# A Multivariate Spatiotemporal Model for County Level Mortality Data in the Contiguous United States

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#### Abstract

We seek to understand the factors that drive mortality in the contiguous United States using data that is indexed by county and year and grouped into 18 different age bins. We propose a model that adds two important contributions to existing mortality studies. First, we treat age as a random effect. This is an improvement over previous models because it allows the model in one age group to borrow information from other age groups. Second, we utilize Gaussian Processes to create nonlinear covariate effects for predictors such as unemployment rate, race, and education level. This allows for a more flexible relationship to be modeled between mortality and these predictors. Understanding that the United States is expansive and diverse, we allow for many of these effects to vary by location. The flexibility in how predictors relate to mortality has not been used in previous mortality studies and will result in a more accurate model and a more complete understanding of the factors that drive mortality. Both the multivariate nature of the model as well as the spatially-varying non-linear predictors will advance the study of mortality and will allow us to better examine the relationships between the predictors and mortality.

Key Words: Gaussian process, INLA, Bayesian modeling, mortality improvement

### 1 Introduction

Life insurers, among others, are interested in modeling and predicting mortality experience for many different groups. There are many ways that the life experiences of individuals vary across a population. Some of these ways are easily quantifiable such as age, sex, and income. Other things are harder to measure such as happiness, health, and social connection. Even though it is not possible to measure all of the ways that these lives differ, it is nonetheless expected that life experiences will tend to be more similar in areas that are geographically closer to one another.

Before the 1990s mortality models were largely deterministic. Dickson et al. (2020) give a good overview of the development of these methods. Now stochastic modeling is the standard practice for mortality modeling with Lee and Carter (1992) proposing one of the earliest stochastic models for mortality. Despite their model's shortcomings, such as lack of smoothness across ages and a lack of spatiotemporal interactions, it is an improvement over purely deterministic methods and has good interpretability.

Many variations on the Lee-Carter model have been proposed; the Cairns-Blake-Dowd model (Cairns et al., 2006) has been widely used for modeling morality improvements. Cairns et al. (2009) provide an overview of the differences and a quantitative comparison between the Carins-Blake-Dowd model and the Lee-Carter model. Melnikov and Romaniuk (2006) and Booth and Tickle (2008) discuss these different models as well as older models and how they have been used for mortality forecasting.

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One thing that is missing from all of these models is spatial correlation. It seems quite reasonable that mortality would be more similar in areas that are geographically closer as there are similarities in areas that are close together that are not easily measured. Thus, including spatial effects into our modeling could improve our understanding of mortality. Many researchers have incorporated spatial and spatiotemporal effects into their mortality modeling. Clayton and Kaldor (1987) and Manton et al. (1989) were some of the first to use Empirical Bayes to account for spatial correlation. A review of Empirical Bayes and fully Bayesian approaches for modeling spatial variation in mortality rates is given by Bernardinelli et al. (1995). Waller et al. (1997) used a hierarchical Bayesian approach with spatial, temporal, and spatiotemporal effects to model county-level lung cancer death rates in the state of Ohio. Xia and Carlin (1998) studied the same data using a similar technique but also incorporating relevant covariates such as age and smoking prevalence.

More recently Ayele et al. (2015) used Gaussian Markov random fields to account for spatial variation in an additive logistic regression model for child mortality rates in Ethiopia. Dwyer-Lindgren et al. (2016) used a Bayesian approach to fit a hierarchical model which looked at the relationship between the effects at adjacent counties and utilized county-level covariates. Alexander et al. (2017) fit a hierarchical model to obtain subnational mortality estimates. Their model, which smoothed across space and time turned out to be a better fit for mortality data, both simulated U.S. data and real French data divided into 19 age groups, than simpler methods. The improvement was especially noticed in areas with low population. Boing et al. (2020) studied geographic variation in longevity at three different geographic levels: state, county, and census tract. They used these geographic levels as random effects in a linear regression model and found that the census tract accounts for the largest portion of the geographical variation in longevity, making longevity inequality in the U.S. a more local phenomenon than is often assumed. In a similar vein, Kim and Subramanian (2016) used a model with two geographic levels (county and state) to study mortality differences by location. They argued that including state in models was superior to using counties alone, due to the fact that legislation, policies, and programs that can impact health are often implemented at the state level. Li and Hyndman (2021) also considered inequalities in mortality across the United States. They first used Lee-Carter models to produce independent state-level forecasts, then used a forecast reconciliation approach to reconcile the state-level and national-level mortality rate forecasts. They projected mortality rates 10 years into the future and found that mortality inequality among states is likely to persist and that mortality improvement rates will slow down in the future. Haberman and Renshaw (2011) compares several mortality modeling approaches with various versions of ways of treating random and fixed effects. Recent papers by Wen et al. (2023) and Cairns et al. (2024) explore neighborhood effects for modeling mortality as functions of socio-economic factors.

Chetty and et al. (2016), Currie and Schwandt (2016), and Ezzati and et al. (2008) model mortality at the county level in the United States, examining the associations between various socioeconomic factors and life expectancy as we do. They employ different methodologies to analyze mortality patterns and disparities at the county level. On the other hand, Rashid et al. Rashid et al. (2021) develop a Bayesian hierarchical model similar to ours, but with data from communities in England. Despite the geographical difference, their model shares similarities with ours in its approach to understanding mortality dynamics and their relationship with socioeconomic factors.

Recently Gibbs et al. (2020) fit a spatiotemporal model to county-level mortality data from the contiguous United States using conditional auto-regressive priors and a county-varying linear time trend to each age group. Here we utilize the same data and build on this model in two significant ways. First, we model all age groups together in a multivariate spatiotemporal model. Using a multivariate approach adds significant complexity to the model but improves the model in a similar way that adding spatial or temporal correlation would, by allowing data at one age group to borrow strength from data at neighboring age groups (Royle and Berliner, 1999; Gelfand, 2021). We address the added complexity by using strong Markov assumptions on the correlations and estimate the model parameters with Integrated Nested Laplace Approximations (INLA) (Blangiardo and Cameletti, 2015). This is one of the first significant applications modeled using multivariate INLA models for spatiotemporal data (Vicente et al., 2020). Incorporating multivariate dependence in spatiotemporal models is rarely done due to the computational difficulty of building models with compounding sources of dependence, so adding this is a significant feature of the model we propose. While using random effects in a mortality model is not new (Biffis, 2005; Loisel and Serant, 2007), this is the first attempt we are aware of that uses all three axes of dependence as random effects.

The other novel element in our mortality model is including non-linear and spatially-varying covariate

effects. This allows for an unprecedented degree of flexibility in how variables such as employment, education level, and others affect mortality. Several previous works have demonstrated the relationships between various socioeconomic variables and mortality rates. Fuchs (2004) discussed several socioeconomic variables that have been shown to be correlated with health, and in particular the difficulties that arise as a result of these variables often being interrelated and correlated, and sometimes difficult to measure. Mackenbach et al. (2008) studied the mortality inequalities in 22 European countries, considering education level and occupation class. They found that mortality and morbidity are inversely correlated with socioeconomic status, but that the magnitude of the inequality varied considerably. Bassanini and Caroli (2015) focused on the relationship between work and health, in terms of both number of hours worked as well as employment status. They found that an excessive number of hours worked was detrimental to health, but also that a reduction in hours worked due to involuntary job loss also had a deleterious impact. Boing et al. (2020) incorporated several socioeconomic and demographic variables in their multiple geographic level model and found that education and income level are the most significant of these variables. Villegas and Haberman (2014) is one of many papers that examine mortality disparity by socio-economic group.

Functionally, we treat the covariates as processes on the covariate space and they are given Gaussian process priors (Shi and Choi, 2011). By allowing these to change across space we remove the restriction that the predictor variables affect mortality equally across the whole country. It was seen in Gibbs et al. (2020) that this improved the model and helped to clarify the importance of the covariates.

The remainder of this article is organized as follows: Section 2 discusses the data that are analyzed in this study, Section 3 introduces the statistical models, methods, and notation that are used to perform the analysis, Section 4 conveys and discusses the results of this analysis, and Section 5 concludes with discussion of model limitations and potential future work.

### 2 Data

In this analysis we use data from Division of Vital Statistics of the National Center for Health Statistics, part of the Centers for Disease Control and Prevention. The data contain information about every death that occurred in the United States from 2000 to 2017 along with demographic information including age, sex, county of residence, county of death, race, and marital status. In this analysis we only use county of residence (and not county of death), and restrict our attention to those counties of residence belonging to the contiguous United States.

Using census data with interpolation, we are able to obtain the age group mortality exposure for all of the counties during this time period. The census data are available with ages being placed in 18 buckets, with the first being for those 0-4 years of age and the last being those individuals who are 85 years old or older. The mortality data are divided into the same age buckets so that for each sex, county, year, and age group, we had an exposure and the number of people who died.

As the census data are only gathered every ten years, the majority of the population data we have are estimates. However, we have exact information on the number of deaths. This mismatch results in some anomalies whereby there are some sex, county, year, age group combinations where the data indicate that there are more people who died than were living in the county at the time. There are also some counties whose populations are quite small; after dividing the counties by sex and into 18 age groups, there are some counties whose population for a given group was zero in a given year. Counties which have an exposure of zero or more deaths than exposures are combined with their neighbor with the largest population. After such modification, we have 3092 counties that we consider in the model. A full list of counties which were combined can be found in Appendix B. We use the term "county" to refer to the subdivisions throughout all states in the United States, including Louisiana where they are actually parishes.

Figures 1a and 2a show the observed mortality for females age 55-59 and 85+, respectively, and provide evidence of some spatial correlation. These are shown later in the paper to compare with the corresponding fitted values. Mortality tends to be higher in the south and lower in the Midwest. Similar trends exist across both sexes and other age groups. Figure 12 in Appendix A shows the observed countrywide mortality rate for each age group and sex combination. In general, males have higher mortality than females. Mortality increases with age, with the exception that the youngest group sees a mortality improvement upon reaching age 5. Across time there appears to be mortality improvement for those individuals 45+.

Covariate Name	Meaning	Measuring Frequency	Source
Unemployment	Unemployment rate	Every County in Every Year	Bureau of Labor and Statistics
Race	Percent of head of households that are white	Every County in 2010	Census Bureau
Home Value	Typical value of a single family home	Every State in Every Year	Zillow
Education	Percent of people 25 or older who have a bachelor's degree or higher	Every State in 2010	Census Bureau
Marital Status	Percent of people 25 or older who are married but not separated	Every State in 2010	Census Bureau
Household Size	Average household size	Every State in 2010	Census Bureau

Table 1: Covariates used in the model along with what they are measuring, frequency of measurement, and the source of the data.

Six different covariates are used in this analysis: Unemployment, Race, Home Value, Education, Marital Status, and Household Size. These covariates are measured at different frequencies (see Table 1). Table 1 also shows the sources of each variable. To clarify a few points, the covariate Unemployment Rate denotes the proportion of the labor force actively seeking employment, calculated as the number of unemployed individuals divided by the total labor force. Head of households who declare two or more races are not counted as white head of households. The covariates of unemployment, race, and home value are divided into 20 different groups based on quantiles. The remaining covariates of education, marital status, and household size are divided into groups where each group consists of the states that have the same value for the covariate. Those covariates have respectively 44, 43, and 29 unique values and so that is the number of resulting groups. The arithmetic average of the values in each group is used as the "location" for that group for calculating a Matérn correlation between points. The covariates appear to also have some strong spatial dependence. In Appendix A, Figures 13 and 14 show unemployment and race values for 2010 are plotted on the map. There is clear spatial dependence in both of these graphs. The unemployment data are complete except for a few counties in Louisiana for 2005 and 2006. (These correspond to the parts of Louisiana that were most heavily affected by hurricane Katrina.) Similarly we are missing home value information for a few states in the early 2000s. See Appendix 3 for a complete list of modifications. These missing data were dealt with by simply letting those affected observations not have an unemployment and home value effect during those times.

### 3 Methods

We employ a multivariate spatiotemporal random effects model to analyze mortality data, a widelyaccepted approach for handling spatiotemporal variables. While Dynamic Linear Models are another viable option, they primarily excel in predictive analyses. In contrast, our aim is to investigate how various covariates influence mortality trends over time and space. The random effects model offers a more intuitive and flexible framework for this analysis (Banerjee et al., 2003; Cressie and Wikle, 2015).

For each sex we fit distinct models, which are identical in form. Let  $y_{akt}$  be the number of deaths that occurred within age group a in county k during year t. In this application,  $a \in \{1, 2, ..., 18\}$ ,  $k \in \{1, 2, ..., 3092\}$ , and  $t \in \{1, 2, ..., 18\}$ . The number of people within age group a in county k during year t is known, and so the binomial likelihood is appropriate for this data:

$$y_{akt}|\pi_{akt} \sim \text{Binomial}(n_{akt}, \pi_{akt})$$
 (1)

Here  $\pi_{akt}$  is the annual mortality probability for someone who is in age group a in county k during year t and  $n_{akt}$  is the corresponding population count. We can then relate  $\pi_{akt}$  to the desired effects using the logit link function:

$$\ln\left(\frac{\pi_{akt}}{1-\pi_{akt}}\right) = \beta_0 + \sum_{i=1}^3 F_i(\mathbf{x}_k) + \sum_{i=1}^3 G_{is}(\mathbf{x}_{kt}) + \phi_k + \delta_t + \psi_a + \gamma_{akt}$$
(2)

The parameter  $\beta_0$  is an overall intercept for the model. Here,  $\mathbf{x}_k$  represents the vector of covariates at the county level, and  $\mathbf{x}_{kt}$  represents the vector of covariates at the county level that are also time-dependent. Each  $F_i$  ( $i \in \{1, 2, 3\}$ ) represents a function of the covariate value for the covariates of education, marital status, and household size. A standard linear effect would be  $F_i(\mathbf{x}_k) = \beta_i \mathbf{x}_k$ . Instead we induce a nonlinear covariate effect for by allowing  $F_i$  to be a more general function of  $\mathbf{x}_k$ , specifically a Gaussian process with

the Matérn correlation with smoothness  $\kappa = 1.5$ . It is a Gaussian process on the covariate space, meaning the finite dimensional distribution of the *n*-tuple  $(F(x_1), F(x_2), \ldots, F(x_n))$  is a multivariate normal where the covariance is based on the distances between covariate values,  $\operatorname{cov}(F(x_1), F(x_2)) = C(|x_2 - x_1|)$ . This is a flexible way to allow the covariates to have nonlinear effects on mortality. Similarly the  $G_{is}$  are the nonlinear covariate effects for unemployment, race, and home value. Once again these are Gaussian Processes with the Matérn correlation function; however, instead of simply having one effect for the entire dataset we allow each state to have its own effect. For the purpose of these effects we define a state to be one of the 48 contiguous states in the United States (all but Alaska and Hawaii) and Washington D.C. Because we are examining county level data, it might make more sense to use county-varying effects instead of state level effects. However, this would not only lead to a nearly unidentifiable model, but it would be an intractable computational challenge. We are nonetheless gaining some advantage over previous models by allowing for some regionalization beyond an overall national effect. We impose a conditional autoregressive prior on the different state effects so that, conditional on all the other states, the effect for a given state only depends upon those states with which it shares a border. For identifiability, all of the individual covariate effects have a sum to zero constraint.

The spatial effect is broken down into two components:

$$\phi_k = u_k + v_k \tag{3}$$

$$\mathbf{u}|\tau_u \sim \mathcal{N}(\mathbf{0}, \tau_u^{-1}\mathbf{I}) \tag{4}$$

$$\mathbf{v}|\tau_v \sim \mathcal{N}(\mathbf{0}, \tau_v^{-1} \mathbf{W}^{-1}) \tag{5}$$

with precision parameters  $\tau_u$  and  $\tau_v$ , where **u** is our iid spatial effect and **v** is our structured spatial effect. For identifiability, **u** and **v** have a sum to zero constraint. Our structured effect follows the type of conditional autoregressive model proposed by Besag et al. (1991). We say that county *i* and county *j* are neighbors if they share a border and denote this relationship as  $i \sim j$ . We denote the number of neighbors of county *i* by  $n_i$ . Note that in our specification, a county is not its own neighbor. **W** is a county adjacency matrix where, for  $i \neq j$ ,  $W_{ij} = 1$  if  $i \sim j$  and  $W_{ij} = 0$  if  $i \approx j$ . The diagonal entries are  $W_{ii} = -n_i$ . This creates a conditional independence where conditioned on all other counties, the effect for a given county only depends upon those counties with which it is a neighbor. This structure is more apparent if we write the effect as:

$$v_i | v_j \sim \mathcal{N}\left(\frac{1}{n_i} \sum_{j:i \sim j} v_j, \frac{1}{n_i} \tau_v^{-1}\right) \quad \text{for } j \neq i$$
 (6)

However, both formulations are equivalent.

The temporal effect is also broken down into two components:

$$\delta_t = b_t + c_t \tag{7}$$

$$\mathbf{b}|\tau_b \sim \mathcal{N}(\mathbf{0}, \tau_b^{-1}\mathbf{I}) \tag{8}$$

$$\mathbf{c}|\tau_c \sim \mathcal{N}(\mathbf{0}, \tau_c^{-1} \mathbf{R}_c^{-1}) \tag{9}$$

with precision parameters  $\tau_b$  and  $\tau_c$  and structure matrix  $\mathbf{R}_c$ , where **b** is the iid temporal effect and **c** is the structured temporal effect. For identifiability, **b** and **c** have a sum to zero constraint. The structured effect follows a random walk of order 1 so that  $\mathbf{R}_c$  is an  $18 \times 18$  tridiagonal matrix of the form:

$$\mathbf{R}_{c} = \begin{vmatrix} -1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & -2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 \end{vmatrix}$$
(10)

This implies that, given  $c_{t-1}$  and  $c_{t+1}$ ,  $c_t$  is conditionally independent of  $c_{t*}$  for all other  $t^*$  not equal to t-1, t, or t+1. More flexible prior dependence could be imposed by using an autoregressive prior instead

of a random walk, but a random walk is computationally faster in the INLA algorithm and with so much data, the results would be quite similar.

We then have an age group effect again broken down into two components:

$$\psi_a = f_a + g_a \tag{11}$$

$$\mathbf{f}|\tau_f \sim \mathcal{N}(\mathbf{0}, \tau_b^{-1}\mathbf{I}) \tag{12}$$

$$\mathbf{g}|\tau_g \sim \mathcal{N}(\mathbf{0}, \tau_c^{-1} \mathbf{R}_g^{-1}) \tag{13}$$

with precision parameters  $\tau_f$  and  $\tau_g$  and structure matrix  $\mathbf{R}_g$ , where **f** is the iid age group effect and **g** is the structured age group effect. For identifiability, **f** and **g** have a sum to zero constraint. The structured effect follows a random walk of order 1 so we have that  $\mathbf{R}_g$  is an  $18 \times 18$  tridiagonal matrix that coincidentally is the same as  $\mathbf{R}_c$  from Equation 10 because the number of age groups and the number of time points happen to be the same. Then, given  $g_{a-1}$  and  $g_{a+1}$ ,  $g_a$  is conditionally independent of  $g_{a*}$  for all other  $a^*$  not equal to a - 1, a, or a + 1.

Note that treating age as an axis of dependence in the same way as time and space is a novel feature of our model. Other approaches have either built age-specific models or have added age as a covariate. What our approach allows for is that the model at one age point borrows strength from nearby ages, the same way the spatial random effect allows the model at a specific location to borrow strength from nearby locations.

Finally we have an iid error term:

$$\gamma_{akt} \stackrel{iid}{\sim} \mathcal{N}(0, \tau_{\gamma}^{-1}) \tag{14}$$

where  $\tau_{\gamma}$  is a precision parameter. All precision parameters ( $\tau$ ) are given identical gamma priors with mean 2000 and variance 4000000. These priors are chosen to be diffuse, although there is a significant amount of data so the prior choice would not noticeably affect posterior results.

Rue et al. (2009) proposed a deterministic method for performing Bayesian inference using the Integrated Nested Laplace Approximation (INLA) which is implemented in the R-INLA package (Lindgren and Rue, 2015). For a given model, the computation time in R-INLA tends to be much faster than traditional MCMC algorithms that have been used for exploring the posterior of a model. As a result, this methodology has become quite popular in recent years. Blangiardo and Cameletti (2015) is a nice textbook on the theory and implementation of basic spatiotemporal inference using the package. Rue et al. (2017) and Bakka et al. (2018) provide reviews of INLA and how it works with spatial data. We use this method for our computation as we have a large problem and hence computational efficiency is important.

Several techniques exist to accelerate computation in large dependent models. Some methods leverage dimension reduction, while others capitalize on sparsity in the data structure. Approaches that circumvent dimension reduction, such as INLA, are generally considered to yield better performance (Stein, 2014). Other methods that avoid dimension reduction, like Local Approximate Gaussian Process (LA-GP) and Nearest Neighbor Gaussian Process (NNGP), are also expected to perform comparably (Gramacy and Apley, 2015; Datta et al., 2016; Heaton et al., 2019). Importantly, we anticipate that the choice of model fitting procedure should not significantly impact the inferential conclusions drawn from the analysis.

Parameters such as precisions of the stuctured and unstructured effects are estimated using the INLA procedure. Several other modeling decisions, such as the structure of the Gaussian processes for the covariate effects and the structure of the random effects were made by evaluating Deviance Information Criterion of the model output for different versions of the model. The most relevant of these DIC comparisons are shown in the Results section below. For this paper we did not try any other model structure besides the random effects model and we did not try other estimation procedures besides INLA. Even with the sparse modeling that INLA allows us, fitting these models was computationally intensive, especially when using the spatially-varying non-linear covariate effects.

### 4 Results

#### 4.1 Model Comparisons

The model described in Section 3 was fit to the data described in Section 2. We tried three different versions of the model. Specifically, we used one version where we eliminated all the covariate effects ( $F_i$ 's

Model	Difference in DIC (Female)	DIfference in DIC (Male)
Full Model	-	-
Only Countrywide	519	390
No Covariates	1,222	514

Table 2: Difference from the full model in the Deviance Information Criterion (DIC) for the three different model versions that were fit to both the male and female data.

and  $G_{is}$ 's), a second version where we used only countrywide effects and no state-specific covariate effects:

$$\ln\left(\frac{\pi_{akt}}{1-\pi_{akt}}\right) = \beta_0 + \sum_{i=1}^6 F_i(\mathbf{x}_k) + \phi_k + \delta_t + \psi_a + \gamma_{akt},\tag{15}$$

and a third version which was the full model described in Section 3. Equation (15) is essentially the same as Equation (2) with all spatially varying covariates,  $G_{is}$  replaced by country-wide covariate effects,  $F_i$ . Table 2 shows the Deviance Information Criterion (DIC) value obtained for each model fit on both sets of data. DIC is a model comparison metric that favors a good fit but also penalizes for the complexity of the model (Spiegelhalter et al., 2002). Since we are interested in seeing if the more complex model with state specific covariate effects produces a better fit than the simpler models, while accounting for model complexity, we use DIC in our comparison of the three models. Since the DIC was lowest for the full model proposed in Section 3, this model was chosen for the remainder of the analysis.

Figures 1b and 2b show the posterior means of the mortality rates for females aged 55-59 and 85+ respectively. The observed mortality rates for the same groups are displayed in Figures 1a and 2a. The fitted values display similar general patterns to those seen in the observed data. One difference that is noted, however, is that the fitted mortality rates exhibit a greater degree of spatial correlation than do the observed ones. This is to be expected, as the fitted model allows for spatial correlations.



(a) The observed mortality rate for each county in the contiguous United States for females aged 55-59 in 2010. Mortality rates are multiplied by 1000 for readability.

1000 Times the Fitted Mortality Rate for Females 55-59 in 2010



(b) The posterior mean of the mortality rate for females aged 55-59 in 2010. The mortality rate is multiplied by 1000 for readability.

Figure 1: Mortality rate and posterior mean for females aged 55-59 in 2010.

#### 4.2 Random Effects

Figures 3 and 4 show the posterior mean of the spatial effect  $(\phi_k)$  for the counties in the United States for the female and male models, respectively. Recall that these effects have sum-to-zero constraints, meaning they are centered around 0. We see that in both cases the spatial patterns of the data appear to be very similar. We also see the same pattern manifesting itself that we saw earlier where there tends to be higher mortality in the South and lower mortality in the upper Midwest.

One important aspect of the results to consider is how mortality is changing over time. Figure 5 shows the posterior mean and 95% credible interval for the temporal effects ( $\delta_t$ ). From 2000 through roughly 2014,

1000 Times the Observed Mortality Rate for Females 85+ in 2010



(a) The observed mortality rate for each county in the contiguous United States for females aged 85+ in 2010. Mortality rates are multiplied by 1000 for readability.

1000 Times the Fitted Mortality Rate for Females 85+ in 2010



(b) The posterior mean of the mortality rate for females aged 85+ in 2010. The mortality rate is multiplied by 1000 for readability.

Figure 2: Mortality rate and posterior mean for females aged 85+ in 2010.

the trend for both males and females is of mortality improvement over time (i.e., a decrease in  $\delta_t$ ), though the patterns are not smooth or monotonic. Around 2014, there begins to be a small uptick in the mortality. One potential cause for this increase could be a spike in the "deaths of despair" that are often discussed in the mortality literature (see, for example, Scutchfield and Keck (2017)).

Figure 6 shows the posterior mean and 95% credible interval for the age group effects ( $\psi_t$ ). Recall that this effect is built into the model as an additional axis of dependence as opposed to building age specific models or even adding it as a covariate. For the plots we assign the 85+ age group a value of 87.5 and use the midpoint for all other age groups. These results are consistent with the results seen in the raw data with respect to the age group. Namely, mortality improves upon aging out of the youngest age group, but then consistently deteriorates as age increases. The pattern is seen to be quite similar between the males and females; the most significant difference between the two patterns is the more pronounced "accident hump" in the males between the ages of 15 and 29.

#### 4.3 Covariate Effects

#### 4.3.1 Non-Spatially Varying Effects

There were three variables used that were the same for every county across the united states. These were education, marital status, and household size. Note that these are still flexible covariate effects, but no advantage was seen in the model results to justify the increased complexity of making these variables spatially varying. Figures 7 and 8 display the posterior means and 95% credible intervals for the covariate effects corresponding to education, marital status, and household size for the female and male models, respectively. In particular, the plots show the effect on log odds of mortality for a given county level variable, as in Equation (2).

Although the entire credible interval contains 0 for each of these covariates, they nonetheless present nice illustrations of the information that can be obtained when the covariates are allowed to have nonlinear effects. As an example, consider the case of the education covariate. If this had been treated as a linear effect, the result would like have been a slight negative slope. However, just having a negative slope does not communicate as much information as our nonlinear effect. It can be seen from the figure that, rather than having a monotonic linear relationship, there is a trend of slow decrease in mortality as education increases, until the percentage of people with a Bachelor's degree hits 25%. Then follows a quick decline in mortality followed by a leveling off. Thus demonstrates the added flexibility and more nuanced conclusions that can be obtained by utilizing nonlinear covariate effects.

#### 4.3.2 Unemployment Effect

The vast majority of the states follow one of two trends. Either there is no effect for unemployment, as is seen for Colorado, or there is a downward trend for low unemployment rates that flattens out, as is seen **Combined Spatial Effect** 



Figure 3: Posterior mean of the spatial effect  $(\phi_k)$  for the model fit to the female data.



### Combined Spatial Effect

Figure 4: Posterior mean of the spatial effect  $(\phi_k)$  for the model fit to the male data.



Figure 5: Posterior means and 95% credible intervals of the temporal effects ( $\delta_t$ ). Male values are in blue and female values are in red.



Figure 6: Posterior mean and 95% Credible interval of the age group effects ( $\psi_t$ ). Male values are in blue and female values are in red. The credible intervals are hardly visible because they are so tight around the estimates.



Figure 7: Posterior mean and 95% credible interval for the covariate effects  $(F_i(x_k))$  for the model fit to the female data. The effects displayed correspond to education (left, i = 1), marital status (center, i = 2), and household size (right, i = 3).



Figure 8: Posterior mean and 95% credible interval for the covariate effects  $(F_i(x_k))$  for the model fit to the male data. The effects displayed correspond to education (left, i = 1), marital status (center, i = 2), and household size (right, i = 3).

for North Carolina. The negative trend that flattens suggests that low unemployment rates leads to higher mortality.

This effect is unintuitive, and in some ways defies current understanding of the relationship between unemployment rates and mortality rates. For instance, Marmot and Wilkinson (2005) discuss how socioeconomic factors, including unemployment, can impact health behaviors and lifestyle choices, potentially leading to higher mortality rates. Access to healthcare also plays a crucial role, as highlighted by McLaughlin (2004), who examine the demand for healthcare among the unemployed and employed during economic uncertainty. The fact that this effect is so consistent among nearly all states could potentially motivate looking closely at why this effect is seen. There is in fact only one state, California, where low unemployment corresponds to lower mortality. Again, the fact that the shape is so different than the others could merit further investigation. Figure 9 shows these unemployment effects for these three states for the female data; the male data show similar patterns. Results for all 48 states and both genders are given in Supplementary Material 1.

#### 4.3.3 Race Effect

The most consistent effect is percentage of white households. The slopes of the curves are consistently negative and in many cases, such as Alabama, the trend is nearly linear. However, there are states such as Illinois where we see the negative slope shift to no effect after a certain point; these differences support having state-specific effects for race. We do see that some effects are much more sharply sloped than others. Montana for example has a large negative slope while Alabama is nearly flat, suggesting very little effect. The effects for the female data for selected states is given in Figure 10; the remainder of the effects for various states can be found in Supplementary Material 2. From these it can be seen that the effects with the sharpest slope are states in the northern United States such as South Dakota, North Dakota, and Minnesota. States with the smallest effects are in the south, such as Mississippi, Louisiana, and Georgia. The effects of race are very similar in the male and female models.

The strong relationship between race and mortality is consistent with previous results; many studies have sought to explain this relationship. Wheaton et al. (2005) highlights the intricate relationship between socioeconomic status and health outcomes, suggesting that counties with higher percentages of white households may benefit from better economic opportunities, leading to improved health indicators. Furthermore, the Institute of Medicine (2003) emphasizes disparities in healthcare access and quality, potentially explaining why counties with higher white household percentages exhibit lower mortality rates due to better access to medical services. Borrell and Crawford's systematic review (2009) underscores variations in health behaviors across racial and ethnic groups, suggesting that cultural differences in lifestyle choices may contribute to the observed mortality trends. Additionally, Morello-Frosch and Shenassa (2006) discuss the impact of environmental factors on health disparities, implying that counties with higher percentages of white households may enjoy better environmental conditions, further contributing to reduced mortality rates.



Figure 9: Posterior mean and 95% credible interval of the unemployment effects  $(G_{1s}(x_{kt}))$  for selected states for the model fit to the female data.



Figure 10: Posterior mean and 95% credible interval of the race effects  $(G_{2s}(x_{kt}))$  for selected states for the model fit to the female data.

#### 4.3.4 Home Value Effect

Home value seems to have a more significant affect on female mortality than male mortality. Looking at the plots for Connecticut and Idaho for males versus females, we see that the effect of home value on mortality for males nearly contains 0 through the whole trend while the effect for females is very clearly an inverted U-shape. These trends are reasonably consistent among most states. The inverted U-shape is found as the effect for nearly all states for females and in many states for males. This suggests that mortality is highest in counties where the average home price is between 150,000 and 200,000. Mortality is lower when average home value is under 150,000, and above 200,000 the mortality tends to either have a negative slope, or to have a brief negative slope and flatten out, as it does for Delaware. The effects for the female data for selected states is given in Figure 11; the remainder of the effects for various states can be found in Supplementary Material 3.

The relationships between socioeconomic factors and health outcomes are complex, often exhibiting nonlinear patterns. For instance, Case and Deaton (2015) demonstrate the alarming trend of rising morbidity and mortality rates among middle-aged white non-Hispanic Americans in the 21st century, underscoring the importance of socioeconomic disparities in shaping health trajectories. In the context of housing policies, Keene and Geronimus (2011) emphasize the need to evaluate the population health impact of public housing demolition and displacement, highlighting the potential adverse effects of housing instability on health outcomes. Geographical factors also play a significant role in health disparities, as evidenced by Kershaw and Albrecht (2014), who explore the impact of metropolitan-level ethnic residential segregation on body mass index (BMI) among US Hispanic adults. Furthermore, Galea and Vlahov (2005) provide a comprehensive overview of urban health, emphasizing the importance of addressing social determinants of health, including housing affordability and neighborhood characteristics. These studies collectively underscore the need for comprehensive approaches to address socioeconomic disparities and promote population health.



Figure 11: Posterior mean and 95% credible interval of the home value effects  $(G_{3s}(x_{kt}))$  for selected states for the model fit to the female data.

### 5 Conclusion

We have fit a multivariate spatiotemporal model to mortality data in the contiguous United States. This model has built on the existing mortality modeling literature in two significant ways. First, we model all age groups together to create a multivariate spatiotemporal model. This allows for the borrowing of information not only across space and time but also across the different age groups of the model.

The other significant contribution is the inclusion of nonlinear and spatially-varying nonlinear covariate effects on mortality. These nonlinear and spatially-varying covariate effects allow us to see how things such as education, unemployment and race affect mortality and how those effects change over space. By including a nonlinear education effect, we were able to see that while mortality generally improves as education increases, there becomes a point where additional education seems to no longer provide additional mortality improvement. This is broadly consistent with the findings of Boing et al. (2020), which reported a positive correlation between life expectancy and education. Allowing the covariates effects to change over space allowed us to observe both how the magnitude of the effect changes across states, as we saw for race and home value, and how the shape of the effect changes across state values, as we saw for unemployment. This flexibility allowed us to observe some otherwise undetectable trends, such as the positive relationship between unemployment and mortality improvement in some areas, a negative relationship in some, and no relationship in others.

There are ways that the model could be improved. By using the Kronecker product on the precision matrices of our structured random effects we could create space-time, space-age, and age-time interactions. This could help to make the model more realistic, as mortality is probably not changing over time and age in the same way at all points in the country. We could also create a model in which the covariates interact with age or time so that rather than looking at how the effect of a covariate, such as unemployment, on mortality changes across space, we could instead explore how it changes across time or the age of the individuals. Both sexes could also be modeled together increasing the dimension of the model and allowing for additional borrowing of strength. Another potential improvement would be accounting for an effect known as spatial confounding, where spatial relationships within predictors might be affecting spatial correlations. By properly accounting for potential spatial confounding we could look more carefully at the precision parameters of the model and draw inference from them.

Our current projects also include examining how clustering the mortality curves by county can help to provide additional insights (Madrigal et al., 2011). Also we are examining how important it would be to disproportionate and often odd sizes of certain county shapes. We acknowledge that this particular study treated large, small, and oddly shaped counties the same and we hope to examine the effect of that in the future.

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### 7 Data Availability Statement

The mortality data can be requested from Division of Vital Statistics of the National Center for Health Statistics. The covariate data and code that support the findings of this study are available from the corresponding author upon reasonable request.

### 8 Author Contributions

Michael Shull: Methodology, Software, Validation, Formal Analysis, Investigation, Data Curation, Writing - Original Draft, Writing - Review & Editing, Visualization. Robert Richardson: Conceptualization, Methodology, Validation, Formal Analysis, Investigation, Resources, Writing - Review & Editing, Supervision, Project administration, Funding acquisition. Chris Groendyke: Methodology, Validation, Formal Analysis, Investigation, Writing - Review & Editing, Visualization. Brian Hartman: Conceptualization, Methodology, Validation, Formal Analysis, Investigation, Resources, Writing - Review & Editing, Supervision, Project administration, Formal Analysis, Investigation, Resources, Writing - Review & Editing, Supervision, Project administration, Funding acquisition.

### 9 Conflicts of Interest

The authors declare no conflicts of interest.

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# Appendix A Mortality and Covariate Plots

Figure 12: The countrywide mortality trends for each age group and sex. The plots on the left are for females and the plots on the right are for males. The top plots are for ages 44 and under, and the bottom plots are for ages 45+.

Unemployment Rate By County in 2010



Figure 13: A map of the contiguous United States showing the unemployment rates in 2010.



Proportion of Heads of Households Who Are White by County in 2010

Figure 14: A map of the contiguous United States showing the proportion of heads of household in the county which are white.

## Appendix B County Adjustments

Table 3 contains the FIPS codes for those counties where we had to adjust the counties for the purpose of the analysis as well as the reason for the adjustment.

Original FIPS	Adjusted FIPS	Reason for Adjustment
08079	08007	Low population
08111	08067	Low population
30055	30085	Low population
30069	30027	Low population
31009	31041	Low population
31075	31033	Low population
46017	46041	Low population
46113	46102	County name and FIPS were changed in 2015
48173	48329	Low population
48259	48275	Inconsistent data
48261	48215	Low population
48269	48275	Low population
48301	48389	Low population
48311	48013	Low population
48443	48465	Low population
49009	49047	Low population
51515	51019	County boundary was adjusted in 2013
51720	51195	Counties were combined

Table 3: Table of FIPS adjustments and justifications

Unemployment data was unavailable in 2005 and 2006 in the state of Louisiana for the 7 counties with the following FIPS codes: 22051, 22071, 22075, 22087, 22089, 22095, and 22103.

We were unable to acquire typical home value data for Montana during 2000-2001 and for North Dakota during 2000-2004.

# Appendix C List of Supplementary Materials

- S1: Results for the female and male models for the non-linear unemployment effect for all 48 states in the model.
- S2: Results for the female and male models for the non-linear race effect for all 48 states in the model, represented by the effect of the percentage of white head of households.
- S3: Results for the female and male models for the non-linear home value effect for all 48 states in the model.