1. A policy has a negative binomial claim count distribution with mean 300 and variance 800. Ground-up severity is given by the following table:

Severity	Probability
40	0.25
80	0.25
120	0.25
200	0.25

You expect severity to increase 50% with no change in frequency. You decide to impose a per claim deductible of 100.

(a) Calculate the expected total claim payment after these changes.

(b) Calculate the standard deviation of the total claim payment after these changes.

2. Losses in a certain line of business follow an exponential distribution with a mean of 100. Currently the policies have no deductibles, limits, or co-insurance rates. Next year, the company is expecting uniform inflation of 15%. Define a policy to go into effect after the inflation which will reduce the expected value of the per-loss random variable to pre-inflation levels

(a) using only an ordinary deductible.

(b) using only a limit.

(c) using only co-insurance.

3. The number of claims experienced by a policyholder is assumed to follow a Poisson distribution with mean λ. For 2025, the book of business had the following claim experience:

No. of Claims	Policyholders
0	4000
1	1000
2	150
3	25
4+	0

(a) Find the maximum likelihood estimate of λ.

(b) Using the groups 0, 1, 2, and 3 or more, perform a chi-square goodness-of-fit test at the 5% significance level. The 95th percentile of the chi-square distribution with 2 degrees of freedom is 5.991.

(c) Suggest a frequency model that may fit these data better, and explain why.

4. A loss is modeled as a continuous spliced distribution as follows:

$$f(x) = \begin{cases} c_1 \exp(-x/100) & \text{for } 0 < x < 100 \\ c_2 \exp(-x/200) & \text{for } x \geq 100 \end{cases}$$

where the density is continuous at 100.

(a) Calculate c_1 and c_2.

(b) Calculate the expected loss.

(c) Calculate the variance of the loss.

(d) Calculate the loss elimination ratio for an ordinary deductible of 100.

5. X is modeled as an inverse exponential random variable with density

$$f(x|\theta) = \frac{1}{x^2}\theta e^{-\theta/x}, \qquad x > 0,$$

where θ has an exponential distribution with mean 4.

(a) Derive $f(x)$, and identify the distribution.

(b) Calculate $\mathbb{E}[X \wedge u]$.

(c) Does $\mathbb{E}[X]$ exist? Classify the tail of X, and state the maximum domain of attraction of X.

6. A reinsurer models large losses using extreme value theory. In a sample of 1000 losses, 60 exceed the threshold $d = 50$. The excess over the threshold, $Y = X - 50 \mid X > 50$, is modeled with a generalized Pareto distribution with $\xi = 0.4$ and $\beta = 20$.

(a) Estimate $\Pr(X > 150)$.

(b) Estimate the 99% VaR of X.

(c) Estimate the 99% Expected Shortfall of X.

(d) State the maximum domain of attraction of X, and which moments of X are finite.

7. The annual number of claims for a policyholder is Poisson with mean λ. The prior distribution of λ is gamma with $\alpha = 2$ and $\theta = 0.3$. A policyholder has 2, 0, and 1 claims in the last three years.

(a) Determine the posterior distribution of λ.

(b) Calculate the Bayesian premium (the expected number of claims next year).

(c) Calculate the probability that the policyholder has no claims next year.

(d) Calculate the Bühlmann credibility premium, and explain why it equals your answer to (b).

(e) Calculate the Bayes estimate of λ under zero-one loss, and a 95% equal-tailed credibility interval for λ. (If Λ is gamma with integer α, then $2\Lambda/\theta$ is chi-square with 2α degrees of freedom; the 2.5th and 97.5th percentiles of the chi-square distribution with 10 degrees of freedom are 3.247 and 20.483.)

8. In one year, 200 policyholders had the following numbers of claims:

Number of Claims	Number of Policyholders
0	120
1	60
2	15
3	5

Each policyholder's number of claims is assumed to be Poisson. Use semiparametric empirical Bayes estimation.

(a) Estimate μ, v, and a.

(b) Calculate the credibility premium for a policyholder who had 2 claims.

9. You are given the following experience for two groups:

	Year 1		Year 2	
Group	Members	Average Loss	Members	Average Loss
1	50	200	70	230
2	30	300	50	250

Use nonparametric empirical Bayes estimation (Bühlmann-Straub).

(a) Estimate the expected process variance v and the variance of the hypothetical means a.

(b) Calculate the credibility premium per member for Group 1.

10. New rates will be in effect for one year beginning July 1, 2027, and all policies are annual. You are given the following for Accident Year 2025:

Earned exposures	10,000
Earned premium at current rate level	6,000,000
Reported losses and LAE as of 12/31/2025	3,200,000
Loss development factor to ultimate	1.30
Projected fixed expenses	400,000
Variable expenses (% of premium)	15%
Underwriting profit (% of premium)	5%
Annual loss trend	5%

(a) Determine the trend period and the projected ultimate losses and LAE.

(b) Calculate the indicated overall rate change using the loss ratio method.

(c) Calculate the indicated average rate using the pure premium method, and confirm it agrees with (b).

11. An insurer writes business in two territories. Territory A is the base territory. Losses have been developed and trended to the new rate period.

Territory	Current Relativity	Earned Exposures	Losses
A	1.00	6000	1,800,000
B	1.25	4000	1,700,000

The current base rate is 500.

(a) Calculate the indicated relativity for Territory B using the loss ratio method.

(b) The insurer adopts the indicated relativity and an overall rate increase of 8%. Calculate the off-balance factor and the new base rate.

(c) Calculate the percentage premium change in each territory.

12. Given the following cumulative paid loss triangle (claims are fully developed at 60 months):

Accident Year	12M	24M	36M	48M	60M
2014	3575	4400	5500	6000	6200
2015	4200	5200	6500	7100	
2016	4875	7000	8200		c
2017	5600	7500	b		
2018	6000			a	

(a) Using the chain ladder method with volume-weighted development factors, estimate the values a, b, and c.

(b) Estimate the total outstanding claims.

(c) The earned premium for AY 2018 is 12,000 and the expected loss ratio is 65%. Calculate the outstanding claims for AY 2018 using (i) the expected loss ratio method and (ii) the Bornhuetter-Ferguson method.

(d) Explain why the Bornhuetter-Ferguson estimate is often preferred for the most recent accident year.

13. An insurer records the cumulative number of settled claims and the incremental claim payments for a portfolio. All claims are settled by the end of Development Year (DY) 2.

Cumulative Settled Claims

AY	DY 0	DY 1	DY 2
0	100	150	160
1	120	175	
2	130		

Incremental Claim Payments

AY	DY 0	DY 1	DY 2
0	30,000	75,000	40,000
1	37,200	90,000	
2	41,600		

(a) Using the chain ladder method, project the number of claims settled in each future development year.

(b) Calculate the average incremental payment per settled claim in each development year.

(c) Estimate the total outstanding claims using the frequency-severity method.

(d) Estimate the total outstanding claims by applying the chain ladder method to the cumulative payments, and compare.

(e) Give one advantage of the frequency-severity method over the chain ladder method.

14. Mack's model is applied to the following cumulative claims triangle:

AY	DY 0	DY 1	DY 2	DY 3
0	1000	1500	1650	1700
1	1100	1700	1850	
2	1200	1750		
3	1300			

(a) Calculate the chain ladder estimate of the outstanding claims for AY 3.

(b) Calculate $\hat{\sigma}_0^2$, $\hat{\sigma}_1^2$, and $\hat{\sigma}_2^2$.

(c) Calculate the process standard deviation of the AY 3 outstanding claims.

(d) Calculate the square root of the estimation error, and the root mean square error of prediction, for AY 3.

(e) Under the Poisson model fitted by maximum likelihood, state $\hat{\mu}_3$ and $\hat{\gamma}_0$.

(f) Now use the overdispersed Poisson model. Estimate the dispersion parameter ϕ using Pearson residuals, and calculate the process standard deviation of the AY 3 outstanding claims. Compare your answer with (c).

15. You are given the following cumulative claims triangle. Claims are fully developed by the end of Development Year (DY) 2. You use the Bühlmann-Straub credibility model to estimate the outstanding claims.

AY	0	1	2
0	300	500	540
1	360	520	
2	250		

(a) Calculate the chain ladder estimates of $\hat{\gamma}_j$ and of the ultimate claims for each accident year.

(b) Calculate $\hat{v}$.

(c) Calculate $\hat{a}$.

(d) Calculate the credibility factors Z_0, Z_1, Z_2 and $\hat{\mu}$.

(e) Calculate the Bühlmann-Straub estimates of the outstanding claims for AY 1 and AY 2, and compare them with the chain ladder estimates.

(f) Give one advantage of the Bühlmann-Straub approach over the Bornhuetter-Ferguson method.