

- $\overline{P}$: average premium/gross rate (per exposure)
- $\overline{L}$, $\overline{E}_L$ and $\overline{E}_F$: losses, LAE, and fixed expenses per exposure
- V and Q_T: (percentage) variable expenses and UW profit

Pure Premium (Loss Cost) Method: Indicated Average Rate

$$\overline{P}_I = \frac{\left(\overline{L} + \overline{E}_L\right) + \overline{E}_F}{1 - V - Q_T} = \frac{\left(\overline{L} + \overline{E}_L\right) + \overline{E}_F}{VPLR}$$

Example: Pure Premium Method

Example

You are given the following information:

Projected Reported Losses (incl. LAE)	30,000,000
Earned Exposure Units	1,000,000
Earned Premium at Current Rate Level	45,000,000
Present Average Manual Rate	45
Fixed Expenses	5,000,000
Variable Permissible Loss Ratio	0.75

Calculate the new average gross rate using the pure premium method. [46.67]

Loss Ratio Method

Loss Ratio Method: Indicated Rate Change

$$\frac{\overline{P}_I}{\overline{P}_C} - 1 = \frac{\frac{L+E_L}{P_C} + \frac{\overline{E}_F}{\overline{P}_C}}{1 - V - Q_T} - 1 = \frac{\frac{L+E_L}{P_C} + F}{VPLR} - 1$$

- P_C: projected **total** premium at current rate level; $\overline{P}_C = P_C/(\text{exposures})$: projected **average** premium per exposure (the bar means "per exposure," as on the previous slide).
- L and E_L: projected total losses and LAE.
- $\overline{E}_F$: fixed expenses per exposure, so $F = \overline{E}_F/\overline{P}_C$ is the fixed expense ratio.
- When fixed expenses are all treated as variable ($F = 0$),

$$\frac{\overline{P}_I}{\overline{P}_C} - 1 = \frac{(L + E_L)/P_C}{PLR} - 1$$

Example: Loss Ratio Method

Example

You are given the following information:

Projected Reported Losses (incl. LAE)	30,000,000
Earned Exposure Units	1,000,000
Earned Premium at Current Rate Level	45,000,000
Present Average Manual Rate	45
Fixed Expenses	5,000,000
Variable Permissible Loss Ratio	0.75

Calculate the new average gross rate using the loss ratio method.
[+3.70%, giving 46.67]

(Exercise: ASTAM F23 Q5(c)-(e))

Summary: Pure Premium vs. Loss Ratio

- The two approaches are equivalent, although the loss ratio method is more commonly used.
- Major differences:
  - Underlying measure
    - PP/LC: pure premium statistics, require *defined exposures*
    - LR: loss ratios, require *premium at current rate level*
    - **Implications**: PP is preferable if premium is not available (e.g., new company) or difficult to calculate at current rate level (e.g., personal auto with multiple rating variables); LR is preferable if exposure data is not available or there are no clearly defined exposures (e.g., commercial general liability)
  - Output: indicated rate vs. indicated rate change

Indicated Rate Differentials

- The previous section outlines the methods for determining the indicated overall average rate change; however, the ultimate goal is to determine new rates for *each rating class*.
- In practice, one specific risk class may be chosen as the base cell, normally the one with the largest amount of exposure.
- The rate for the base cell is referred to as the **base rate**.
- Other rate cells will be defined by a variety of risk classification variables (i.e., class, territory, etc.). For each variable, there will be a vector of **differentials** (or relativities), with the base characteristic assigned a differential of 1.
- Our objective is to determine the required differential changes.

Rate Differentials: Methods

- There are two different methods for determining the new relativity.
  - Loss Ratio Method:

$$\text{Indicated Relativity}_i = \text{Current Relativity}_i \frac{\text{LR}_i}{\text{LR}_{\text{base}}}$$

  - (Adjusted) Pure Premium Method:

$$\text{Indicated Relativity}_i = \frac{\text{Adj. Pure Premium}_i}{\text{Adj. Pure Premium}_{\text{base}}}$$

  - The pure premium should be exposure weighted, unless the distribution of all cross variables in any cell is *homogeneous*.

$$\text{Adj. Pure Premium}_i = \frac{L_i}{\text{Avg Relativity}_i \cdot \text{Exposure}_i}$$

    where L_i is the (developed, trended) loss for level i of the variable being analyzed, and Avg Relativity_i is the exposure-weighted average relativity of the *other* rating variables among the exposures in level i. Dividing by it removes their effect.

Example: New Class Differentials

Example

Determine new differentials for Class B and Class C, given the following information and assuming full credibility in all classes.

Class	Existing Differential	Experience Period Loss Ratio at Current Rates	Experience Period Loss Cost
A	1.00	0.65	129
B	0.85	0.71	120
C	1.21	0.66	157

[Loss ratio method: B 0.929, C 1.229; pure premium method: B 0.930, C 1.217]

New Base Rate

- While we separately determined the overall rate change and new relativities, this two-stage process can result in **off-balance**.
- We need to further adjust the base rate such that under the new relativities, the overall rate change is exactly as indicated.
- The new base rate can be determined using
  - Current Base Rate $\times$ Overall Rate Change/Off-Balance Factor:

$$\text{Off-Balance Factor} = \frac{\text{Indicated Avg Relativities}}{\text{Current Avg Relativities}}$$

  - Indicated (New) Overall Rate/Indicated Average Relativities
- The average relativities are **exposure-weighted** (volume-weighted) averages over all rating cells, not straight averages of the relativities.

Example: New Base Rate

Example

Assume that the overall rate change is +10% and the existing base (Class A) rate is 100. The class differentials (relativities) are

Class	Existing Differential	Proposed Differential	Earned Exposures
A	1.00	1.00	410
B	0.85	0.93	395
C	1.21	1.23	195

Determine the new base rate. [106.16]

Example: ASTAM Spring 2023 Q5(a)(b)

Example

You are conducting a ratemaking analysis for a line of business that is sold in only two geographical areas, Territory A and Territory B. You are given the following information:

(i)

Territory	Age Group	Calendar Year 2022 Earned Exposures	Accident Year 2022 Losses as of December 31, 2022
A	1	5,000	600,000
A	2	2,200	242,000
B	1	3,100	434,000
B	2	1,300	169,000

Territory	Current Differential
A	1.00
B	1.20

Age Group	Current Differential
1	1.00
2	0.95

Example: ASTAM Spring 2023 Q5(a)(b) (cont.)

Example

(ii) The current base rate for Territory A and Age Group 1 is 200.

(iii) Age Group differentials will not change.

(a) Show that the new differential for Territory B is 1.17 to the nearest 0.01, using the loss ratio method. You should calculate the value to the nearest 0.001. [1.171]

(b) State why losses do not need to be developed to ultimate values when calculating indicated differentials in part (a). [development affects both territories alike, so it cancels]

(Exercises: ASTAM Sample Q2 & S24 Q2(a)-(d), S25 Q2(a)-(c))

Example: ASTAM Spring 2023 Q5(c)(d)

Example

The company has decided to implement the following new territory differentials:

Territory	New Differential
A	1.00
B	1.15

(c) Calculate the new Territory A, Age Group 1 base rate needed to give an overall rate increase of 7%. [217.84]

(d) (i) Calculate the percentage increase in premiums in Territory B. [4.38%]

(ii) State with reasons whether you would expect the proportion of policies sold in Territory A to increase, decrease, or stay the same, as a result of the rate changes. [decrease: Territory A rises 8.92%]

(Exercises: ASTAM S24 Q2(e) & S25 Q2(d))