Stat 344
Life Contingencies I

Chapter 4: Life insurance

Review of (actuarial) interest theory — notation

- We use i to denote an annual effective rate of interest.
- The one year present value (discount) factor is denoted by $v = 1/(1+i)$.
- $i^{(m)}$ is an annual nominal rate of interest, convertible m times per year.
- The annual discount rate (a.k.a., interest rate in advance) is denoted by d.
- $d^{(m)}$ is an annual nominal rate of discount, convertible m times per year.
- The force of interest is denoted by δ (or δ_t if it varies with time).

Review of (actuarial) interest theory — relationships

To accumulate for n periods, we can multiply by any of the quantities below; to discount for n periods, we would divide by any of them.

n Period Accumulation Factors

$$(1+i)^n = \left(1+\frac{i^{(m)}}{m}\right)^{mn} = (1-d)^{-n} = \left(1-\frac{d^{(r)}}{r}\right)^{-nr} = e^{\delta n}$$

If the force of interest varies with time, we can discount from time n back to time 0 by multiplying by

$$e^{-\int_0^n \delta_t \, dt}$$

Valuation of life insurance benefits

The timing of life insurance benefits generally depends on the survival status of the insured individual. Since the future lifetime of the insured individual is a *random variable*, the present value of life insurance benefits will also be a *random variable*.

We'll commonly denote the random variable representing the PV of a life insurance benefit by Z.

- Unless otherwise specified, assume a benefit amount of \$1.

We're often interested in various properties (e.g., mean, variance) of Z.

- The mean of Z is referred to as the **expected value of the present value**, **expected present value (EPV)**, **actuarial present value (APV)**, or simply **actuarial value**.

Whole life insurance — Benefits paid at moment of death

The first type of life insurance we'll consider is **whole life** insurance.

- Consider the case where the benefit is paid at the moment of death (this is sometimes referred to as the **continuous** case).

For this case, the present value of the benefit is $Z = v^{T_x} = e^{-\delta T_x}$.

The corresponding EPV is denoted by $\overline{A}_x$

EPV for Whole Life Insurance — Continuous Case

$$\overline{A}_x = E[Z] = E\left[e^{-\delta T_x}\right] = \int_0^{\infty} e^{-\delta t} \, {}_t p_x \, \mu_{x+t} \, dt$$

We can calculate the second moment for Z similarly:

$$E\left[Z^2\right] = E\left[\left(e^{-\delta T_x}\right)^2\right] = \int_0^\infty e^{-(2\delta)t}\, {}_tp_x\, \mu_{x+t}\, dt$$

We can find this second moment by computing the expectation at twice the force of interest, 2δ. When we calculate the expectation at twice the force of interest, we denote it with the symbol ${}^2\overline{A}_x$.

- Then for this case, we have $E\left[Z^2\right] = {}^2\overline{A}_x$.

Then we can calculate the variance of Z:

$$V[Z] = E\left[Z^2\right] - E\left[Z\right]^2 = {}^2\overline{A}_x - \left(\overline{A}_x\right)^2$$

We may also be interested in various percentiles or functions of Z — all the usual rules of random variables apply.

Assume that a particular individual, currently age x, has a future lifetime described by a random variable with density

$$f_x(t) = \frac{1}{60} \quad \text{for} \quad 0 < t < 60$$

This person wants to purchase insurance that will provide a benefit of \$1 at the moment of death. Assuming a force of interest of $\delta = 0.06$: $\bar{A}_x = \int_0^{60} e^{-0.06t} \frac{1}{60} dt$ $\quad {}^2\bar{A}_x = \int_0^{60} e^{-0.12t} \frac{1}{60} dt$

EPV = 0.27
Var = 0.0659

1. Find the EPV and variance for this death benefit.
2. Find the minimum value H such that $P(Z \leq H) \geq 0.9$. 0.6977
3. Find the minimum single premium H that an insurance company must charge in order to be at least 90% certain that this premium will be adequate to fund the death claim, should their assets accumulate at $\delta = 0.06$. 0.6977

$\int_0^t \frac{1}{60} dt = 0.1$

$\frac{t}{60} = 0.1 \quad t = 6$

$e^{-\delta(6)} = 0.6977$

Whole life insurance — Benefits paid at end of the year of death

Next we consider a whole life insurance in which the death benefit is paid at the end of the year in which the insured dies (this is sometimes called the **annual** case).

K_x is the time corresponding to the beginning of the year of death; $K_x + 1$ is the end of the year of death.

Since the benefit is paid at the end of the year of death, the present value of the benefit is $Z = v^{K_x+1}$.

Then this is a *discrete* random variable.

- What does its pmf look like?

Whole life insurance — Benefits paid at end of the year of death

We can find the mean and variance of this random variable:

EPV and Variance for Whole Life Insurance — Annual Case

$$E[Z] = E\left[v^{K_x+1}\right] = \sum_{k=0}^{\infty} v^{k+1}\ {}_{k|}q_x = A_x$$

$$E[Z^2] = E\left[\left(v^{K_x+1}\right)^2\right] = \sum_{k=0}^{\infty} \left(v^2\right)^{(k+1)}\ {}_{k|}q_x = {}^2A_x$$

$$V[Z] = E\left[Z^2\right] - E[Z]^2 = {}^2A_x - (A_x)^2$$

Whole life insurance example

Consider a \$50,000 whole life insurance policy issued to (x), with death benefit paid at the end of the year of death. Let Z be the present value of the death benefit RV.

We're given:

$$q_x = 0.01 \quad q_{x+1} = 0.02 \quad q_{x+2} = 0.03 \quad q_{x+3} = 0.04 \quad i = 10\%$$

Find $\Pr[36{,}000 \leq Z \leq 42{,}000]$

$Z = 50000\,(1.1)^{-(K_x+1)}$

$K_x + 1 = 2 \qquad Z = 41{,}322$

$\qquad\quad 3 \qquad Z = 37{,}565$

$\Pr(K_x = 1 \text{ or } 2) = p_x \; {}_2q_{x+1}$

$= 0.99\,(1 - 0.98(0.97))$

Whole life insurance — Benefits paid m^{th}ly

Now we consider the case where the whole life death benefit is paid at the end of the $1/m$ period of a year in which the insured dies.

For this case, the present value of the benefit is $Z = v^{K_x^{(m)} + \frac{1}{m}}$.

Then for this discrete random variable we have:

EPV and Variance for Whole Life Insurance — m^{th}ly Case

$$E[Z] = E\left[v^{K_x^{(m)} + \frac{1}{m}}\right] = \sum_{k=0}^{\infty} v^{\frac{k+1}{m}} \; {}_{\frac{k}{m}|\frac{1}{m}}q_x = A_x^{(m)}$$

$$E[Z^2] = E\left[\left(v^{K_x^{(m)} + \frac{1}{m}}\right)^2\right] = \sum_{k=0}^{\infty} \left(v^2\right)^{\left(\frac{k+1}{m}\right)} \; {}_{\frac{k}{m}|\frac{1}{m}}q_x = {}^2A_x^{(m)}$$

$$V[Z] = E\left[Z^2\right] - E[Z]^2 = {}^2A_x^{(m)} - \left(A_x^{(m)}\right)^2$$

General strategy for calculating EPV

We can always find the EPV of *any* life-contingent payment (not just life insurance benefits) by summing over all possible payment times the product of:

1. The amount of the payment
2. An appropriate present value (discount) factor
3. The probability that the payment will be made

All of the EPV formulas for life insurance benefits we've seen are specific cases of this principle.

Note that this formula only works for calculating EPVs.

Recursion formulas

Using the summations given above, we can derive various **recursion** formulas for the expected present value of life insurance benefits, i.e., formulas relating successive values of the EPV.

- We'll encounter these types of formulas in many contexts.
- These formulas are useful for a number of reasons.

For the annual case, we have

EPV of Whole Life Insurance — Annual Case

$$A_x = v\, q_x + v\, p_x\, A_{x+1}$$

For the m^{th}ly case, we have

EPV of Whole Life Insurance — m^{th}ly Case

$$A_x^{(m)} = v^{1/m}\, {}_{\frac{1}{m}}q_x + v^{1/m}\, {}_{\frac{1}{m}}p_x\, A^{(m)}_{x+\frac{1}{m}}$$

Relating the whole life EPV values

Note that in order to calculate A_x, we only need the information in a life table.

However, in order to calculate $\overline{A}_x$, we need the full survival model.

- If we're not given this information (i.e., if we only have a life table), then we'll have to make some sort of fractional age assumption as before.

Making the UDD assumption gives the relationships:

Relationships Between Whole Life EPV Values under UDD

$$\overline{A}_x \overset{UDD}{=} \frac{i}{\delta} A_x \qquad A_x^{(m)} \overset{UDD}{=} \frac{i}{i^{(m)}} A_x$$

These relationships are often used as approximations, but are only exact under UDD.

Numerical EPV values for whole life insurance

x	A_x	$A_x^{(12)}$	$\bar{A}_x$
20	$4,922$	$5,033$	$5,043$
40	$12,106$	$12,379$	$12,404$
60	$29,028$	$29,683$	$29,743$
80	$59,293$	$60,641$	$60,764$
100	$87,068$	$89,158$	$89,341$

Table 4.3 from Dickson et al.: EPV values for a whole life insurance with a death benefit of $100,000$, using Makeham's mortality model and $i = 5\%$.

- Note the pattern between the values for the three cases, at each given age.
- Using the UDD approximations, we can calculate approximate values for $A_x^{(12)}$ and $\bar{A}_x$. Then we can compare them to the actual values shown in the table.

$$4922\left(\frac{0.05}{\log(1.05)}\right) = 5044$$

SULT Whole Life Example

Let Z be the PV of a \$100,000 whole life insurance (annual case) issued to (45). Let $i = 5\%$ and mortality be given by the Standard Ultimate Life Table (SULT).

(a) Calculate $E[Z]$ $\quad 100000 A_{45}$

$(100000^2)\,{}^2A_{45} - (100000 A_{45})^2$

(b) Calculate the standard deviation of Z. $\quad 100000\sqrt{{}^2A_{45} - (A_{45})^2}$

(c) Recalculate $E[Z]$ for the monthly case, i.e., if the death benefit was payable at the end of the month of death. Use the UDD fractional age assumption.

$$100000\, A_{45}^{(12)} = \left(\frac{i}{i^{(12)}}\right) A_{45}\,(100000)$$

Term life insurance — Benefits paid at moment of death

The next type of life insurance we'll consider is (n-year) **term life** insurance.

- First consider the continuous case, where the benefit is paid at the moment of death.

For this case, the present value of the benefit is

$$Z = \begin{cases} v^{T_x} & \text{if } T_x < n \\ 0 & \text{if } T_x \geq n \end{cases}$$

The corresponding EPV is denoted by $\bar{A}^{1}_{x:\overline{n}|}$

EPV for n-year Term Life Insurance — Continuous Case

$$\bar{A}^{1}_{x:\overline{n}|} = E[Z] = \int_0^n e^{-\delta t}\, {}_tp_x\, \mu_{x+t}\, dt$$

We can also calculate the second moment for Z:

$$E\left[Z^2\right] = \int_0^n e^{-2\delta t} \, {}_t p_x \, \mu_{x+t} \, dt = {}^2\bar{A}^{1}_{x:\overline{n}|}$$

Some term life insurance example problems:

1. How would you expect the EPV for a term insurance to vary as n increases?
2. Redo the whole life example, using the same survival model as before, but this time assuming that the person wishes to purchase a 20-year term insurance (with benefit payable at the moment of death) rather than a whole life insurance policy. Compare the answers for the two insurances.

Term life insurance — Benefits paid at end of the year of death

Next we consider the annual case for an n-year term life insurance.

For this case, the present value of the benefit is

$$Z = \begin{cases} v^{K_x+1} & \text{if } K_x \leq n-1 \\ 0 & \text{if } K_x \geq n \end{cases}$$

Then the EPV is denoted by $A^{1}_{x:\overline{n|}}$

EPV for Term Life Insurance — Annual Case

$$A^{1}_{x:\overline{n|}} = E[Z] = \sum_{k=0}^{n-1} v^{k+1}\ {}_{k|}q_x$$

Term life insurance — Benefits paid m^{th}ly

Now we consider the case where the term life death benefit is paid at the end of the $1/m$ period of a year in which the insured dies.

For this case, the present value of the benefit is

$$Z = \begin{cases} v^{K_x^{(m)} + \frac{1}{m}} & \text{if } K_x^{(m)} \le n - \frac{1}{m} \\ 0 & \text{if } K_x^{(m)} \ge n \end{cases}$$

Then the EPV is denoted by $A^{(m)}{}^{1}_{x:\overline{n}|}$

EPV for Term Life Insurance — m^{th}ly Case

$$A^{(m)}{}^{1}_{x:\overline{n}|} = E[Z] = \sum_{k=0}^{nm-1} v^{(k+1)/m} \, {}_{\frac{k}{m}|\frac{1}{m}}q_x$$

Term life insurance example and relationships

A person age x wishes to purchase a 3-year term life insurance policy with benefit amount $\$400,000$ payable at the end of the year of death.

Find the EPV of this benefit, assuming that

$$p_x = 0.97 \qquad p_{x+1} = 0.96 \qquad p_{x+2} = 0.94 \qquad i = 0.10$$

$$400{,}000\left(0.03\left(\frac{1}{1.1}\right) + 0.97(0.04)\left(\frac{1}{1.1}\right)^2 + 0.97(0.96)(0.06)\left(\frac{1}{1.1}\right)^3\right) = \boxed{40{,}526.52}$$

It turns out that the relationships between EPV values derived for whole life insurance also work for term life insurance.

- They're exact under UDD and approximations otherwise.

Relationships Between Term Life EPV Values under UDD

$$\bar{A}^{1}_{x:\overline{n}|} \overset{UDD}{=} \frac{i}{\delta} A^{1}_{x:\overline{n}|} \qquad A^{(m)\,1}_{x:\overline{n}|} \overset{UDD}{=} \frac{i}{i^{(m)}} A^{1}_{x:\overline{n}|}$$

Pure endowment

A **pure endowment** is a type of contract that pays a benefit at the end of a fixed time period (e.g., n years) if the policyholder is still alive at that time.

This type of policy is not typically sold by itself, but is nonetheless important:

- It can be combined with other types of insurance.
- It can be used to find the EPV of life contingent payments.

For this case, the present value of the benefit is

$$Z = \begin{cases} 0 & \text{if } T_x < n \\ v^n & \text{if } T_x \geq n \end{cases}$$

Pure endowment

The EPV of an n-year pure endowment is denoted by $A_{x:\overset{\,1}{\overline{n}|}}$

- Note that there are *not* separate continuous and $m^{th}ly$ cases for a pure endowment.
- The alternate (more convenient) notation ${}_nE_x$ is also used to denote the EPV of an n-year pure endowment.

We will commonly use ${}_nE_x$ as a sort of general "life-contingent discount factor".

EPV for Pure Endowment

$$A_{x:\overset{\,1}{\overline{n}|}} = {}_nE_x = E[Z] = v^n \, {}_np_x$$

SULT Pure Endowment Example

Let Z by the PV of a \$100,000 10-year pure endowment issued to (45). Let $i = 5\%$ and mortality be given by the Standard Ultimate Life Table (SULT).

(a) Calculate $E[Z]$ 60,655 ${}_{10}E_{45} \cdot 100000$

(b) Redo part (a) using $i = 9\%$

$100000 \left(\frac{1}{1.09}\right)^{10} \frac{l_{55}}{l_{45}}$

or $100000 \left({}_{10}E_{45}\right)\left(1.05^{10}\right)\left(1.09^{-10}\right)$

41734.44

Endowment insurance

Endowment insurance combines term insurance with a pure endowment.

- An n-year endowment insurance pays a benefit if the insured dies within n years.
- It also pays a benefit (of the same amount) at the end of n years if the person is alive at that point.

In the case where the death benefit portion is paid at the moment of death, the present value of the entire benefit (assuming as usual a benefit amount of $1) is

$$\begin{aligned} Z &= \begin{cases} v^{T_x} & \text{if } T_x < n \\ v^n & \text{if } T_x \geq n \end{cases} \\ &= v^{\min(T_x, n)} \end{aligned}$$

Endowment insurance

For the continuous case, the EPV is denoted $\bar{A}_{x:\overline{n}|}$, which we can write in terms of other EPV symbols:

EPV of Endowment Insurance — Continuous Case

$\bar{A}_{x:\overline{n}|} = E[Z] = \bar{A}^{1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}$

For the annual case, the PV of the benefit is

$$Z = \begin{cases} v^{K_x+1} & \text{if } K_x \le n-1 \\ v^n & \text{if } K_x \ge n \end{cases}$$
$$= v^{\min(K_x+1,n)}$$

Then the EPV for this case is denoted by $A_{x:\overline{n}|}$

EPV of Endowment Insurance — Annual Case

$A_{x:\overline{n}|} = E[Z] = A^{1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}$

Endowment insurance

For the m^{th}ly case, the PV of the benefit is

$$Z = \begin{cases} v^{K_x^{(m)}+\frac{1}{m}} & \text{if } K_x^{(m)} \leq n - \frac{1}{m} \\ v^n & \text{if } K_x^{(m)} \geq n \end{cases}$$
$$= v^{\min\left(K_x^{(m)}+\frac{1}{m},\, n\right)}$$

Then the EPV for the m^{th}ly case is denoted by $A^{(m)}_{x:\overline{n}|}$

EPV of Endowment Insurance — $m^{th}ly$ Case

$$A^{(m)}_{x:\overline{n}|} = E[Z] = A^{(m)1}_{x:\overline{n}|} + A_{x:\overset{1}{\overline{n}|}}$$

Relationships for endowment insurance

It's important to note that the UDD relationships we developed for the whole life and term insurance work *only* for death benefits, not for endowment benefits.

Therefore, in order to apply the UDD approximation, we must first split the term insurance benefit from the endowment portion:

Relationships Between Term Life EPV Values under UDD

$$\bar{A}_{x:\overline{n}|} \overset{UDD}{=} \frac{i}{\delta} A^{1}_{x:\overline{n}|} + A_{x:\overset{\;\;1}{\overline{n}|}} \qquad A^{(m)}_{x:\overline{n}|} \overset{UDD}{=} \frac{i}{i^{(m)}} A^{1}_{x:\overline{n}|} + A_{x:\overset{\;\;1}{\overline{n}|}}$$

As before, these relationships are exact under the UDD assumption, and approximate otherwise.

SULT Term Example

Let Z by the PV of a \$100,000 20-year (annual case) term insurance issued to (45). Let $i = 5\%$ and mortality be given by the Standard Ultimate Life Table (SULT).

(a) Calculate $E[Z]$
(b) Calculate $P(Z > 0)$
(c) Recalculate $E[Z]$ for the continuous case, i.e., if the death benefit was payable at the moment of death. Use the UDD fractional age assumption.

Deferred insurance benefits

In all of the types of insurances we've discussed thus far, the death benefit period starts immediately at the time of purchase.

- It's also possible to **defer** the coverage until some future time.

For example, consider the continuous case of a whole life insurance. Suppose we wanted to defer this insurance for u years. The PV of the benefit would be

$$Z = \begin{cases} 0 & \text{if } T_x < u \\ v^{T_x} & \text{if } T_x \geq u \end{cases}$$

We denote this deferment of benefits in much the same way as we did for a deferred mortality probability.

EPV for Deferred Whole Life Insurances

$${}_{u|}\bar{A}_x = {}_uE_x\,\bar{A}_{x+u} \qquad {}_{u|}A_x = {}_uE_x\,A_{x+u} \qquad {}_{u|}A_x^{(m)} = {}_uE_x\,A_{x+u}^{(m)}$$

Deferred insurance benefits — term insurance

Similarly, we can also consider deferred term life insurances. If we were to defer an n year continuous term insurance by u years, the PV of the benefit would be

$$Z = \begin{cases} 0 & \text{if } T_x < u \text{ or } T_x \geq u + n \\ v^{T_x} & \text{if } u \leq T_x < u + n \end{cases}$$

The corresponding EPV is denoted by ${}_{u|}\bar{A}^{1}_{x:\overline{n}|}$

EPV for Deferred Term Life Insurance — Continuous Case

$${}_{u|}\bar{A}^{1}_{x:\overline{n}|} = {}_{u}E_x \, \bar{A}^{\;1}_{x+u:\overline{n}|}$$

The annual and m^{th}ly cases work similarly.

Deferred insurance benefits — relationships

Using the principle of benefit deferment allows us to develop some useful relationships among the various insurance EPV values:

$$ {}_{u|}A^{1}_{x:\overline{n}|} = A^{1}_{x:\overline{u+n}|} - A^{1}_{x:\overline{u}|} \qquad A^{1}_{x:\overline{n}|} = \sum_{r=0}^{n-1} {}_{r|}A^{1}_{x:\overline{1}|} $$

$$ A_x = \sum_{r=0}^{\infty} {}_{r|}A^{1}_{x:\overline{1}|} \qquad A_x = A^{1}_{x:\overline{n}|} + {}_{n|}A_x $$

There are analogous continuous and m^{th}ly versions of these relationships as well.

Another SULT Term Example

$A^{1}_{45:\overline{17}|} = A^{1}_{45:\overline{20}|} - {}_{17|}A^{1}_{45:\overline{3}|}$

$A_{45:\overline{20}|} - {}_{20}E_{45} - \left(v^{18}{}_{17|}q_{45} + v^{19}{}_{18|}q_{45} + v^{20}{}_{19|}q_{45}\right)$

Let Z by the PV of a \$100,000 17-year (annual case) term insurance issued to (45). Let $i = 5\%$ and mortality be given by the Standard Ultimate Life Table (SULT).

$A^{1}_{45:\overline{17}|} = A_{45} - {}_{17|}A_{45} = A_{45} - v^{17}{}_{17}p_{45}A_{62}$

$= A_{45} - v^{17}\dfrac{\ell_{62}}{\ell_{45}}A_{62}$

- (a) Calculate $E[Z]$ 1849
- (b) Calculate the standard deviation of Z 10,599.96
- (c) Redo part (a) for a 30-year term product, leaving everything else the same.

5034.76

$Var(Z) = 100000^2\left({}^2A^{1}_{45:\overline{17}|} - \left(A^{1}_{45:\overline{17}|}\right)^2\right)$

${}^2A_{45:\overline{17}|} = {}^2A_{45} - {}_{17|}{}^2A_{45}$

$= {}^2A_{45} - {}_{17}E_{45}^{2}\,{}^2A_{62}$

$= {}^2A_{45} - \dfrac{\ell_{62}}{\ell_{45}}\left(\dfrac{1}{1.05}\right)^{2\cdot 17}{}^2A_{62}$

Variable benefits — Arithmetically Increasing

We can find the EPV for benefits with various patterns.

One pattern that deserves special mention is the case of an **arithmetically increasing** benefit, one in which the benefit increases by a constant amount each year:

$$(\bar{I}\bar{A})^{\,1}_{x:\overline{n}|} = \int_0^n t\, e^{-\delta t}\, {}_tp_x\, \mu_{x+t}\, dt$$

$$(\bar{I}\bar{A})_x = \int_0^\infty t\, e^{-\delta t}\, {}_tp_x\, \mu_{x+t}\, dt$$

$$(IA)^{\,1}_{x:\overline{n}|} = \sum_{k=0}^{n-1} (k+1)\, v^{k+1}\, {}_{k|}q_x$$

$$(IA)_x = \sum_{k=0}^{\infty} (k+1)\, v^{k+1}\, {}_{k|}q_x$$

Variable benefits — Geometrically Increasing

Another pattern that could be useful is that of a geometrically increasing benefit, one in which the benefit increases by a constant percentage each year.

Example: Consider a 20-year term insurance issued to (x) in which the death benefit is paid at the end of the year of death. The amount of the death benefit is $\$100,000$ if the insured dies in the first year, and rises by 3% each subsequent year. Find an expression for the expected present value of this benefit.

We could use similar logic to find the EPV of a benefit that was arithmetically / geometrically decreasing.

Valuing Insurance Benefits under a Select-and-Ultimate Mortality Model

We can calculate present values and EPVs for all of the insurances we've seen using a select-and-ultimate mortality model; we simply have to use the correct mortality values.

For example, under a 2-year select-and-ultimate mortality model, the EPV of a whole life insurance issued to $[x]$ would be

$$A_{[x]} = v\, q_{[x]} + v^2\, p_{[x]}\, q_{[x]+1} + v^3\, p_{[x]}\, p_{[x]+1}\, q_{x+2} \\ + v^4\, p_{[x]}\, p_{[x]+1}\, p_{x+2}\, q_{x+3} = \cdots$$

and we can also write

$$A_{[x]} = A^{\,1}_{[x]:\overline{n}|} + {}_nE_{[x]}\, A_{x+n}$$

(assuming $n \geq 2$ here)

Example (AMLCR Exercise 4.1)

x	ℓ_x	A_x
35	100,000.00	0.151375
36	99,737.15	0.158245
37	99,455.91	0.165386
38	99,154.72	0.172804
39	98,831.91	0.180505
40	98,485.68	0.188492

$\frac{i}{\delta} A^{1}_{35:\overline{5}|} + {}_5E_{35}$

Assuming $i = 0.06$, calculate

1. ${}_5E_{35}$ $\quad .735942 = v^5 \, {}_5p_{35} = v^5 \, \ell_{40}/\ell_{35}$
2. $A^{\;1}_{35:\overline{5}|}$ $\quad 0.012656 \quad A_{35} - {}_5E_{35} A_{40}$
3. ${}_{5|}A_{35}$ $\quad 0.138719 \quad {}_5E_{35} A_{40}$
4. $\bar{A}_{35:\overline{5}|}$ assuming UDD

0.748974

SOA Example Multiple Choice #3

$$E(Z) = \int_0^\infty b_t e^{-\delta t} {}_tp_x \mu_{x+t} dt$$

$$= \int_0^\infty e^{0.06t} e^{-0.08t} e^{-0.05t} 0.05 dt$$

$$E(Z^2) = \int_0^\infty \left[e^{0.06t} e^{-0.08t}\right]^2 e^{-0.05t} 0.05$$

For a special whole life insurance on (x), payable at the moment of death:

- $\mu_{x+t} = 0.05, t > 0$
- $\delta = 0.08$
- The death benefit at time t is $b_t = e^{0.06t}, t > 0$.
- Z is the present value random variable for this insurance at issue.

Calculate $Var[Z]$. [0.04535]

$${}_tp_x = e^{-\int_0^t \mu_{x+s} ds}$$

For a group of individuals all age x, 25% are smokers (s), 75% are nonsmokers (ns), $i = 0.02$

k	q^{s}_{x+k}	q^{ns}_{x+k}
0	0.10	0.05
1	0.20	0.10
2	0.30	0.15

Calculate $10,000A^{1}_{x:\overline{2}|}$ for an individual chosen at random from this group. [0.1730]

$$10000 A^{1}_{x:\overline{2}|} = 10000\left(0.25 A^{1\,(s)}_{x:\overline{2}|} + 0.75 A^{1\,(ns)}_{x:\overline{2}|}\right)$$

$$A^{1\,(s)}_{x:\overline{2}|} = v q^{(s)}_{x} + v^2 p^{(s)}_{x} q^{(s)}_{x+1}$$

SOA Example Multiple Choice #17

$A_{40} = vq_{40} + vp_{40}A_{41}$ $A_{41} = 0.2165$

$A_{41} - A_{40} = A_{41} - (vq_{40} + vp_{40}A_{41}) = (1 - vp_{40})A_{41} - vq_{40}$

For a whole life insurance of 1 on (41) with death benefit payable at the end of year of death, you are given:

- $i = 0.05$
- $p_{40} = 0.9972$
- $A_{41} - A_{40} = 0.00822$
- ${}^2A_{41} - {}^2A_{40} = 0.00433$
- Z is the present value random variable for this insurance.

$^2A_{40} = v^2 q_{40} + v^2 p_{40}\, {}^2A_{41}$

$0.07193 = {}^2A_{41}$

Calculate $Var[Z]$. [0.02544]

$E(Z) = A_{41}$

SOA Example Multiple Choice #34

Assuming $i = 0.03$, you are given the following select and ultimate mortality table with a select period of three years.

x	$q_{[x]}$	$q_{[x]+1}$	$q_{[x]+2}$	q_{x+3}	$x+3$
60	0.09	0.11	0.13	0.15	63
61	0.10	0.12	0.14	0.16	64
62	0.11	0.13	0.15	0.17	65
63	0.12	0.14	0.16	0.18	66
64	0.13	0.15	0.17	0.19	67

Calculate ${}_{2|}A^{\;1}_{[60]:\overline{2}|}$. [0.19]

$= v^3 \, {}_2p_{[60]} \, q_{[60]+2} + v^4 \, {}_3p_{[60]} \, q_{63}$

$= v^3 \left[(0.91)(0.89)(0.13)\right] + v^4 \left[(0.91)(0.89)(0.87)(0.15)\right]$

SOA Example Written Answer #7

For a special deferred term insurance on (40) with death benefits payable at the end of the year of death, you are given:

- The death benefit is 0 in years 1-10; 1000 in years 11-20; 2000 in years 21-30; 0 thereafter.
- Mortality follows the Illustrative Life Table.
- $i = 0.06$.
- The random variable Z is the present value, at age 40, of the death benefits.
- $E[Z] = 107$.

$$Z = \begin{cases} 0 & K_{40} < 10 \\ 1000\, v^{K_{40}+1} & 10 \le K_{40} < 20 \\ 2000\, v^{K_{40}+1} & 20 \le K_{40} < 30 \\ 0 & 30 \le K_{40} \end{cases}$$

$1000\, v^t = 400 \quad t = 15.73$

$2000\, v^t = 400 \quad t = 27.62$

1. Write an expression for Z in terms of K_{40}.
2. Calculate $\Pr(Z = 0)$. [0.75] $\quad {}_{10}q_{40} + {}_{30}p_{40}$
3. Calculate $\Pr(Z > 400)$. [0.1392]
4. Calculate $Var[Z]$. [36,046]