Asset Returns from a Distance-Based Random Partition Model

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Introduction

When pricing an insurance product, many factors affect market returns. Policyholder behavior (lapses, mortality, morbidity, etc.), policy riders (guarantees, look-backs, ratchets, etc.), industry and government forces (competition and regulation), among others can all depend on the asset returns and affect the total risk exposure from the market. In these complicated settings, an analytical solution is often unavailable and the asset returns will need to be simulated. Regime-switching models are important in finance and actuarial science because they are often successful in simulating asset returns. These models fit the complicated nature of market risk exposure more effectively and naturally than other models. Although regime-switching models have been successful, we propose a more flexible model that clusters data based on both the value of the data as well as its temporal proximity to other observations.

Methodology

Let S_t be the price of an asset (e.g., a stock index) at time t (for $t = 1, \ldots, n$) and y_t is the return on the asset, defined as $y_t = \frac{S_t}{S_{t-1}}$. We model the distribution from which the y_t are drawn as a mixture of distributions of the form $F(\theta)$, with the mixing distribution over θ being G. The prior for this mixing distribution will be a modified Dirichlet process with mass parameter α and base distribution G_0.

Our model modifies the Dirichlet process and incorporates the temporal distance between observations to inform the clustering process. For $i \neq k$ we use the following distance metric to inform the model:

$$d_{ij} = \begin{cases} |i-j| & \text{if } |i-j| \leq 1 \\ 1000 & \text{if } |i-j| > 1 \end{cases}$$

This distance metric places a higher probability of an observation clustering with adjacent observations and accounts for the temporal nature of the data in the clustering process.

Results

To demonstrate this model's ability to cluster data based on both the value of the asset return and its temporal proximity to other returns, we used data from the S&P 500 Index. A comparison

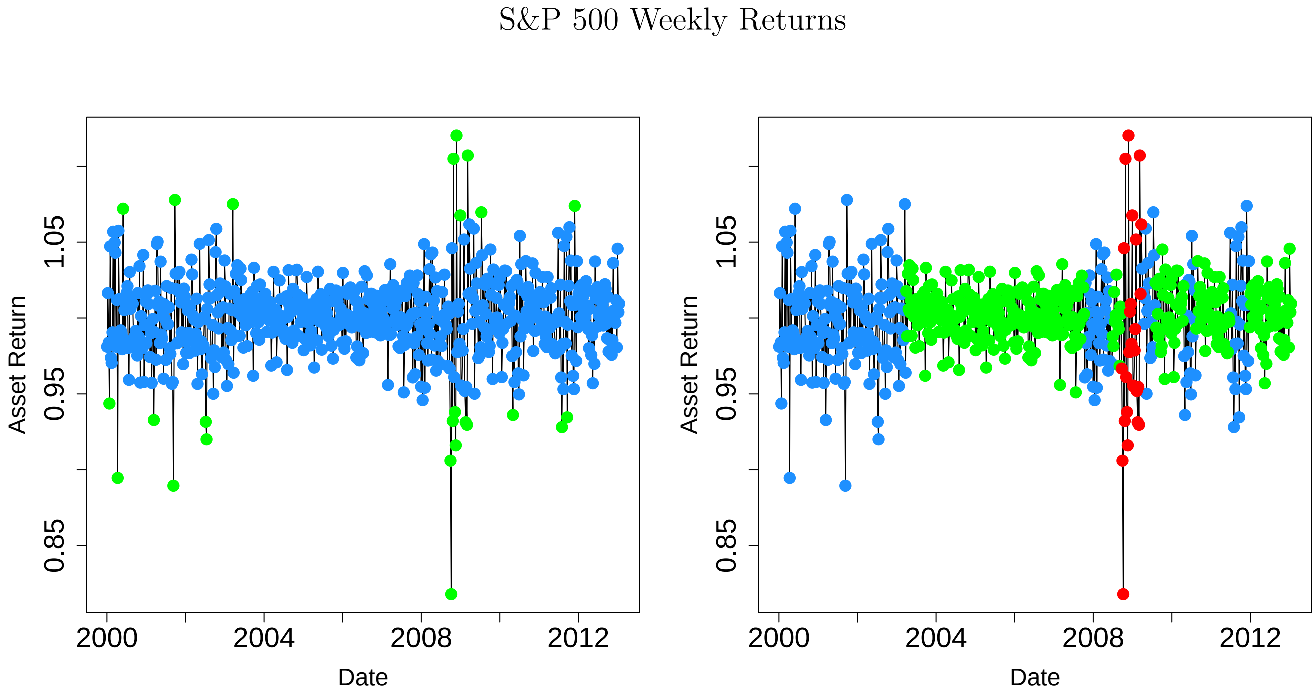

Figure 1: Classic Dirichlet Clustering (left), Modified Dirichlet Clustering (right)

of the classic Dirichlet process and our modified clustering model are shown in Figure 1. In this temporal data structure, the Dirichlet process successfully clusters asset returns into two clusters of extreme returns and more typical return values. Conversely, our model successfully incorporates the temporal structure of the data to identify periods of time with mild, moderate, and extreme levels of volatility (green, blue, and red, respectively).

To test the effectiveness of the model in forecasting, weekly asset returns from a random sample of 91 S&P 500 corporations were obtained for the years 1992-2013. Using our model, 50 weeks of data were forecast using data prior to 2012. The results of our model were then compared to the actual market results in 2012-2013. In order to benchmark the effectiveness of the model, forecasting was also performed by existing regime-switching models. The results of this analysis indicate that our model produces equivalent results to the regime-switching models.

Although our model is less efficient than existing regime-switching models, it has some potential advantages. This model is capable of providing greater flexibility in autocorrelation decay (i.e. the ability to increase short-term autocorrelation by increasing the mass parameter). Further investigation is needed to determine if our model has significant advantages regime switching models for short-term forecasting.

Conclusion

This model successfully clusters asset returns both on the value of the asset return and the return's temporal proximity to other returns. In particular, our model was successful in capturing clusters of observations over time with different volatilities. This model produced predictions comparable to current regime-switching models. The increased flexibility in autocorrelation decay may be useful in special cases performing short-term forecasting.