1. Claims follow an exponential distribution with a mean of 500. The policy has an ordinary deductible of 100.
   (a) Calculate the loss elimination ratio for this deductible.
   (b) Calculate the expected loss amount (the expected value of the per-loss random variable).
   (c) Now assume that the losses experience 10% inflation. What is the expected loss amount after inflation?
   (d) By what percent did the expected loss amount grow? Why is that value different than 10% (give two reasons)?
   (e) Calculate the loss elimination ratio after inflation, and explain why it changed.
   (f) Ignoring inflation, the insurer also offers a deductible of 250. Using the deductible of 100 as the base, calculate the deductible relativity for the 250 deductible.

2. Claims follow a Pareto distribution with $\alpha = 4$ and $\theta = 1000$. A policy has an ordinary deductible of 100, a maximum covered loss of 1500, and a coinsurance rate of 80%. Next year, losses are subject to 10% uniform inflation, and the policy terms do not change. For next year:
   (a) What is the most the company would pay on any single claim?
   (b) What is the expected value of the per-loss random variable?
   (c) What is the expected value of the per-payment random variable?
   (d) What is the expected percent of the total loss covered by the insurer?
   (e) Ignoring inflation, the company prices policies with no deductible and no coinsurance using a basic limit of 1500. Calculate the increased limit factor for a limit of 3000.

3. Suppose that the number of claims follows a zero-modified Poisson distribution with $\lambda = 2$. Under this distribution, the mean number of claims is 1.50.
   (a) Calculate the probability that the number of claims will be greater than one.
   (b) A deductible is introduced, and each claim now results in a payment with probability 0.4. Calculate the probability that there are no payments.

4. You have a portfolio of 100 independent policyholders. Each policyholder has at most one claim per year, and has a 20% chance of not having any claims for the year. Given that they do have a claim, claim amounts follow an exponential distribution with a mean of 1000.
   (a) Calculate the expected aggregate loss for this portfolio.
   (b) Calculate the variance of the aggregate loss.
   (c) Assume further that you knew that 80 of the policyholders would have a claim and the other 20 would not. Calculate the expected value and the variance of the aggregate loss in this situation.

5. You sell a single auto insurance policy. The number of claims on this policy is Poisson distributed with a mean of 2. The size of each claim follows a zero-truncated Poisson distribution with $\lambda = 0.8$. S is the aggregate claim amount for this policy.

   Calculate
   (a) $\Pr(S = 0)$
   (b) $\Pr(S = 3)$

(c) the net stop-loss premium with an aggregate deductible of 2.

6. You have a portfolio of 2000 calculator warranty policyholders (do not ever buy or sell these in real life). For each policyholder, the number of claims in a year is Poisson with mean 0.01, and each claim pays the calculator's replacement value. The replacement values differ as follows:

Replacement Value	Number of Policyholders
25	1000
35	800
75	200

   (a) Explain why the aggregate claims S have a compound Poisson distribution, and give the Poisson parameter and the claim amount distribution.

   (b) Calculate the expected value and variance of the aggregate claims.

   (c) Using the normal approximation, estimate the probability that the total claims will exceed 700.

7. The number of losses follows a Poisson distribution with mean 0.8. Loss amounts (in thousands) follow an exponential distribution with mean 3. You discretize the loss amounts with a span of $h = 1$.

   (a) Using local moment matching with one moment, calculate f_0, f_1, and f_2, the probabilities of the discretized loss amounts 0, 1, and 2.

   (b) Using the method of rounding, calculate f_0, f_1, and f_2.

   (c) Using the distribution from (a) and the recursive formula, calculate the probability that the discretized aggregate losses are no more than 2.

   (d) Which discretization preserves the mean loss? What is the mean of the discretized aggregate loss in (c)?

8. Assume that claim frequency (N) follows a Poisson(λ) distribution. You take a sample of 1000 policies and find 975 total claims.

   (a) Find the method of moments estimate of λ.

   (b) Find the maximum likelihood estimate of λ.

   (c) Estimate the variance of the MLE, and construct an approximate 95% confidence interval for λ.

   (d) Use the delta method to construct an approximate 95% confidence interval for $\Pr(N = 0)$.

9. Ground-up losses follow an exponential distribution with mean θ. A policy has an ordinary deductible of 200 and a maximum covered loss of 2000. Losses of 200 or less are not reported. The insurer recorded six ground-up losses:

$$350 \quad 600 \quad 900 \quad 1200 \quad 2000^+ \quad 2000^+$$

   where the last two losses exceeded 2000 (so the insurer paid the maximum).

   (a) Write the log-likelihood function for θ.

   (b) Find the maximum likelihood estimate of θ.

(c) Using the observed information, estimate the variance of $\hat{\theta}$ and construct an approximate 95% normal confidence interval for θ.

(d) The normal interval in (c) can include values close to zero. Construct an approximate 95% log-transformed confidence interval for θ.

(e) Construct an approximate 95% likelihood-based confidence interval for θ.

10. Losses follow an exponential distribution with mean θ. A random sample of 20 losses is distributed as follows:

Claim Size	Frequency
0–1000	13
1000–∞	7

(a) Calculate the maximum likelihood estimate of θ.

(b) Calculate the maximum likelihood estimate of $\Pr(X > 2000)$.

11. You observe the following five losses:

100 250 400 900 1500

You test H_0: the losses follow an exponential distribution with mean 600.

(a) Calculate the Kolmogorov-Smirnov test statistic.

(b) The critical value at the 5% significance level is $1.36/\sqrt{n}$. State the conclusion of the test.

12. Four models are fitted by maximum likelihood to 200 claim amounts:

Model	Number of Parameters	Log-likelihood
Exponential	1	−1520.4
Gamma	2	−1517.9
Weibull	2	−1518.6
Transformed Gamma	3	−1516.8

The 95th percentiles of the chi-square distribution with 1 and 2 degrees of freedom are 3.841 and 5.991.

(a) Use the likelihood ratio test at the 5% level to compare the exponential and gamma models.

(b) Use the likelihood ratio test at the 5% level to compare the gamma and transformed gamma models.

(c) Which model is selected by the Akaike Information Criterion?

(d) Which model is selected by the Schwarz Bayesian Criterion?

(e) Explain why the likelihood ratio test cannot be used to compare the gamma and Weibull models.