1. A policy has a negative binomial claim count distribution with mean 300 and variance 800. Ground-up severity is given by the following table:

Severity	Probability
40	0.25
80	0.25
120	0.25
200	0.25

You expect severity to increase 50% with no change in frequency. You decide to impose a per claim deductible of 100.

(a) Calculate the expected total claim payment after these changes.

Solution: Inflated losses are 60, 120, 180, 300, so the payments per loss are 0, 20, 80, 200, with mean $\mathbb{E}[Y^L] = 75$. The expected total payment is $300(75) = 22{,}500$.

(b) Calculate the standard deviation of the total claim payment after these changes.

Solution: $\mathbb{E}[(Y^L)^2] = 0.25(0 + 400 + 6400 + 40{,}000) = 11{,}700$, so $\text{Var}(Y^L) = 11{,}700 - 75^2 = 6075$. Then

$$\begin{aligned}\text{Var}(S) &= \mathbb{E}[N]\text{Var}(Y^L) + \text{Var}(N)\,\mathbb{E}[Y^L]^2 \\ &= 300(6075) + 800(5625) = 6{,}322{,}500,\end{aligned}$$

so the standard deviation is 2514.5.

2. Losses in a certain line of business follow an exponential distribution with a mean of 100. Currently the policies have no deductibles, limits, or co-insurance rates. Next year, the company is expecting uniform inflation of 15%. Define a policy to go into effect after the inflation which will reduce the expected value of the per-loss random variable to pre-inflation levels

(a) using only an ordinary deductible.

Solution: After inflation, losses are exponential with mean 115. Solve $115e^{-d/115} = 100$: $d = 115 \ln 1.15 = 16.07$.

(b) using only a limit.

Solution: Solve $\mathbb{E}[X \wedge u] = 115(1 - e^{-u/115}) = 100$: $u = 115 \ln(115/15) = 234.24$.

(c) using only co-insurance.

Solution: $\alpha(115) = 100$ gives $\alpha = 0.8696$.

3. The number of claims experienced by a policyholder is assumed to follow a Poisson distribution with mean λ. For 2025, the book of business had the following claim experience:

No. of Claims	Policyholders
0	4000
1	1000
2	150
3	25
4+	0

(a) Find the maximum likelihood estimate of λ.

Solution: $\hat{\lambda} = \bar{x} = \frac{1000+300+75}{5175} = 0.26570$.

(b) Using the groups 0, 1, 2, and 3 or more, perform a chi-square goodness-of-fit test at the 5% significance level. The 95th percentile of the chi-square distribution with 2 degrees of freedom is 5.991.

Solution: Expected counts $5175\,p_k$: 3967.51, 1054.17, 140.05, 13.27 (all at least 5).

$$\chi^2 = \frac{(4000-3967.51)^2}{3967.51} + \frac{(1000-1054.17)^2}{1054.17} + \frac{(150-140.05)^2}{140.05} + \frac{(25-13.27)^2}{13.27} = 14.12.$$

With $4-1-1=2$ degrees of freedom, $14.12 > 5.991$, so we reject the Poisson model.

(c) Suggest a frequency model that may fit these data better, and explain why.

Solution: A negative binomial: the sample variance (0.282) exceeds the sample mean (0.266), and there are more policyholders with 3 claims than the Poisson predicts.

4. A loss is modeled as a continuous spliced distribution as follows:

$$f(x) = \begin{cases} c_1 \exp(-x/100) & \text{for } 0 < x < 100 \\ c_2 \exp(-x/200) & \text{for } x \geq 100 \end{cases}$$

where the density is continuous at 100.

(a) Calculate c_1 and c_2.

Solution: Continuity: $c_1 e^{-1} = c_2 e^{-0.5}$, so $c_2 = c_1 e^{-0.5}$. Total probability: $c_1(100)(1-e^{-1}) + c_2(200)e^{-0.5} = 1$, i.e., $c_1(63.212 + 73.576) = 1$. So $c_1 = 0.0073106$ and $c_2 = 0.0044341$.

(b) Calculate the expected loss.

Solution: $\int_0^{100} x\, c_1 e^{-x/100}\,dx + \int_{100}^{\infty} x\, c_2 e^{-x/200}\,dx = 19.32 + 161.36 = 180.68$.

(c) Calculate the variance of the loss.

Solution: $\mathbb{E}[X^2] = 71{,}098.9$, so $\mathrm{Var}(X) = 71{,}098.9 - 180.68^2 = 38{,}452.7$.

(d) Calculate the loss elimination ratio for an ordinary deductible of 100.

Solution: $\Pr(X > 100) = c_2(200)e^{-0.5} = 0.53788$, so

$$\mathbb{E}[X \wedge 100] = 19.32 + 100(0.53788) = 73.11, \qquad \text{LER} = 73.11/180.68 = 0.4046.$$

5. X is modeled as an inverse exponential random variable with density

$$f(x|\theta) = \frac{1}{x^2}\theta e^{-\theta/x}, \qquad x > 0,$$

where θ has an exponential distribution with mean 4.

(a) Derive $f(x)$, and identify the distribution.

Solution:

$$f(x) = \int_0^\infty \frac{\theta}{x^2}e^{-\theta/x}\,\frac{1}{4}e^{-\theta/4}\,d\theta = \frac{1}{4x^2}\cdot\frac{1}{(1/x+1/4)^2} = \frac{4}{(x+4)^2},$$

a Pareto distribution with $\alpha = 1$ and $\theta = 4$.

(b) Calculate $\mathbb{E}[X \wedge u]$.

Solution: $\mathbb{E}[X \wedge u] = \int_0^u \frac{4}{x+4}\,dx = 4\ln\left(\frac{u+4}{4}\right)$.

(c) Does $\mathbb{E}[X]$ exist? Classify the tail of X, and state the maximum domain of attraction of X.

Solution: No: $\mathbb{E}[X \wedge u] \to \infty$ as $u \to \infty$. The tail is very heavy (a Pareto has finite kth moments only for $k < \alpha = 1$), and X is in the Fréchet MDA with $\xi = 1/\alpha = 1$.

6. A reinsurer models large losses using extreme value theory. In a sample of 1000 losses, 60 exceed the threshold $d = 50$. The excess over the threshold, $Y = X - 50 \mid X > 50$, is modeled with a generalized Pareto distribution with $\xi = 0.4$ and $\beta = 20$.

(a) Estimate $\Pr(X > 150)$.

Solution: $\Pr(X > x) = \overline{F}_X(d)\left(1 + \frac{\xi}{\beta}(x-d)\right)^{-1/\xi}$, with $\overline{F}_X(d) \approx 60/1000 = 0.06$:

$$\Pr(X > 150) = 0.06(1 + 0.02 \times 100)^{-2.5} = 0.06(3)^{-2.5} = 0.00385.$$

(b) Estimate the 99% VaR of X.

Solution: $Q_{0.99} = d + \frac{\beta}{\xi}\left[\left(\frac{\overline{F}_X(d)}{0.01}\right)^{\xi} - 1\right] = 50 + 50\left(6^{0.4} - 1\right) = 102.38.$

(c) Estimate the 99% Expected Shortfall of X.

Solution: $\text{ES}_{0.99} = \frac{Q_{0.99}+\beta-\xi d}{1-\xi} = \frac{102.38+20-20}{0.6} = 170.64$.

(d) State the maximum domain of attraction of X, and which moments of X are finite.

Solution: $\xi > 0$, so X is in the Fréchet MDA. Moments of order $k < 1/\xi = 2.5$ are finite, so the mean and variance exist but the third moment does not.

7. The annual number of claims for a policyholder is Poisson with mean λ. The prior distribution of λ is gamma with $\alpha = 2$ and $\theta = 0.3$. A policyholder has 2, 0, and 1 claims in the last three years.

(a) Determine the posterior distribution of λ.

Solution: Gamma with $\alpha^* = 2 + 3 = 5$ and $\theta^* = (1/0.3 + 3)^{-1} = 0.15789$.

(b) Calculate the Bayesian premium (the expected number of claims next year).

Solution: $\alpha^*\theta^* = 5(0.15789) = 0.7895$.

(c) Calculate the probability that the policyholder has no claims next year.

Solution: The predictive distribution is negative binomial with $r = 5$ and $\beta = 0.15789$: $(1.15789)^{-5} = 0.4805$.

(d) Calculate the Bühlmann credibility premium, and explain why it equals your answer to (b).

Solution: $\mu = v = \alpha\theta = 0.6$ and $a = \alpha\theta^2 = 0.18$, so $k = 3.333$ and $Z = \frac{3}{6.333} = 0.4737$. The premium is $0.4737(1) + 0.5263(0.6) = 0.7895$. The Poisson–gamma model is conjugate with a linear posterior mean, so Bühlmann credibility is exact.

(e) Calculate the Bayes estimate of λ under zero-one loss, and a 95% equal-tailed credibility interval for λ. (If Λ is gamma with integer α, then $2\Lambda/\theta$ is chi-square with 2α degrees of freedom; the 2.5th and 97.5th percentiles of the chi-square distribution with 10 degrees of freedom are 3.247 and 20.483.)

Solution: The posterior mode is $(\alpha^* - 1)\theta^* = 4(0.15789) = 0.6316$. Since $2\lambda/\theta^* = 12.667\lambda$ is chi-square with 10 degrees of freedom, the interval is

$$\left(\frac{3.247}{12.667},\ \frac{20.483}{12.667}\right) = (0.2563,\ 1.6171).$$

8. In one year, 200 policyholders had the following numbers of claims:

Number of Claims	Number of Policyholders
0	120
1	60
2	15
3	5

Each policyholder's number of claims is assumed to be Poisson. Use semiparametric empirical Bayes estimation.

(a) Estimate μ, v, and a.

Solution: $\hat{\mu} = \hat{v} = \bar{x} = 105/200 = 0.525$. The sample variance is

$$s^2 = \frac{165 - 200(0.525)^2}{199} = 0.552136, \qquad \text{so} \qquad \hat{a} = s^2 - \hat{v} = 0.027136.$$

(b) Calculate the credibility premium for a policyholder who had 2 claims.

Solution: $\hat{k} = 0.525/0.027136 = 19.347$, so $\hat{Z} = 1/(1 + 19.347) = 0.0491$ and the premium is $0.0491(2) + 0.9509(0.525) = 0.5975$.

9. You are given the following experience for two groups:

	Year 1		Year 2	
Group	Members	Average Loss	Members	Average Loss
1	50	200	70	230
2	30	300	50	250

Use nonparametric empirical Bayes estimation (Bühlmann-Straub).

(a) Estimate the expected process variance v and the variance of the hypothetical means a.

Solution: $\bar{X}_1 = 217.5$ ($m_1 = 120$), $\bar{X}_2 = 268.75$ ($m_2 = 80$), $\bar{X} = 238$.

$$\hat{v} = \frac{50(17.5^2) + 70(12.5^2) + 30(31.25^2) + 50(18.75^2)}{1 + 1} = 36{,}562.5,$$

$$\hat{a} = \frac{120(20.5^2) + 80(30.75^2) - \hat{v}(2 - 1)}{200 - (120^2 + 80^2)/200} = \frac{126{,}075 - 36{,}562.5}{96} = 932.42.$$

(b) Calculate the credibility premium per member for Group 1.

Solution: $\hat{k} = 39.212$, so $\hat{Z}_1 = \frac{120}{159.212} = 0.7537$ and $\hat{Z}_2 = \frac{80}{119.212} = 0.6711$. The credibility-weighted mean is $\hat{\mu} = \frac{0.7537(217.5) + 0.6711(268.75)}{0.7537 + 0.6711} = 241.64$, so

$$P_1 = 0.7537(217.5) + 0.2463(241.64) = 223.45.$$

10. New rates will be in effect for one year beginning July 1, 2027, and all policies are annual. You are given the following for Accident Year 2025:

Earned exposures	10,000
Earned premium at current rate level	6,000,000
Reported losses and LAE as of 12/31/2025	3,200,000
Loss development factor to ultimate	1.30
Projected fixed expenses	400,000
Variable expenses (% of premium)	15%
Underwriting profit (% of premium)	5%
Annual loss trend	5%

(a) Determine the trend period and the projected ultimate losses and LAE.

Solution: From the AY 2025 average accident date (7/1/2025) to the average accident date under the new rates (7/1/2028): 3 years.

$$\text{Projected losses} = 3{,}200{,}000(1.30)(1.05^3) = 4{,}815{,}720.$$

(b) Calculate the indicated overall rate change using the loss ratio method.

Solution: $\dfrac{4{,}815{,}720/6{,}000{,}000 + 400{,}000/6{,}000{,}000}{1 - 0.15 - 0.05} - 1 = \dfrac{0.80262 + 0.06667}{0.80} - 1 = +8.66\%$.

(c) Calculate the indicated average rate using the pure premium method, and confirm it agrees with (b).

Solution: $\overline{P}_I = \dfrac{481.572 + 40}{0.80} = 651.97$. The current average premium is 600, and $651.97/600 - 1 = +8.66\%$.

11. An insurer writes business in two territories. Territory A is the base territory. Losses have been developed and trended to the new rate period.

Territory	Current Relativity	Earned Exposures	Losses
A	1.00	6000	1,800,000
B	1.25	4000	1,700,000

The current base rate is 500.

(a) Calculate the indicated relativity for Territory B using the loss ratio method.

Solution: Premium at current rates: A 500(6000) = 3,000,000, B 625(4000) = 2,500,000. Loss ratios 0.60 and 0.68, so the new relativity for B is 1.25(0.68/0.60) = 1.4167.

(b) The insurer adopts the indicated relativity and an overall rate increase of 8%. Calculate the off-balance factor and the new base rate.

Solution: Exposure-weighted average relativities:

$$\text{current} = \frac{6000 + 1.25(4000)}{10{,}000} = 1.1, \qquad \text{new} = \frac{6000 + 1.4167(4000)}{10{,}000} = 1.16667.$$

Off-balance factor = 1.16667/1.1 = 1.0606, and the new base rate is 500(1.08)/1.0606 = 509.14.

(c) Calculate the percentage premium change in each territory.

Solution: A: 509.14/500 − 1 = +1.83%. B: 509.14(1.4167) = 721.29 versus 625, i.e., +15.41%.

12. Given the following cumulative paid loss triangle (claims are fully developed at 60 months):

Accident Year	12M	24M	36M	48M	60M
2014	3575	4400	5500	6000	6200
2015	4200	5200	6500	7100	
2016	4875	7000	8200		c
2017	5600	7500	b		
2018	6000			a	

(a) Using the chain ladder method with volume-weighted development factors, estimate the values a, b, and c.

Solution: $\hat{f}$ = 24100/18250 = 1.32055, 20200/16600 = 1.21687, 13100/12000 = 1.09167, 6200/6000 = 1.03333.

$$a = 6000(1.32055)(1.21687)(1.09167) = 10{,}525.4,$$
$$b = 7500(1.21687) = 9126.5,$$
$$c = 8200(1.09167)(1.03333) = 9250.1.$$

(b) Estimate the total outstanding claims.

Solution: Ultimates: 7336.7, 9250.1, 10,295.2, 10,876.3 for AY 2015–2018, so the reserves are 236.7 + 1050.1 + 2795.2 + 4876.3 = 8958.2.

(c) The earned premium for AY 2018 is 12,000 and the expected loss ratio is 65%. Calculate the outstanding claims for AY 2018 using (i) the expected loss ratio method and (ii) the Bornhuetter-Ferguson method.

Solution: The prior ultimate is $\mu = 0.65(12{,}000) = 7800$. (i) ELR: 7800 − 6000 = 1800. (ii) $\hat{\lambda}_0 = 1.81271$, so $\hat{\beta}_0 = 0.55166$ and the BF reserve is $(1 - 0.55166)(7800) = 3497.0$.

(d) Explain why the Bornhuetter-Ferguson estimate is often preferred for the most recent accident year.

Solution: For an immature year, little of the ultimate has emerged ($\hat{\beta}_0 = 0.55$), and the chain ladder multiplies any randomness in the first value by a large factor. BF gives credibility $\hat{\beta}_0$ to the data and $1 - \hat{\beta}_0$ to the prior, so it is more stable.

13. An insurer records the cumulative number of settled claims and the incremental claim payments for a portfolio. All claims are settled by the end of Development Year (DY) 2.

Cumulative Settled Claims

AY	DY 0	DY 1	DY 2
0	100	150	160
1	120	175	
2	130		

Incremental Claim Payments

AY	DY 0	DY 1	DY 2
0	30,000	75,000	40,000
1	37,200	90,000	
2	41,600		

(a) Using the chain ladder method, project the number of claims settled in each future development year.

Solution: $\hat{f}_0 = \frac{150+175}{100+120} = 1.47727$ and $\hat{f}_1 = \frac{160}{150} = 1.06667$. Projected cumulative counts: AY 1 DY 2: $175(1.06667) = 186.667$; AY 2 DY 1: $130(1.47727) = 192.045$, DY 2: $192.045(1.06667) = 204.848$. Future settlements: AY 1: 11.667 (DY 2); AY 2: 62.045 (DY 1) and 12.803 (DY 2).

(b) Calculate the average incremental payment per settled claim in each development year.

Solution: Observed settlements by DY: AY 0: 100, 50, 10; AY 1: 120, 55; AY 2: 130.

$$\hat{s}_0 = \frac{108{,}800}{350} = 310.86, \qquad \hat{s}_1 = \frac{165{,}000}{105} = 1571.43, \qquad \hat{s}_2 = \frac{40{,}000}{10} = 4000.$$

(c) Estimate the total outstanding claims using the frequency-severity method.

Solution: AY 1: $11.667(4000) = 46{,}667$. AY 2: $62.045(1571.43) + 12.803(4000) = 97{,}500 + 51{,}212 = 148{,}712$. Total: 195,379.

(d) Estimate the total outstanding claims by applying the chain ladder method to the cumulative payments, and compare.

Solution: Cumulative payments: AY 0: 30,000, 105,000, 145,000; AY 1: 37,200, 127,200; AY 2: 41,600. $\hat{f}_0 = 232{,}200/67{,}200 = 3.45536$ and $\hat{f}_1 = 145{,}000/105{,}000 = 1.38095$, so the reserves are

$$127{,}200(0.38095) + 41{,}600(3.45536 \times 1.38095 - 1) = 48{,}457 + 156{,}902 = 205{,}359,$$

about 5% higher than the frequency-severity estimate.

(e) Give one advantage of the frequency-severity method over the chain ladder method.

Solution: It separates the number of claims from their cost, so it shows whether changes come from claim frequency or severity, and severities can be adjusted directly for inflation or trend. It also uses the claim count information, which is often more stable than the payment amounts.

14. Mack's model is applied to the following cumulative claims triangle:

AY	DY 0	DY 1	DY 2	DY 3
0	1000	1500	1650	1700
1	1100	1700	1850	
2	1200	1750		
3	1300			

(a) Calculate the chain ladder estimate of the outstanding claims for AY 3.

Solution: $\hat{f}_0 = 4950/3300 = 1.5$, $\hat{f}_1 = 3500/3200 = 1.09375$, $\hat{f}_2 = 1700/1650 = 1.03030$.

$$\hat{C}_{3,3} = 1300(1.5)(1.09375)(1.03030) = 2197.44, \quad \text{so the reserve is } 897.44.$$

(b) Calculate $\hat{\sigma}_0^2$, $\hat{\sigma}_1^2$, and $\hat{\sigma}_2^2$.

Solution:

$$\hat{\sigma}_0^2 = \tfrac{1}{2}\left[1000(0)^2 + 1100(0.04545)^2 + 1200(0.04167)^2\right] = 2.1780,$$
$$\hat{\sigma}_1^2 = 1500(0.00625)^2 + 1700(0.00551)^2 = 0.11029,$$
$$\hat{\sigma}_2^2 = \min\{\hat{\sigma}_1^4/\hat{\sigma}_0^2,\ \hat{\sigma}_0^2,\ \hat{\sigma}_1^2\} = 0.005585.$$

(c) Calculate the process standard deviation of the AY 3 outstanding claims.

Solution: With $\hat{C}_{3,0} = 1300$, $\hat{C}_{3,1} = 1950$, $\hat{C}_{3,2} = 2132.81$:

$$2197.44^2\left[\frac{2.1780}{1.5^2(1300)} + \frac{0.11029}{1.09375^2(1950)} + \frac{0.005585}{1.03030^2(2132.81)}\right] = 3835.8,$$

so the process standard deviation is 61.93.

(d) Calculate the square root of the estimation error, and the root mean square error of prediction, for AY 3.

Solution: With $S_0 = 3300$, $S_1 = 3200$, $S_2 = 1650$, the estimation error is

$$2197.44^2 \sum_{j=0}^{2} \frac{\hat{\sigma}_j^2}{\hat{f}_j^2 S_j} = 1571.0,$$

with square root 39.64. The root MSEP is $\sqrt{3835.8 + 1571.0} = 73.53$.

(e) Under the Poisson model fitted by maximum likelihood, state $\hat{\mu}_3$ and $\hat{\gamma}_0$.

Solution: The Poisson MLEs reproduce the chain ladder: $\hat{\mu}_3 = 2197.44$ and $\hat{\gamma}_0 = \hat{\beta}_0 = 1/(1.5 \times 1.09375 \times 1.03030) = 0.5916$.

(f) Now use the overdispersed Poisson model. Estimate the dispersion parameter ϕ using Pearson residuals, and calculate the process standard deviation of the AY 3 outstanding claims. Compare your answer with (c).

Solution: The incremental claims are AY 0: 1000, 500, 150, 50; AY 1: 1100, 600, 150; AY 2: 1200, 550; AY 3: 1300. With $\hat{\gamma}_j = 0.5916,\ 0.2958,\ 0.0832,\ 0.0294$, the fitted values $\hat{\mu}_i\hat{\gamma}_j$ (Pearson residuals) are:

AY	0	1	2	3
0	1005.7 (−0.18)	502.9 (−0.13)	141.4 (0.72)	50.0 (0)
1	1127.6 (−0.82)	563.8 (1.52)	158.6 (−0.68)	
2	1166.7 (0.98)	583.3 (−1.38)		
3	1300.0 (0)			

$\sum r_{i,j}^2 = 6.888$ with $n - p = 10 - 7 = 3$, so $\hat{\phi} = 2.296$, and the process standard deviation is $\sqrt{2.296(897.44)} = 45.39$. This is smaller than Mack's estimate (61.93); the two models make different variance assumptions, and both are imprecise with so few data.

15. You are given the following cumulative claims triangle. Claims are fully developed by the end of Development Year (DY) 2. You use the Bühlmann-Straub credibility model to estimate the outstanding claims.

AY	0	1	2
0	300	500	540
1	360	520	
2	250		

(a) Calculate the chain ladder estimates of $\hat{\gamma}_j$ and of the ultimate claims for each accident year.

Solution: $\hat{f}_0 = \frac{500+520}{300+360} = 1.54545$ and $\hat{f}_1 = \frac{540}{500} = 1.08$, so $\hat{\beta}_0 = 0.59913$, $\hat{\beta}_1 = 0.92593$, $\hat{\beta}_2 = 1$, and $\hat{\gamma}_j = 0.5991,\ 0.3268,\ 0.0741$. Ultimates: 540, 561.6, 417.27.

(b) Calculate $\hat{v}$.

Solution: The ratios $X_{i,j}/\hat{\gamma}_j$ are 500.73, 612.00, 540.00 (AY 0) and 600.87, 489.60

(AY 1).

$$\hat{s}_0^2 = \tfrac{1}{2}\left[0.59913(500.73-540)^2 + 0.32680(612.00-540)^2\right] = 1309.1,$$
$$\hat{s}_1^2 = 0.59913(600.87-561.6)^2 + 0.32680(489.60-561.6)^2 = 2618.2,$$

so $\hat{v} = (1309.1 + 2618.2)/2 = 1963.6$.

(c) Calculate $\hat{a}$.

Solution: $\hat{\beta}_\bullet = 2.52505$ and $\bar{C} = (540+520+250)/2.52505 = 518.80$. The weighted sum of squares is $\sum_i \hat{\beta}_{2-i}(\hat{C}_{i,2} - \bar{C})^2 = 8321.3$, so

$$\hat{a} = \frac{8321.3 - 2(1963.6)}{2.52505 - (1 + 0.85734 + 0.35896)/2.52505} = \frac{4394.0}{1.64733} = 2667.3.$$

(d) Calculate the credibility factors Z_0, Z_1, Z_2 and $\hat{\mu}$.

Solution: $\hat{v}/\hat{a} = 0.73618$, so

$$Z_0 = \frac{1}{1.73618} = 0.5760, \qquad Z_1 = \frac{0.92593}{1.66211} = 0.5571, \qquad Z_2 = \frac{0.59913}{1.33531} = 0.4487,$$

and

$$\hat{\mu} = \frac{0.5760(540) + 0.5571(561.6) + 0.4487(417.27)}{1.5817} = 512.79.$$

(e) Calculate the Bühlmann-Straub estimates of the outstanding claims for AY 1 and AY 2, and compare them with the chain ladder estimates.

Solution:

$$C_{1,2}^{BS} = 0.5571(561.6) + 0.4429(512.79) = 539.98,$$
$$C_{2,2}^{BS} = 0.4487(417.27) + 0.5513(512.79) = 469.94,$$

so the reserves are 19.98 (AY 1) and 219.94 (AY 2), versus chain ladder reserves of 41.60 and 167.27. Each ultimate is pulled toward $\hat{\mu}$, most strongly for the least developed AY.

(f) Give one advantage of the Bühlmann-Straub approach over the Bornhuetter-Ferguson method.

Solution: It needs no external (subjective) prior: $\hat{\mu}$ is estimated from the triangle, and the credibility factors reflect both the within-AY and between-AY variability.