

# A Multivariate Spatiotemporal Model for County Level Mortality Data in the Contiguous United States

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## Abstract

Using a number of modern predictive modeling methods, we seek to understand the factors that drive mortality in the contiguous United States. The mortality data we use is indexed by county and year as well as grouped into 18 different age bins. We propose a model that adds two important contributions to existing mortality studies. First, instead of building mortality models separately by age or treating age as a fixed covariate, we treat age as a random effect. This is an improvement over previous models because it allows the model in one age group to borrow strength and information from other age groups that are nearby. The result is a multivariate spatiotemporal model and is estimated using Integrated Nested Laplace Approximations (INLA). Second, we utilize Gaussian Processes to create nonlinear covariate effects for predictors such as unemployment rate, race, and education level. This allows for a more flexible relationship to be modeled between mortality and these important predictors. Understanding that the United States is expansive and diverse, we also allow for many of these effects to vary by location. The amount of flexibility of our model in how predictors relate to mortality has not been used in previous mortality studies and will result in a more accurate model and a more complete understanding of the factors that drive mortality. Both the multivariate nature of the model as well as the non-linear predictors that have an interaction with space will advance the study of mortality beyond what has been done previously and will allow us to better examine the often complicated relationships between the predictors and mortality in different regions.

Key Words: JEL:C23, Gaussian process, INLA, Bayesian modeling, mortality improvement

## 1 Introduction

Life insurers, among others, are interested in modeling and predicting mortality experience for many different groups. There are many ways that the life experiences of individuals vary across a population. Some of these ways are easily quantifiable such as age, sex, and income. Other things are harder to measure such as happiness, health, and social connection. Even though it is not possible to measure all of the ways that these lives vary, it is nonetheless expected that life experiences will tend to be more similar in areas that are geographically closer to one another.

Before approximately 30 years ago mortality models were largely deterministic. Dickson et al. (2020) give a good overview of the development of these methods. Now stochastic modeling is the standard practice for mortality modeling with Lee and Carter (1992) proposing one of the earliest stochastic models for mortality. Despite their model's shortcomings, such as lack of smoothness across ages and a lack of spatiotemporal interactions, it is an improvement over purely deterministic methods and has good interpretability.

Many variations on the Lee-Carter model have been proposed; the Cairns-Blake-Dowd model (Cairns et al. (2006)) has been widely used for modeling mortality improvements. Cairns et al. (2009) provide an

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overview of the differences and a quantitative comparison between the Carins-Blake-Dowd model and the Lee-Carter model. Melnikov and Romaniuk (2006) and Booth and Tickle (2008) discuss these different models as well as older models and how they have been used for mortality forecasting.

One thing that is missing from all of these models is spatial correlation. It seems quite reasonable that mortality would be similar in areas that are geographically closer as there are similarities in areas that are close together that are not easily measured. Thus, including spatial effects into our modeling could improve our understanding of mortality. Many researchers have incorporated spatial and spatio-temporal effects into their mortality modeling. Clayton and Kaldor (1987) and Manton et al. (1989) were some of the first to use Empirical Bayes to account for spatial correlation. A review of Empirical Bayes and fully Bayesian approaches for modeling spatial variation in mortality rates is given by Bernardinelli et al. (1995). Waller et al. (1997) used a hierarchical Bayesian approach with spatial, temporal, and spatio-temporal effects to model county-level lung cancer death rates in the state of Ohio. Xia and Carlin (1998) studied the same data using a similar technique but also incorporating relevant covariates such as age and smoking prevalence.

More recently Ayele et al. (2015) used Gaussian Markov random fields to account for spatial variation in an additive logistic regression model for child mortality rates in Ethiopia. Dwyer-Lindgren et al. (2016) used a Bayesian approach to fit a hierarchical model. Their model looked at the relationship between the effects at adjacent counties and utilized county-level covariates. Alexander et al. (2017) fit a hierarchical model to obtain subnational mortality estimates. Their model, which smoothed across space and time turned out to be a better fit for mortality data, both simulated U.S. data and real French data divided into 19 age groups, than simpler methods. The improvement was especially noticed in areas with low population.

Recently Gibbs et al. (2020) fit a spatio-temporal model to county-level mortality data from the contiguous United States using conditional auto-regressive priors and a county-varying linear time trend to each age group. Here we utilize the same data and build on this model in two significant ways.

First we model all age groups together in a multivariate spatio-temporal model. Using a multivariate approach adds significant complexity to the model but improves the model in a similar way that adding spatial or temporal correlation would, by allowing data at one age group to borrow strength from data at neighboring age groups (Royle and Berliner, 1999; Gelfand, 2021). We address the added complexity by using strong Markov assumptions on the correlations and estimate the model parameters with Integrated Nested Laplace Approximations (INLA) (Blangiardo and Cameletti, 2015). This is one of the first significant applications modeled using multivariate INLA models for spatio-temporal data (Vicente et al., 2020).

The other novel element in our mortality model is including non-linear and spatially-varying covariate effects. This allows for an unprecedented degree of flexibility in how variables such as employment, education level, and others affect mortality. Functionally, the covariates are treated as processes on the covariate space and are given Gaussian process priors (Shi and Choi, 2011). By allowing these to vary across space we remove the restriction that the predictor variables affect mortality equally across the whole country. It was seen in Gibbs et al. (2020) that this improved the model and helped interpret the importance of the covariates.

The remainder of this article is organized as follows: Section 2 discusses the data that are analyzed in this study, Section 3 introduces the statistical models, methods, and notation that are used to perform the analysis, Section 4 conveys and discusses the results of this analysis, and Section 5 concludes with discussion of model limitations and potential future work.

## 2 Data

In this analysis we use data from Division of Vital Statistics of the National Center for Health Statistics, part of the Centers for Disease Control and Prevention. The data contain information about every death that occurred in the United States from 2000 to 2017 along with demographic information including age, sex, county of residence, county of death, race, and marital status. In this analysis we only use county of residence (and not county of death), and restrict our attention to those counties of residence belonging to the contiguous United States.

Using census data with interpolation, we are able to obtain the age group mortality exposure for all of the counties during this time period. The census data are available with ages being placed in 18 buckets, with the first being for those 0-4 years of age and the last being those individuals who are 85 years old or older. The mortality data are divided into the same age buckets so that for each sex, county, year, and age

## 1000 Times the Observed Mortality Rate for Females 55-59 in 2010

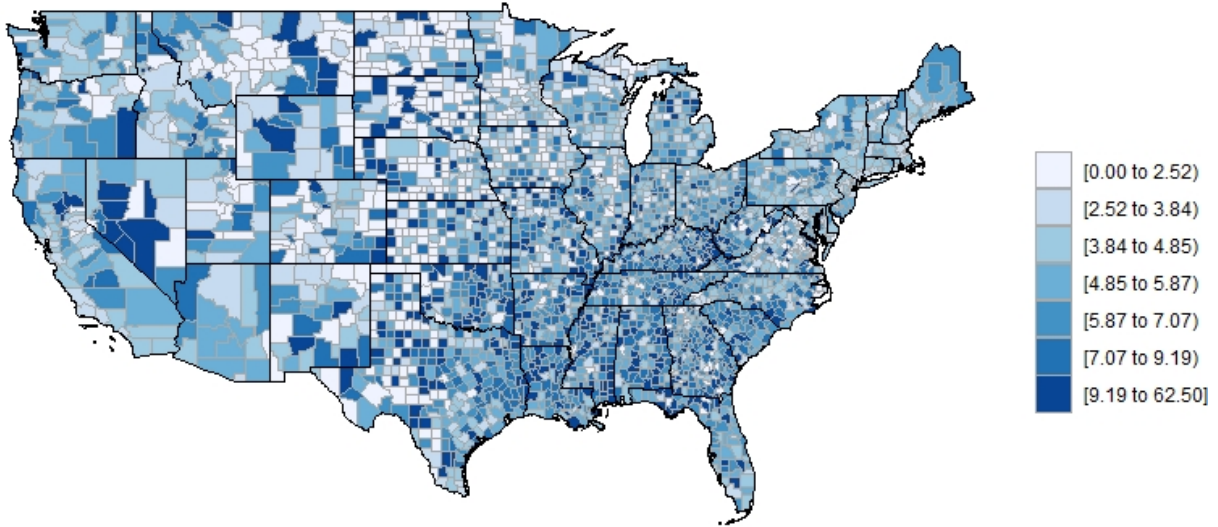


Figure 1: The observed mortality rate for each county in the contiguous United States for females aged 55-59 in 2010. Mortality rates are multiplied by 1000 for readability.

group, we had an exposure and the number of people who died.

As the census data are only gathered every ten years, the majority of the population data we have are estimates. However, we have exact information on the number of deaths. This mismatch results in some anomalies whereby there are some sex, county, year, age group combinations where the data indicate that there are more people who died than were living in the county at the time. There are also some counties whose populations are quite small; after dividing the counties by sex and into 18 age groups, there are some counties whose population for a given group was zero in a given year. Counties which have an exposure of zero or more deaths than exposures are combined with their neighbor with the largest population. After such modification, we have 3092 counties that we consider in the model. A full list of counties which were combined can be found in Appendix A. We use the term “county” to refer to the subdivisions throughout all states in the United States, including Louisiana where they are actually parishes.

Figures 1 and 2 show the observed mortality for females age 55-59 and 85+, respectively, and provide evidence of some spatial correlation. Mortality tends to be higher in the south and lower in the Midwest. Similar trends exist across both sexes and other age groups. Figure 3 shows the observed countrywide mortality rate for each age group and sex combination. In general, males have higher mortality than females. Mortality increases with age, with the exception that the youngest group sees a mortality improvement upon reaching age 5. Across time there appears to be mortality improvement for those individuals 45+.

Six different covariates are used in this analysis: Unemployment, Race, Home Value, Education, Marital Status, and Household Size. These covariates are measured at different frequencies (see Table 1). The covariates of unemployment, race, and home value are divided into 20 different groups based on quantiles. The remaining covariates of education, marital status, and household size are divided into groups where each group consists of the states that have the same value for the covariate. Those covariates have respectively 44, 43, and 29 unique values and so that is the number of resulting groups. The arithmetic average of the values in each group is used as the “location” for that group for calculating a Matérn correlation between points. The covariates appear to also have some strong spatial dependence. In Figures 4 and 5, unemployment and race values for 2010 are plotted on the map. There is clear spatial dependence in both of these graphs. The unemployment data are complete except for a few counties in Louisiana for 2005 and 2006 (see Appendix B). (These correspond to the parts of Louisiana that were most heavily affected by hurricane Katrina.) Similarly

## 1000 Times the Observed Mortality Rate for Females 85+ in 2010

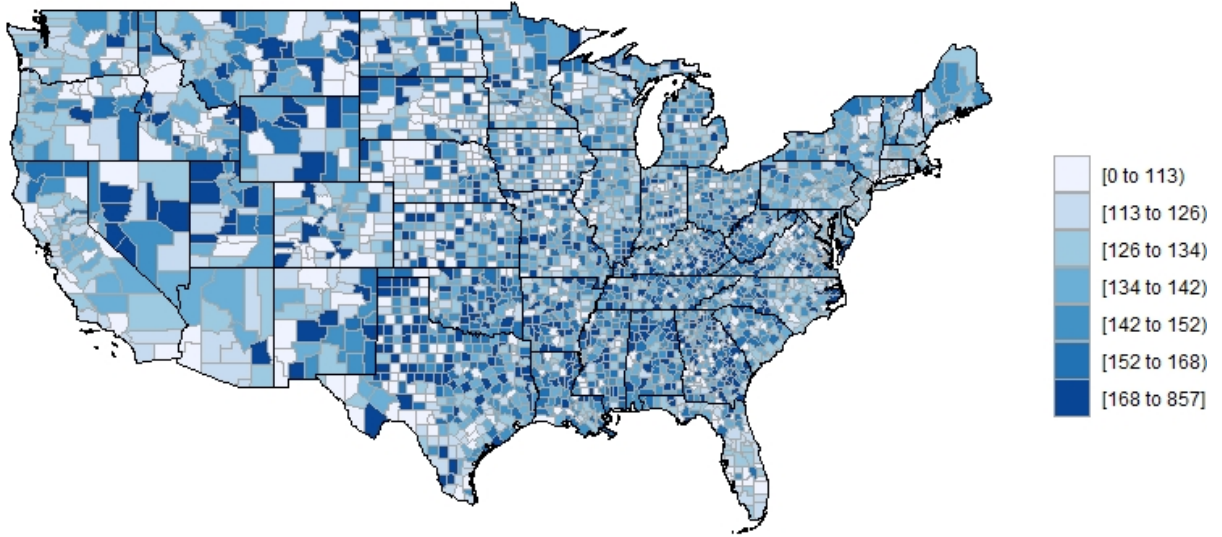


Figure 2: The observed mortality rate for each county in the contiguous United States for females aged 85+ in 2010. Mortality rates are multiplied by 1000 for readability.

| Covariate Name | Meaning  | Measuring Frequency        | Source                         |
|----------------|--|----------------------------|--------------------------------|
| Unemployment   | Unemployment rate  | Every County in Every Year | Bureau of Labor and Statistics |
| Race           | Percent of head of households that are white                         | Every County in 2010       | Census Bureau                  |
| Home Value     | Typical value of a home  | Every State in Every Year  | Zillow                         |
| Education      | Percent of people 25 or older who have a bachelor's degree or higher | Every State in 2010        | Census Bureau                  |
| Marital Status | Percent of people 25 or older who are married but not separated      | Every State in 2010        | Census Bureau                  |
| Household Size | Average household size   | Every State in 2010        | Census Bureau                  |

Table 1: Covariates used in the model along with what they are measuring, frequency of measurement, and the source of the data.

we are missing home value information (Appendix C) for a few states in the early 2000s. These missing data were dealt with by simply letting those affected observations not have an unemployment and home value effect during those times.

### 3 Methods

For each sex we fit a distinct model, which are identical in form. Let  $y_{akt}$  be the number of deaths that occurred within age group  $a$  in county  $k$  during year  $t$ . In this application,  $a \in \{1, 2, \dots, 18\}$ ,  $k \in \{1, 2, \dots, 3092\}$ , and  $t \in \{1, 2, \dots, 18\}$ . The number of people within age group  $a$  in county  $k$  during year  $t$  is known, and so the binomial likelihood is appropriate for this data:

$$y_{akt} | \pi_{akt} \sim \text{Binomial}(n_{akt}, \pi_{akt})$$

Here  $\pi_{akt}$  is the annual mortality probability for someone who is in age group  $a$  in county  $k$  during year  $t$  and  $n_{akt}$  is the corresponding population count. We can then relate  $\pi_{akt}$  to the desired effects using the logit link function:

$$\ln\left(\frac{\pi_{akt}}{1 - \pi_{akt}}\right) = \beta_0 + \sum_{i=1}^3 F_i(\mathbf{x}_k) + \sum_{i=1}^3 G_{is}(\mathbf{x}_{kt}) + \phi_k + \delta_t + \psi_a + \gamma_{akt}$$

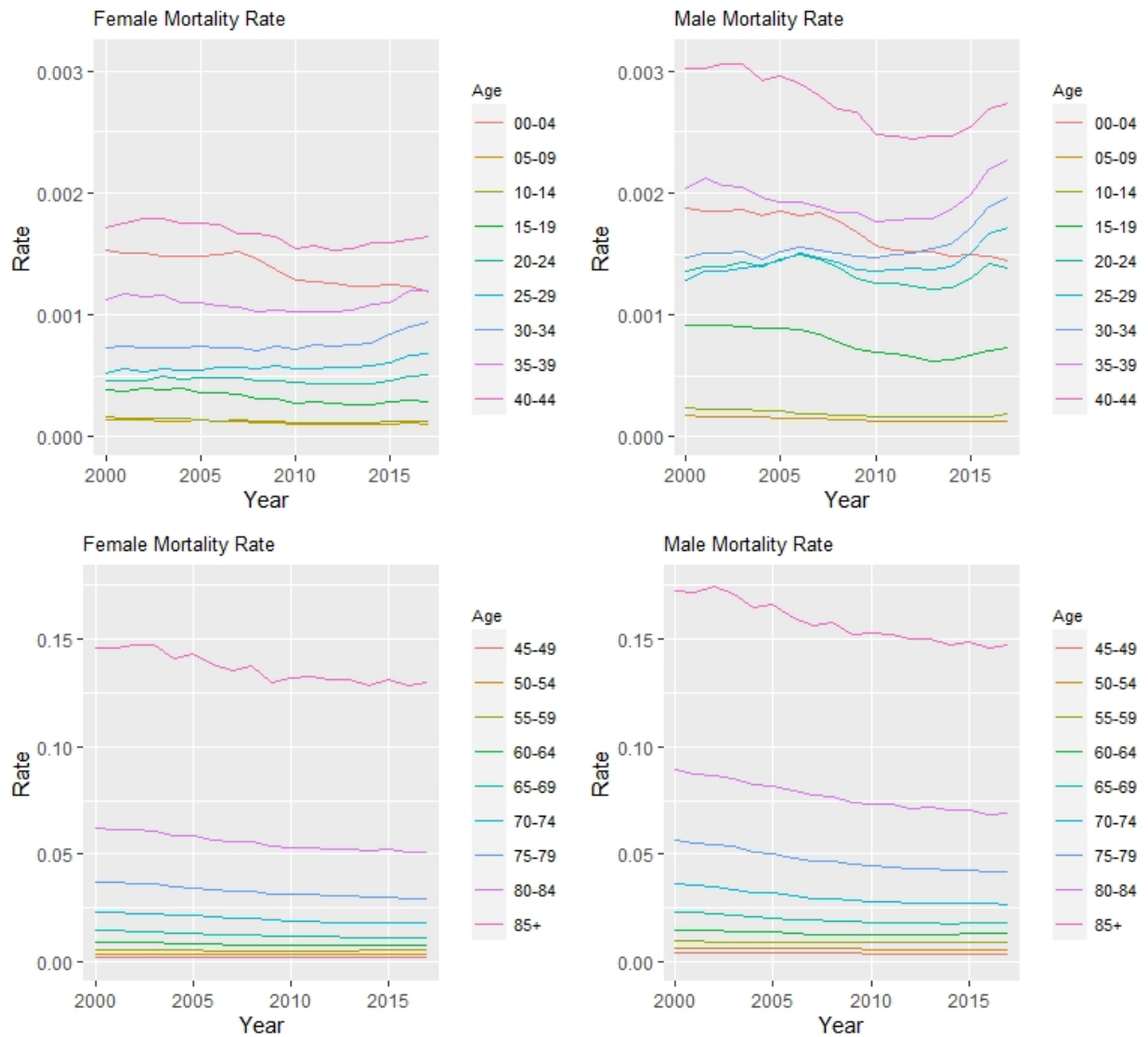


Figure 3: The countrywide mortality trends for each age group and sex. The plots on the left are for females and the plots on the right are for males. The top plots are for ages 44 and under, and the bottom plots are for ages 45+.

### Unemployment Rate By County in 2010

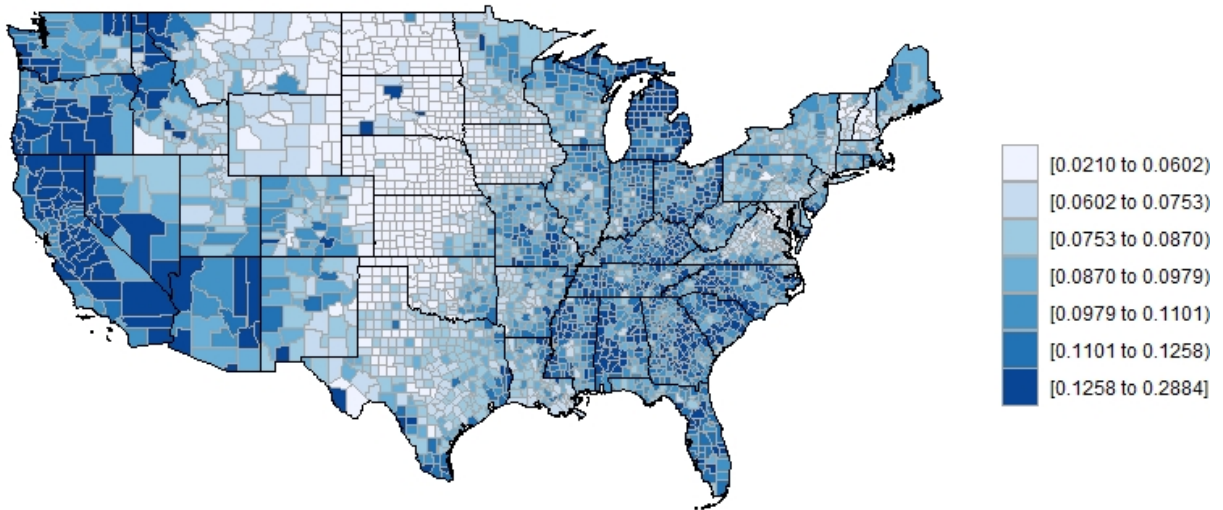


Figure 4: A map of the contiguous United States showing the unemployment rates in 2010.

### Proportion of Heads of Households Who Are White by County in 2010

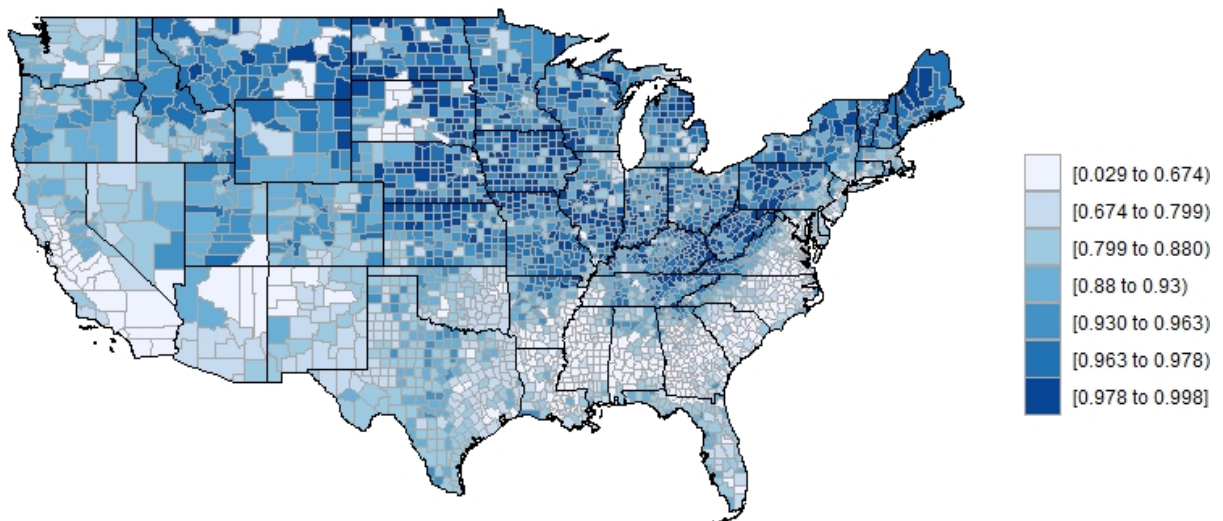


Figure 5: A map of the contiguous United States showing the proportion of heads of household in the county which are white.

The parameter  $\beta_0$  is an overall intercept for the model. Each  $F_i$  ( $i \in \{1, 2, 3\}$ ) is a nonlinear covariate effect for the covariates of education, marital status, and household size. For each of these covariate effects we use a Gaussian process with the Matérn correlation function. This is a flexible way to allow the covariates to have nonlinear effects on mortality. Similarly the  $G_{i,s}$  are the nonlinear covariate effects for unemployment, race, and home value. Once again these are Gaussian Processes with the Matérn correlation function; however, instead of simply having one effect for the entire dataset we allow each state to have its own effect. For the purpose of these effects we define a state to be one of the 48 contiguous states in the United States (all but Alaska and Hawaii) and Washington D.C. We impose a conditional autoregressive prior on the different state effects so that, conditional on all the other states, the effect for a given state only depends upon those states with which it shares a border. For identifiability, all of the individual covariate effects have a sum to zero constraint.

The spatial effect is broken down into two components:

$$\begin{aligned}\phi_k &= u_k + v_k \\ \mathbf{u}|\tau_u &\sim \mathcal{N}(\mathbf{0}, \tau_u^{-1}\mathbf{I}) \\ \mathbf{v}|\tau_v &\sim \mathcal{N}(\mathbf{0}, \tau_v^{-1}\mathbf{W}^{-1})\end{aligned}$$

with precision parameters  $\tau_u$  and  $\tau_v$ , where  $\mathbf{u}$  is our iid spatial effect and  $\mathbf{v}$  is our structured spatial effect. For identifiability,  $\mathbf{u}$  and  $\mathbf{v}$  have a sum to zero constraint. Our structured effect follows the type of conditional autoregressive model proposed by Besag et al. (1991). We say that county  $i$  and county  $j$  are neighbors if they share a border and denote this relationship as  $i \sim j$ . We denote the number of neighbors of county  $i$  by  $n_i$ . Note that in our specification, a county is not its own neighbor.  $\mathbf{W}$  is a county adjacency matrix where, for  $i \neq j$ ,  $W_{ij} = 1$  if  $i \sim j$  and  $W_{ij} = 0$  if  $i \not\sim j$ . The diagonal entries are  $W_{ii} = -n_i$ . This creates a conditional independence where conditioned on all other counties, the effect for a given county only depends upon those counties with which it is a neighbor. This structure is more apparent if we write the effect as:

$$v_i|v_j \sim \mathcal{N}\left(\frac{1}{n_i} \sum_{j:i \sim j} v_j, \frac{1}{n_i} \tau_v^{-1}\right) \quad \text{for } j \neq i$$

However, both formulations are equivalent.

The temporal effect is also broken down into two components:

$$\begin{aligned}\delta_t &= b_t + c_t \\ \mathbf{b}|\tau_b &\sim \mathcal{N}(\mathbf{0}, \tau_b^{-1}\mathbf{I}) \\ \mathbf{c}|\tau_c &\sim \mathcal{N}(\mathbf{0}, \tau_c^{-1}\mathbf{R}_c^{-1})\end{aligned}$$

with precision parameters  $\tau_b$  and  $\tau_c$  and structure matrix  $\mathbf{R}_c$ , where  $\mathbf{b}$  is the iid temporal effect and  $\mathbf{c}$  is the structured temporal effect. For identifiability,  $\mathbf{b}$  and  $\mathbf{c}$  have a sum to zero constraint. The structured effect follows a random walk of order 1 so that  $\mathbf{R}_c$  is an  $18 \times 18$  tridiagonal matrix of the form:

$$\mathbf{R}_c = \begin{bmatrix} -1 & 1 & 0 & \cdots & 0 & 0 & 0 \\ 1 & -2 & 1 & \cdots & 0 & 0 & 0 \\ 0 & 1 & -2 & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -2 & 1 & 0 \\ 0 & 0 & 0 & \cdots & 1 & -2 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 1 & -1 \end{bmatrix} \quad (1)$$

This implies that, given  $c_{t-1}$  and  $c_{t+1}$ ,  $c_t$  is conditionally independent of  $c_{t^*}$  for all other  $t^*$  not equal to  $t-1, t$ , or  $t+1$ .

We then have an age group effect again broken down into two components:

$$\begin{aligned}\psi_a &= f_a + g_a \\ \mathbf{f}|\tau_f &\sim \mathcal{N}(\mathbf{0}, \tau_f^{-1}\mathbf{I}) \\ \mathbf{g}|\tau_g &\sim \mathcal{N}(\mathbf{0}, \tau_g^{-1}\mathbf{R}_g^{-1})\end{aligned}$$

| Model            | DIC (Female) | DIC (Male) |
|------------------|--------------|------------|
| Full Model       | 3,817,853    | 4,266,276  |
| Only Countrywide | 3,818,372    | 4,266,666  |
| No Covariates    | 3,819,075    | 4,266,790  |

Table 2: Deviance Information Criterion (DIC) for the three different model versions that were fit to both the male and female data.

with precision parameters  $\tau_f$  and  $\tau_g$  and structure matrix  $\mathbf{R}_g$ , where  $\mathbf{f}$  is the iid age group effect and  $\mathbf{g}$  is the structured age group effect. For identifiability,  $\mathbf{f}$  and  $\mathbf{g}$  have a sum to zero constraint. The structured effect follows a random walk of order 1 so we have that  $\mathbf{R}_g$  is an  $18 \times 18$  tridiagonal matrix that coincidentally is the same as  $\mathbf{R}_c$  from Equation 1 because the number of age groups and the number of time points happen to be the same. Then, given  $g_{a-1}$  and  $g_{a+1}$ ,  $g_a$  is conditionally independent of  $g_{a^*}$  for all other  $a^*$  not equal to  $a - 1, a$ , or  $a + 1$ .

Finally we have an iid error term:

$$\gamma_{akt} \stackrel{iid}{\sim} \mathcal{N}(0, \tau_\gamma^{-1})$$

where  $\tau_\gamma$  is a precision parameter. All precision parameters ( $\tau$ ) are given identical gamma priors with mean 2000 and variance 4000000.

Rue et al. (2009) proposed a deterministic method for performing Bayesian inference using the Integrated Nested Laplace Approximation (INLA) which is implemented in software called R-INLA (Lindgren and Rue, 2015). For a given model, the computation time in R-INLA tends to be much faster than traditional MCMC algorithms that have been used for exploring the posterior of a model. As a result, this methodology has become quite popular in recent years. Blangiardo and Cameletti (2015) is a nice textbook on the theory and implementation of basic spatiotemporal inference using the package. Rue et al. (2017) and Bakka et al. (2018) provide reviews of INLA and how it works with spatial data. We use this method for our computation as we have a large problem and want to be able to conduct our inference quickly.

## 4 Results

The model described in Section 3 was fit to the data described in Section 2. We tried three different versions of the model. Specifically, we used one version where we eliminated all the covariate effects ( $F_i$ 's and  $G_{is}$ 's), a second version where we used only countrywide effects and no state-specific covariate effects:

$$\ln \left( \frac{\pi_{akt}}{1 - \pi_{akt}} \right) = \beta_0 + \sum_{i=1}^6 F_i(\mathbf{x}_k) + \phi_k + \delta_t + \psi_a + \gamma_{akt},$$

and a third version which was the full model described in Section 3. Table 2 shows the Deviance Information Criterion (DIC) value obtained for each model fit on both sets of data. Since the DIC was lowest for the full model proposed in Section 3, this model was chosen for the remainder of the analysis.

Figures 6 and 7 show the posterior mean of the spatial effect ( $\phi_k$ ) for the counties in the United States. We see that in both cases the spatial patterns of the data appear to be very similar. We also see the same pattern manifesting itself that we saw earlier where there tends to be higher mortality in the South and lower mortality in the upper Midwest.

One important aspect of the results to consider is how mortality is changing over time. Figure 8 shows the posterior mean and 95% credible interval for the temporal effects ( $\delta_t$ ). From 2000 through roughly 2014, the trend for both males and females is of mortality improvement over time (i.e., a decrease in  $\delta_t$ ), though the patterns are not smooth or monotonic. Around 2014, there begins to be a small uptick in the mortality. One potential cause for this increase could be a spike in the “deaths of despair” that are often discussed in the mortality literature (see, for example, Scutchfield and Keck (2017)).

Figure 9 shows the posterior mean and 95% credible interval for the age group effects ( $\psi_t$ ). These results are consistent with the results seen in the raw data with respect to the age group. Namely, mortality improves



Combined Spatial Effect

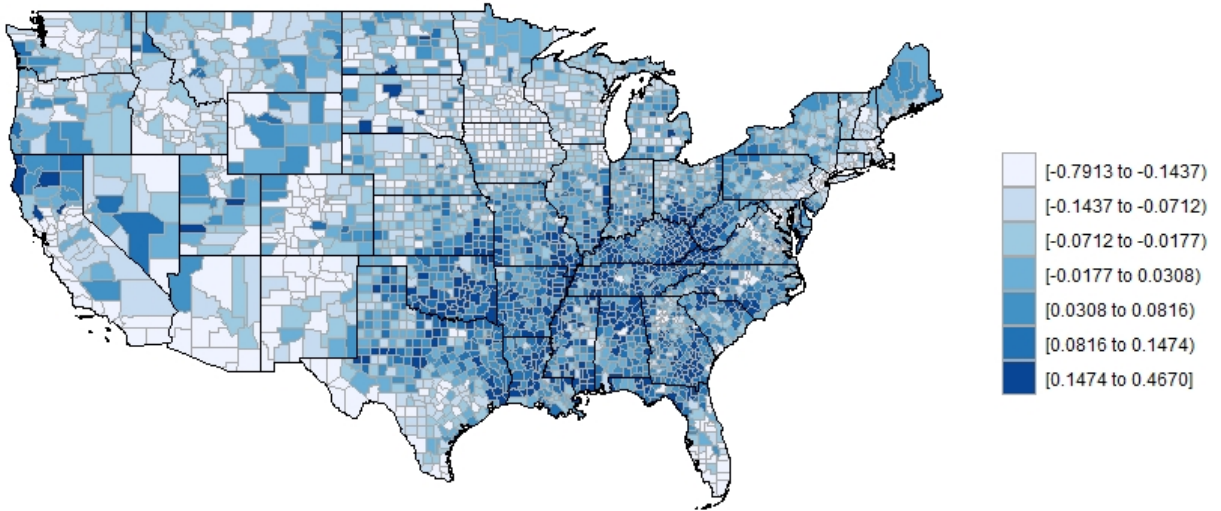


Figure 6: Posterior mean of the spatial effect ( $\phi_k$ ) for the model fit to the female data.

Combined Spatial Effect

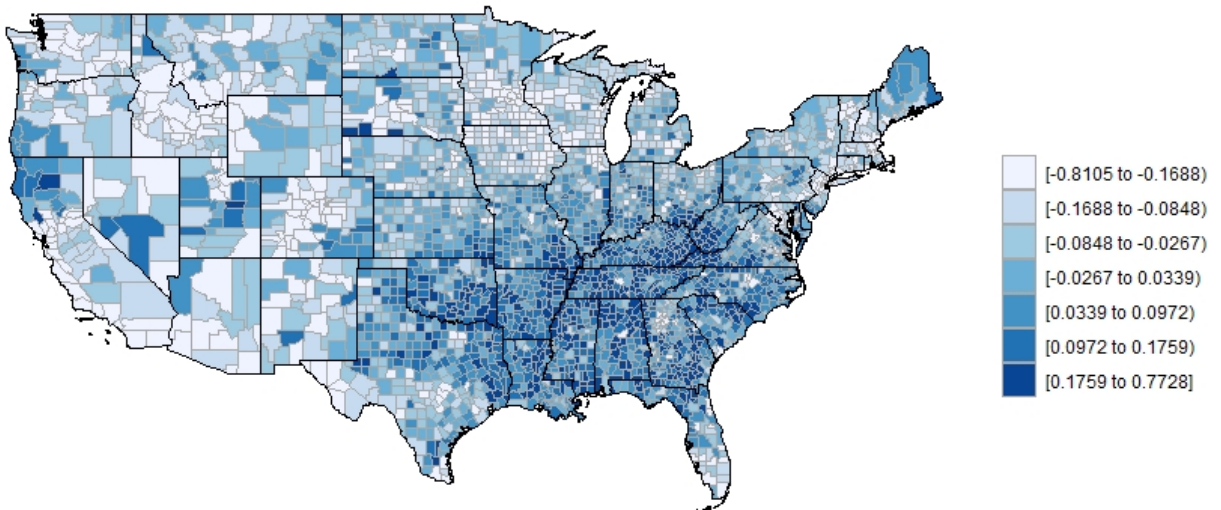


Figure 7: Posterior mean of the spatial effect ( $\phi_k$ ) for the model fit to the male data.

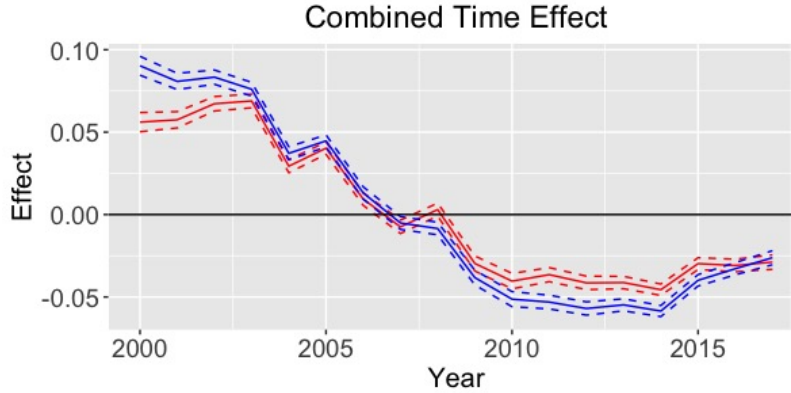


Figure 8: Posterior mean and 95% Credible interval of the temporal effects ( $\delta_t$ ). Male values are in blue and female values are in red.

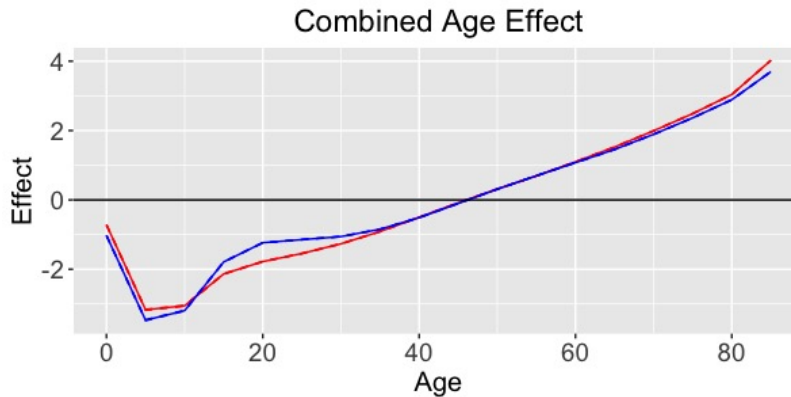


Figure 9: Posterior mean and 95% Credible interval of the age group effects ( $\psi_t$ ). Male values are in blue and female values are in red.

upon aging out of the youngest age group, but then consistently deteriorates as age increases. The pattern is seen to be quite similar between the males and females.

Figures 10 and 11 display the posterior means and 95% credible intervals for the covariate effects corresponding to education, marital status, and household size. Although the entire credible interval contains 0 for each of these covariates, they nonetheless present nice illustrations of the information that can be obtained when the covariates are allowed to have nonlinear effects. As an example, consider the case of the education covariate. If this had been treated as a linear effect, the result would like have been a slight negative slope. However, just having a negative slope does not communicate as much information as our nonlinear effect. It can be seen from the figure that, rather than having a monotonic linear relationship, there is a trend of slow decrease in mortality as education increases, until the percentage of people with a Bachelor's degree hits 25%. Then follows a quick decline in mortality followed by a leveling off. Thus demonstrates the added flexibility and more nuanced conclusions that can be obtained by utilizing nonlinear covariate effects.

Figure 12 plots the posterior means and 95% credible intervals for the unemployment effects in the states of California, Colorado, and North Carolina. These plots show the interaction between space (geographic location) and the impact of unemployment on female mortality. In Colorado, it can be seen that the relationship between mortality and unemployment is minimal. North Carolina and California, on the other hand, each have significant nonlinear effects; moreover, the shape of these effects are quite different between the two states. Similar plots showing these state unemployment effects for each of the states can be found in Appendix D. Similar ideas for the covariates corresponding to race and home value can be seen in Figures 13 and 14, respectively. For example, the effect of race on female mortality can be seen to have a significantly

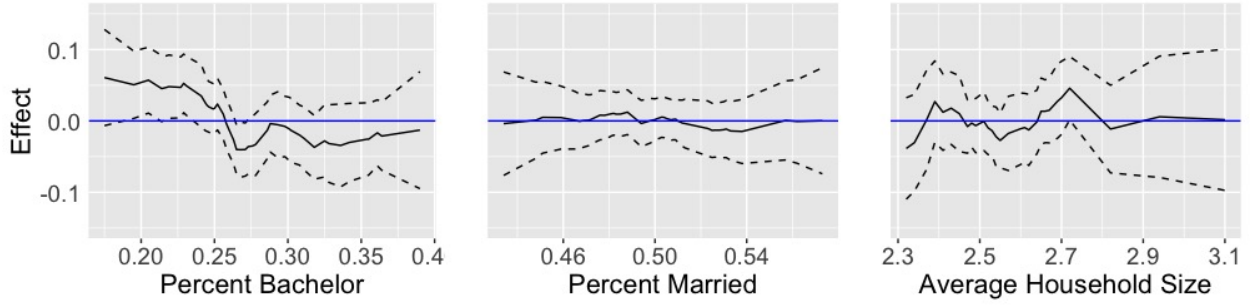


Figure 10: Posterior mean and 95% credible interval for the covariate effects ( $F_i(x_k)$ ) for the model fit to the female data. The effects displayed correspond to education (left,  $i = 1$ ), marital status (center,  $i = 2$ ), and household size (right,  $i = 3$ ).

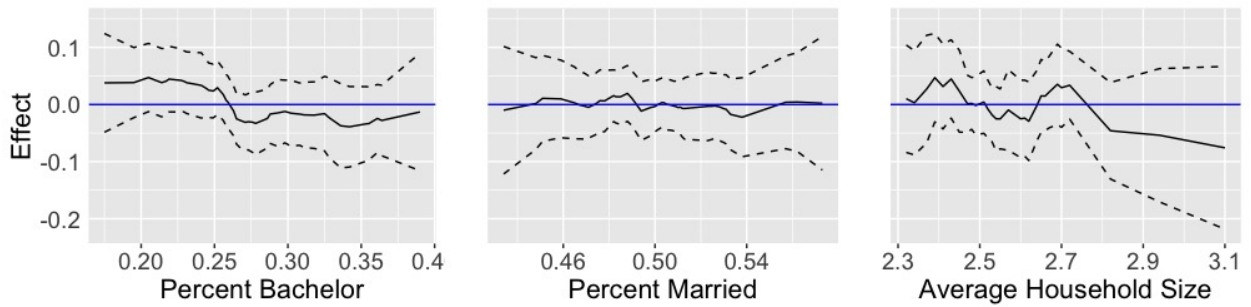


Figure 11: Posterior mean and 95% credible interval for the covariate effects ( $F_i(x_k)$ ) for the model fit to the male data. The effects displayed correspond to education (left,  $i = 1$ ), marital status (center,  $i = 2$ ), and household size (right,  $i = 3$ ).

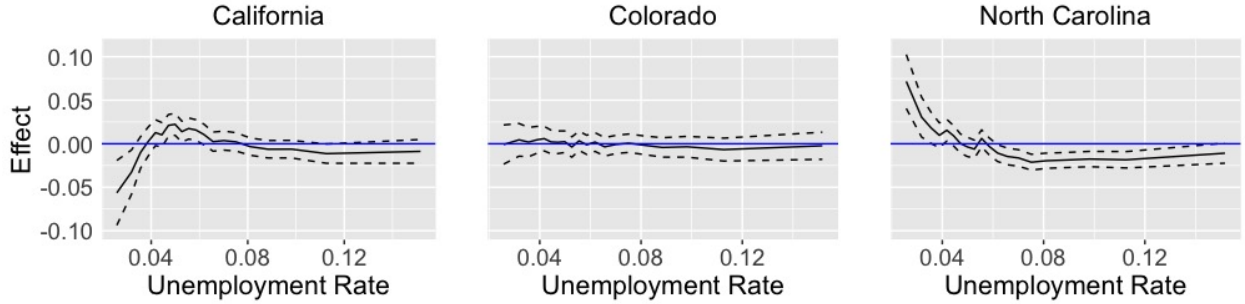


Figure 12: Posterior mean and 95% credible interval of the unemployment effects ( $G_{1s}(x_{kt})$ ) for selected states for the model fit to the female data.

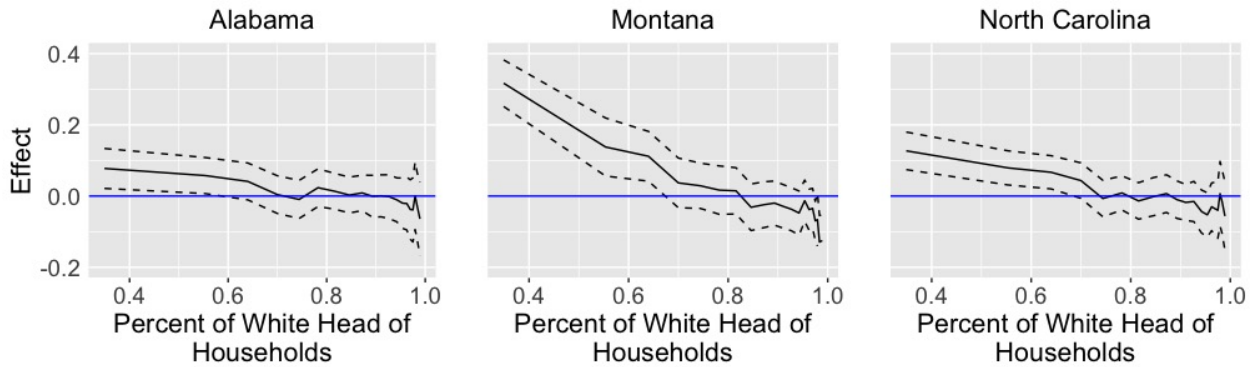


Figure 13: Posterior mean and 95% credible interval of the race effects ( $G_{2s}(x_{kt})$ ) for selected states for the model fit to the female data.

steeper slope in the state of Montana than it does in either Alabama or North Carolina. All of the state effects for race and home value can be found in Appendices E and F respectively.

Figures 15 and 16 show the posterior means of the mortality rates for females aged 55-59 and 85+ respectively. These are the same groups corresponding to the observed mortality rates displayed in Figures 1 and 2 in Section 2. The fitted values display similar general patterns to those seen in the observed data. One difference that is noted, however, is that the fitted mortality rates exhibit a greater degree of spatial correlation than do the observed ones. This is to be expected, as the fitted model allows for spatial correlations.

## 5 Conclusion

We have fit a multivariate spatio-temporal model to mortality data in the contiguous United States. This model has built on the existing mortality modeling literature in two significant ways. First, we model all age groups together to create a multivariate spatio-temporal model. This allows for the borrowing of information not only across space and time but also across the different age groups of the model.

The other significant contribution is the inclusion of nonlinear and spatially-varying nonlinear covariate effects on mortality. These nonlinear and spatially-varying covariate effects allow us to see how things such as education, unemployment and race affect mortality and how those effects change over space. By including a nonlinear education effect, we were able to see that while mortality generally improves as education increases, there becomes a point where additional education seems to no longer provide additional mortality improvement. Allowing the covariates effects to change over space allowed us to observe both how the magnitude of the effect changes across states, as we saw for race and home value, and how the shape of the effect changes across state values, as we saw for unemployment. This flexibility allowed us to observe

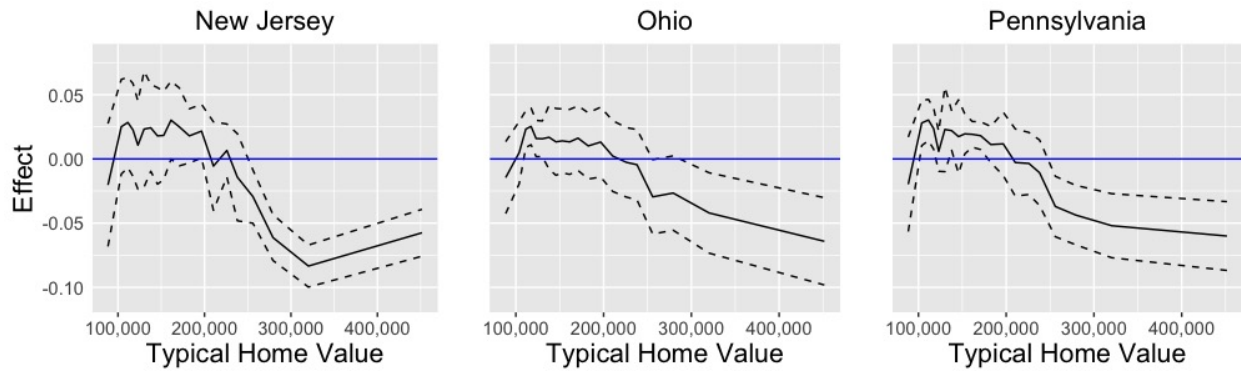


Figure 14: Posterior mean and 95% credible interval of the home value effects ( $G_{3s}(x_{kt})$ ) for selected states for the model fit to the female data.

1000 Times the Fitted Mortality Rate for Females 55-59 in 2010

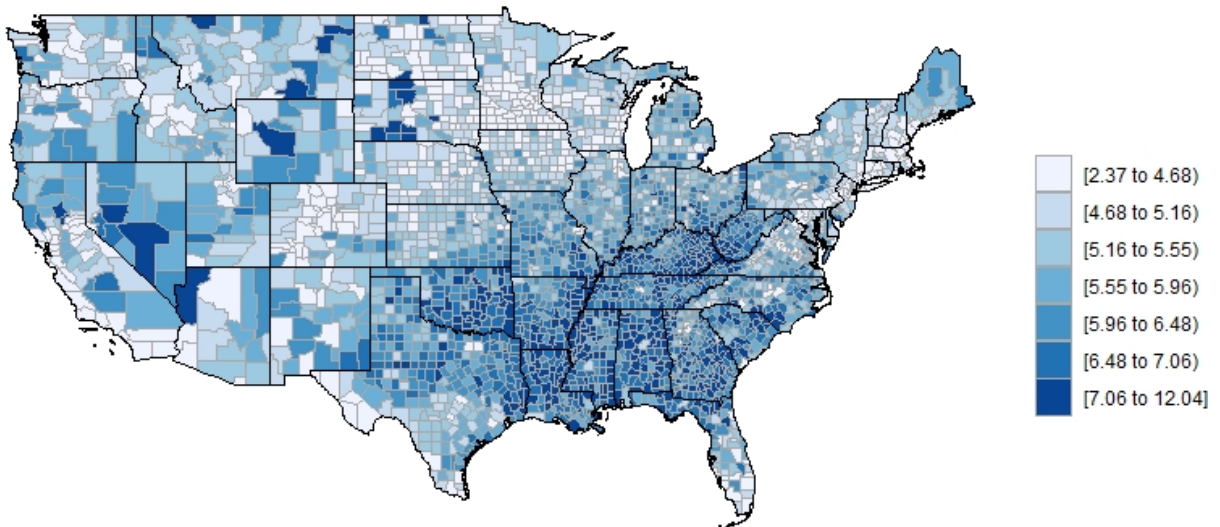


Figure 15: The posterior mean of the mortality rate for females aged 55-59 in 2010. The mortality rate is multiplied by 1000 for readability.

## 1000 Times the Fitted Mortality Rate for Females 85+ in 2010

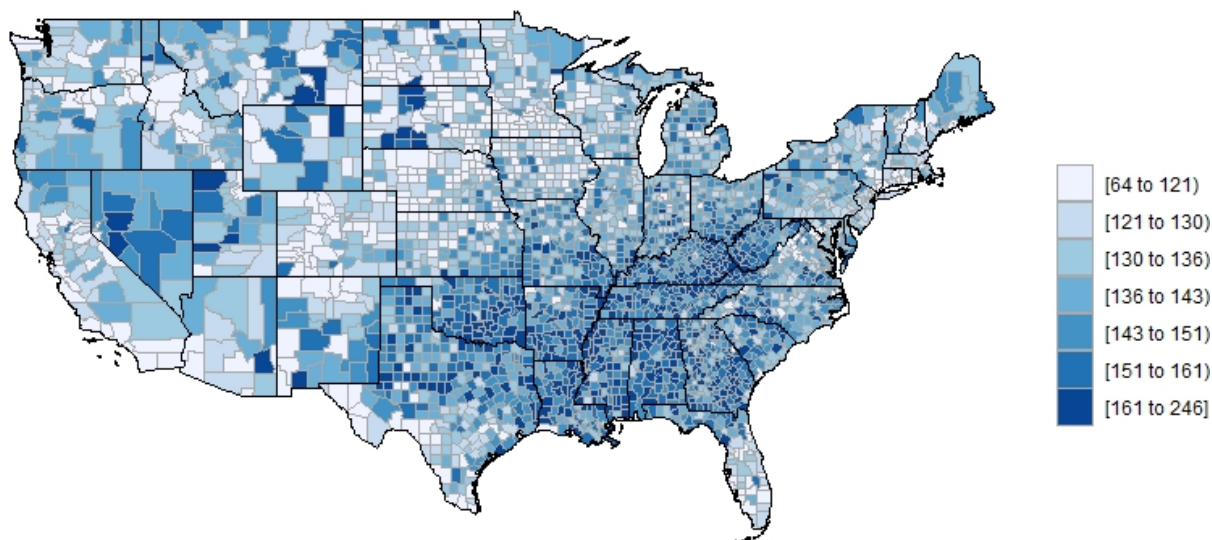


Figure 16: The posterior mean of the mortality rate for females aged 85+ in 2010. The mortality rate is multiplied by 1000 for readability.

some otherwise undetectable trends, such as the positive relationship between unemployment and mortality improvement in some areas, a negative relationship in some, and no relationship in others.

There are ways that the model could be improved. By using the Kronecker product on the precision matrices of our structured random effects we could create space-time, space-age, and age-time interactions. This could help to make the model more realistic, as mortality is probably not changing over time and age in the same way at all points in the country. We could also create a model in which the covariates interact with age or time so that rather than looking at how the effect of a covariate, such as unemployment, on mortality changes across space, we could instead explore how it changes across time or the age of the individuals. Both sexes could also be modeled together increasing the dimension of the model and allowing for additional borrowing of strength.

## 6 Acknowledgements

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## 7 Author Contributions

**Michael Shull:** Methodology, Software, Validation, Formal Analysis, Investigation, Data Curation, Writing - Original Draft, Writing - Review & Editing, Visualization. **Robert Richardson:** Conceptualization, Methodology, Validation, Formal Analysis, Investigation, Resources, Writing - Review & Editing, Supervision, Project administration, Funding acquisition. **Chris Groendyke:** Methodology, Validation, Formal Analysis, Investigation, Writing - Review & Editing, Visualization. **Brian Hartman:** Conceptualization, Methodology, Validation, Formal Analysis, Investigation, Resources, Writing - Review & Editing, Supervision, Project administration, Funding acquisition.

## 8 Conflicts of Interest

The authors declare no conflicts of interest.

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## Appendix A County Adjustments

Table 3 contains the FIPS codes for those counties where we had to adjust the counties for the purpose of the analysis as well as the reason for the adjustment.

| Original FIPS | Adjusted FIPS | Reason for Adjustment                     |
|---------------|---------------|---|
| 08079         | 08007         | Low population                            |
| 08111         | 08067         | Low population                            |
| 30055         | 30085         | Low population                            |
| 30069         | 30027         | Low population                            |
| 31009         | 31041         | Low population                            |
| 31075         | 31033         | Low population                            |
| 46017         | 46041         | Low population                            |
| 46113         | 46102         | County name and FIPS were changed in 2015 |
| 48173         | 48329         | Low population                            |
| 48259         | 48275         | Inconsistent data                         |
| 48261         | 48215         | Low population                            |
| 48269         | 48275         | Low population                            |
| 48301         | 48389         | Low population                            |
| 48311         | 48013         | Low population                            |
| 48443         | 48465         | Low population                            |
| 49009         | 49047         | Low population                            |
| 51515         | 51019         | County boundary was adjusted in 2013      |
| 51720         | 51195         | Counties were combined                    |

Table 3: Table of FIPS adjustments and justifications



## Appendix B Counties with Unavailable Unemployment Data

Unemployment data was unavailable in 2005 and 2006 in the state of Louisiana for the 7 counties with the following FIPS codes: 22051, 22071, 22075, 22087, 22089, 22095, and 22103.

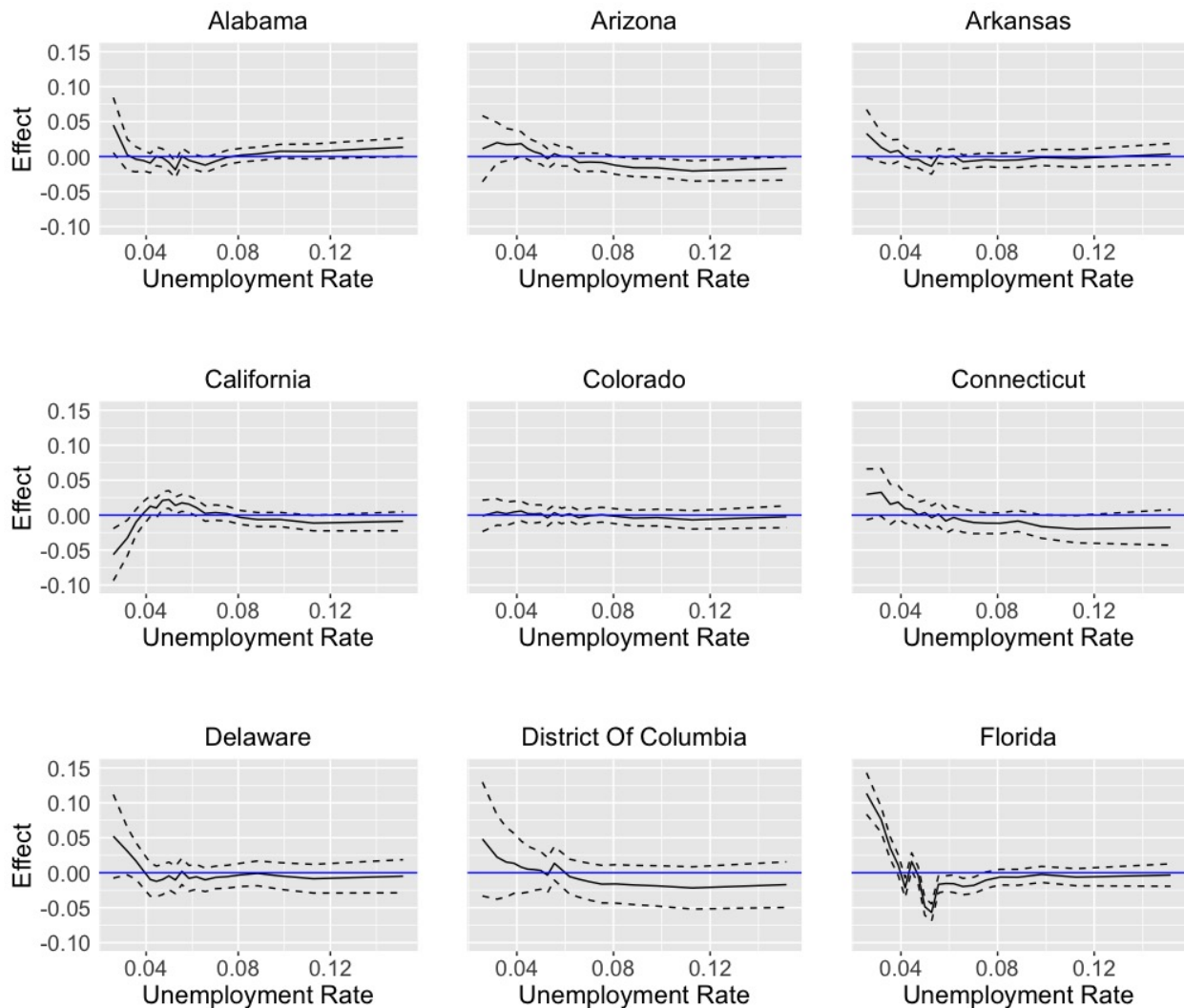
## Appendix C States with Unavailable Home Value Data

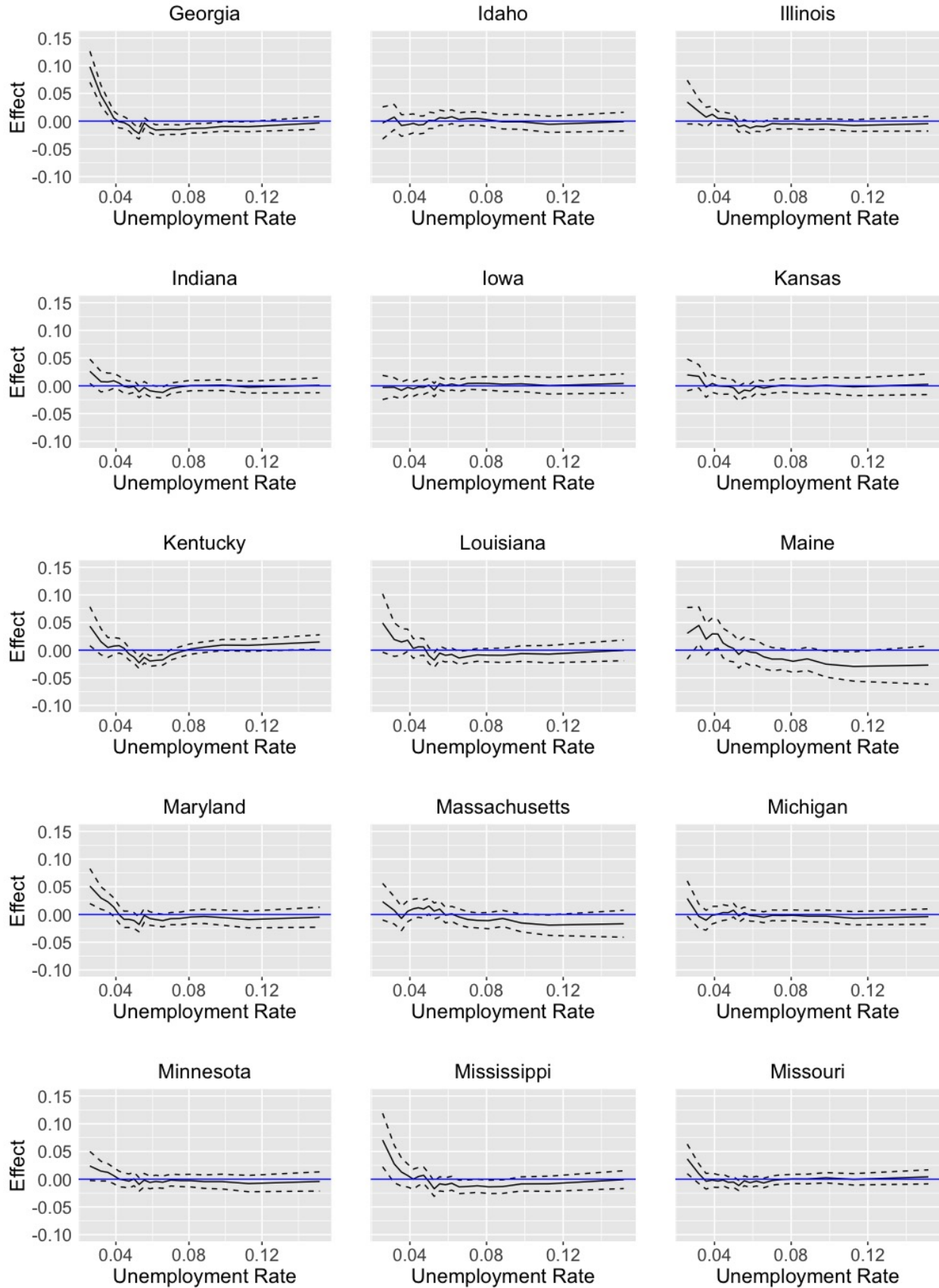
We were unable to acquire typical home value data for Montana during 2000-2001 and for North Dakota during 2000-2004.

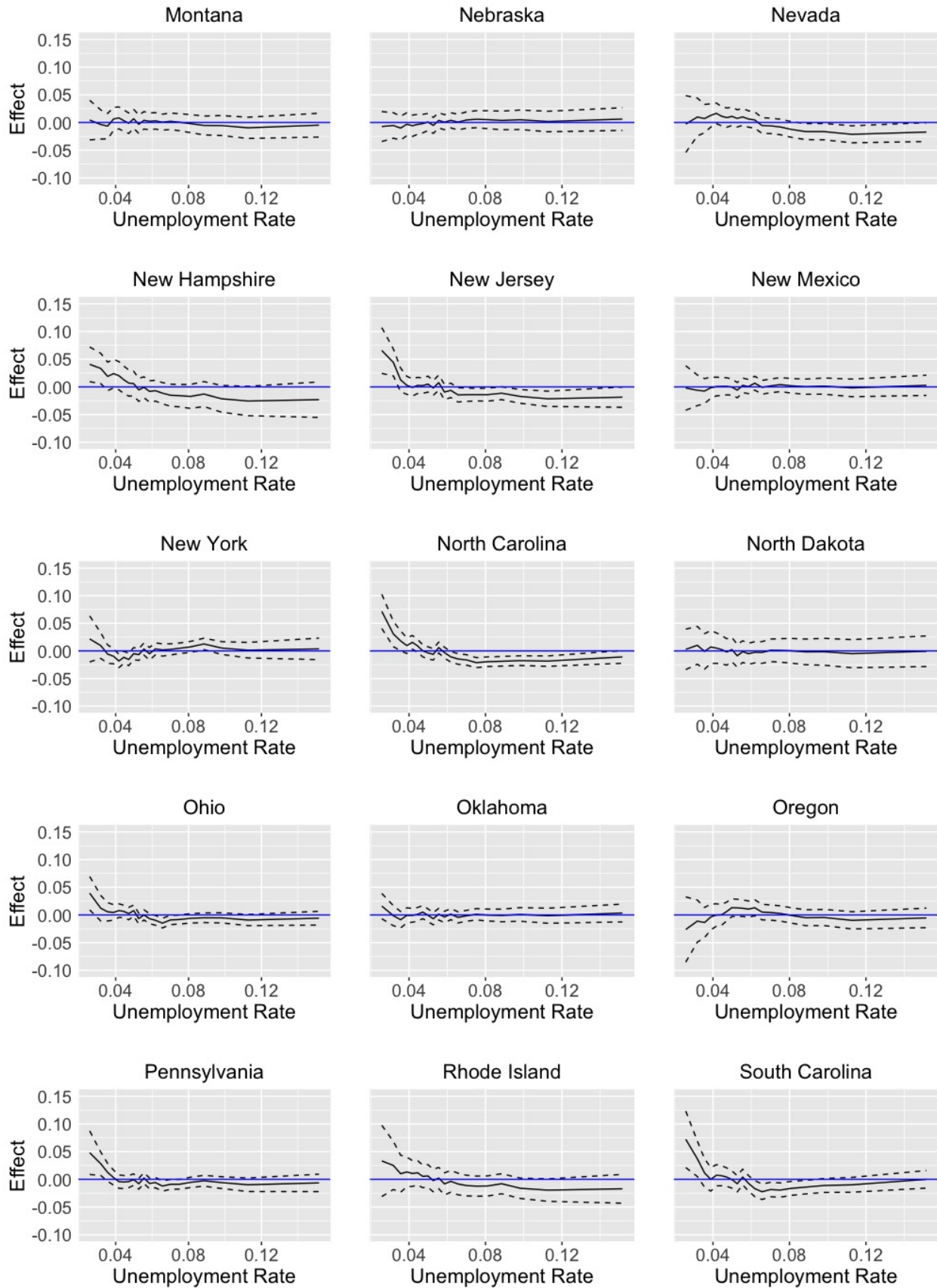
## Appendix D State Unemployment Effects

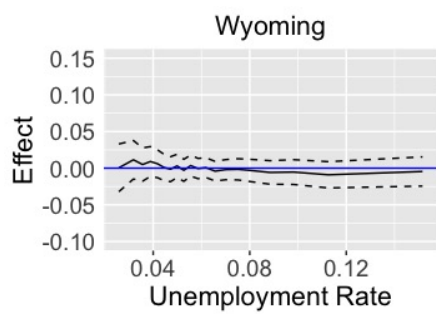
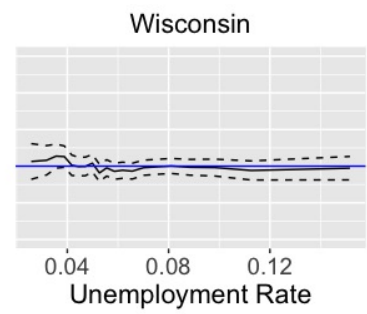
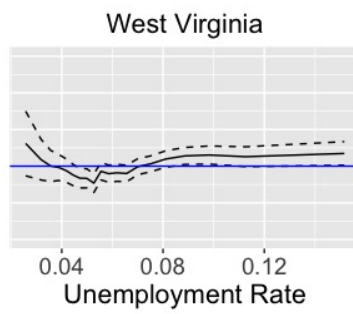
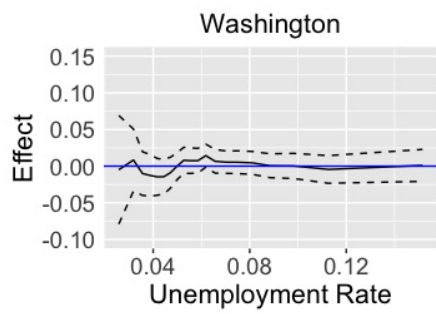
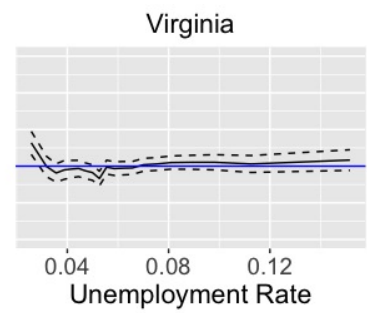
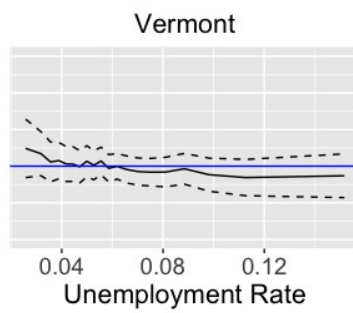
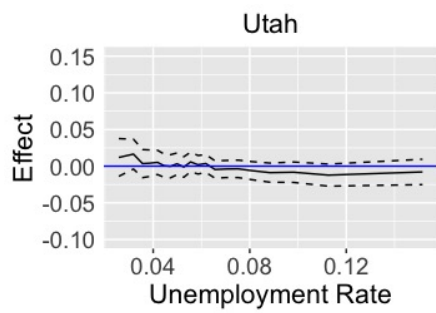
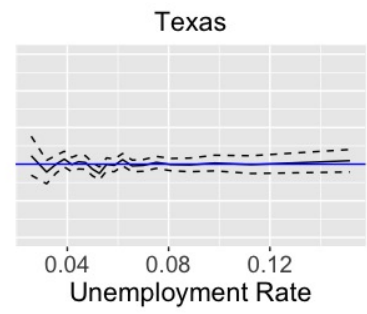
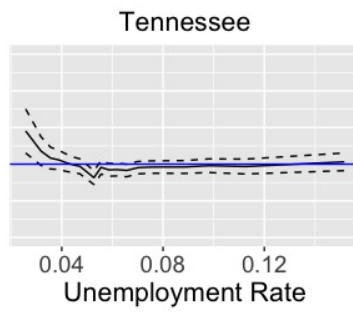
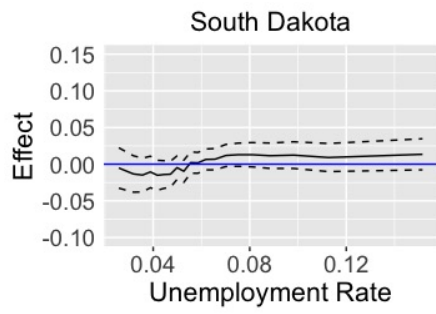
### D.1 Female Model

Here are the state unemployment effects for all 49 states considered (the contiguous U.S. and Washington D.C.) for the Female model. We have the effect for each state plotted along with 95% credible interval.



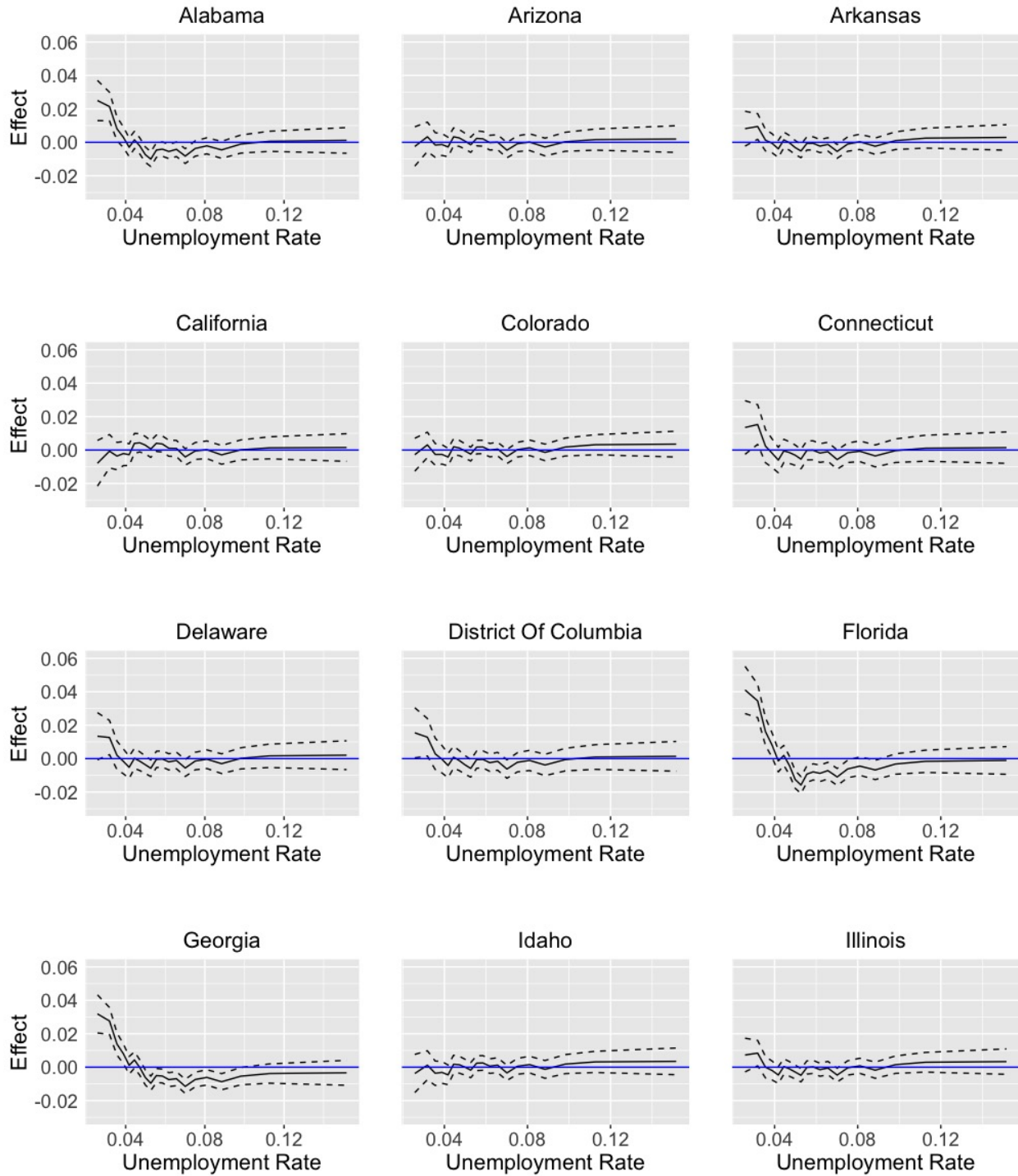


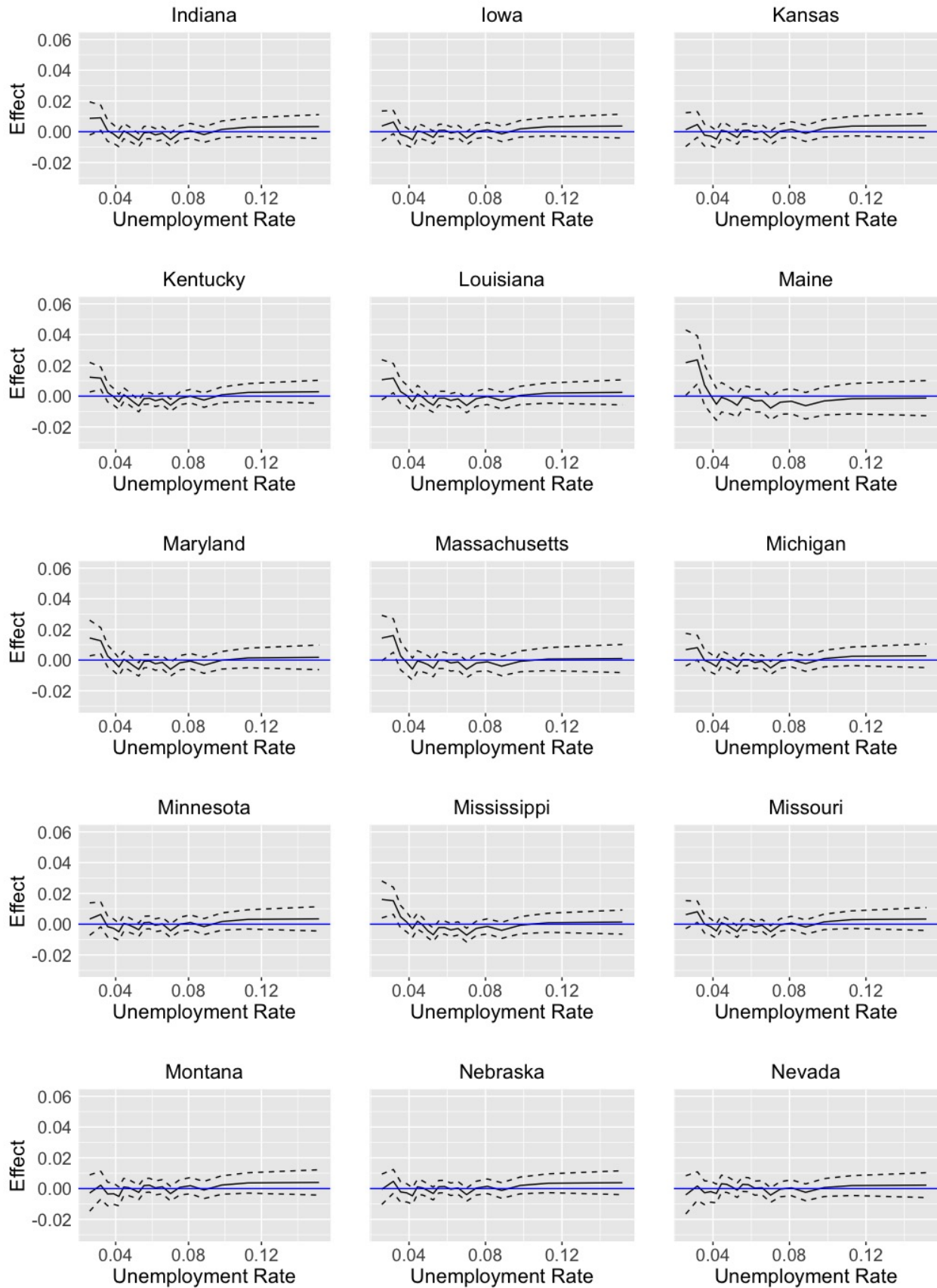


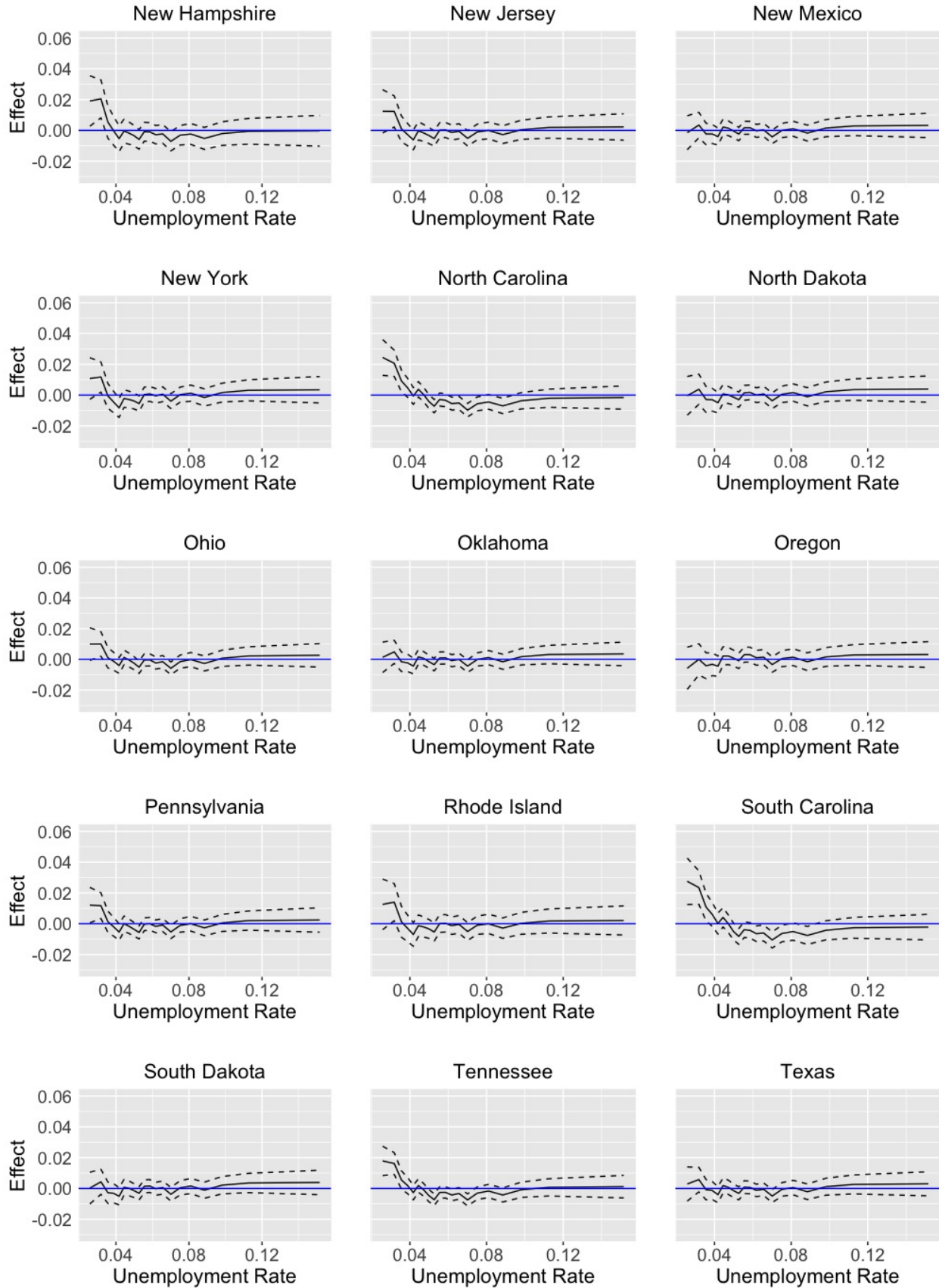


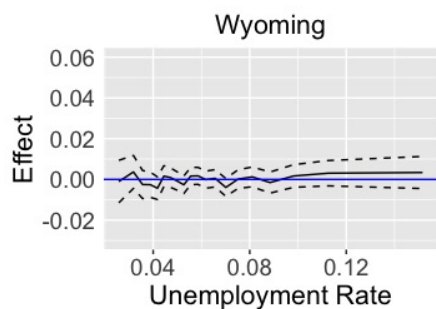
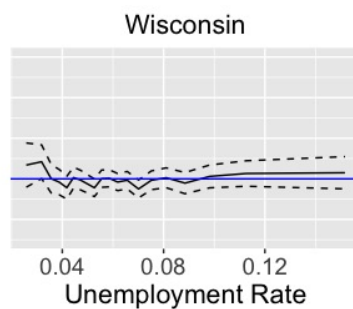
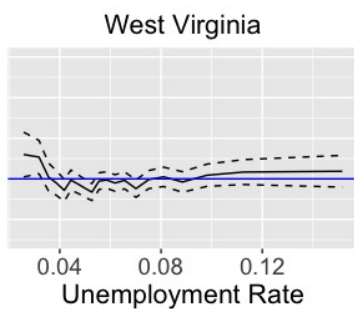
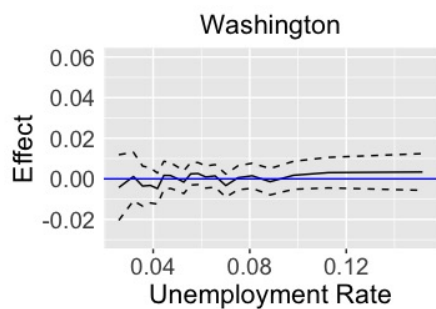
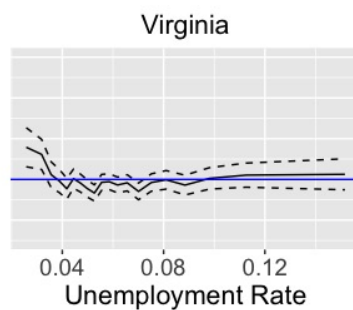
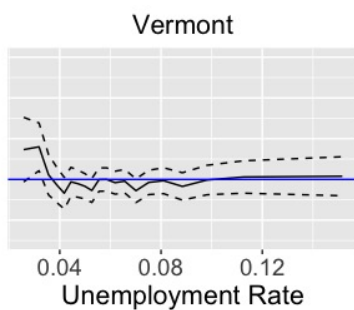
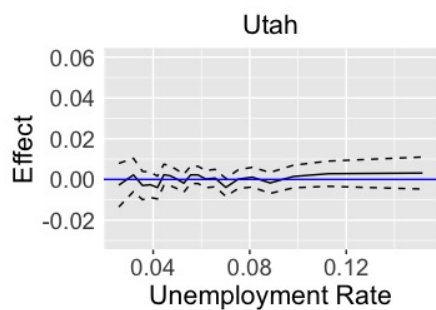
## D.2 Male Model

Here are the state unemployment effects for all 49 states considered (the contiguous U.S. and Washington D.C.) for the Male model. We have the effect for each state plotted along with 95% credible interval.







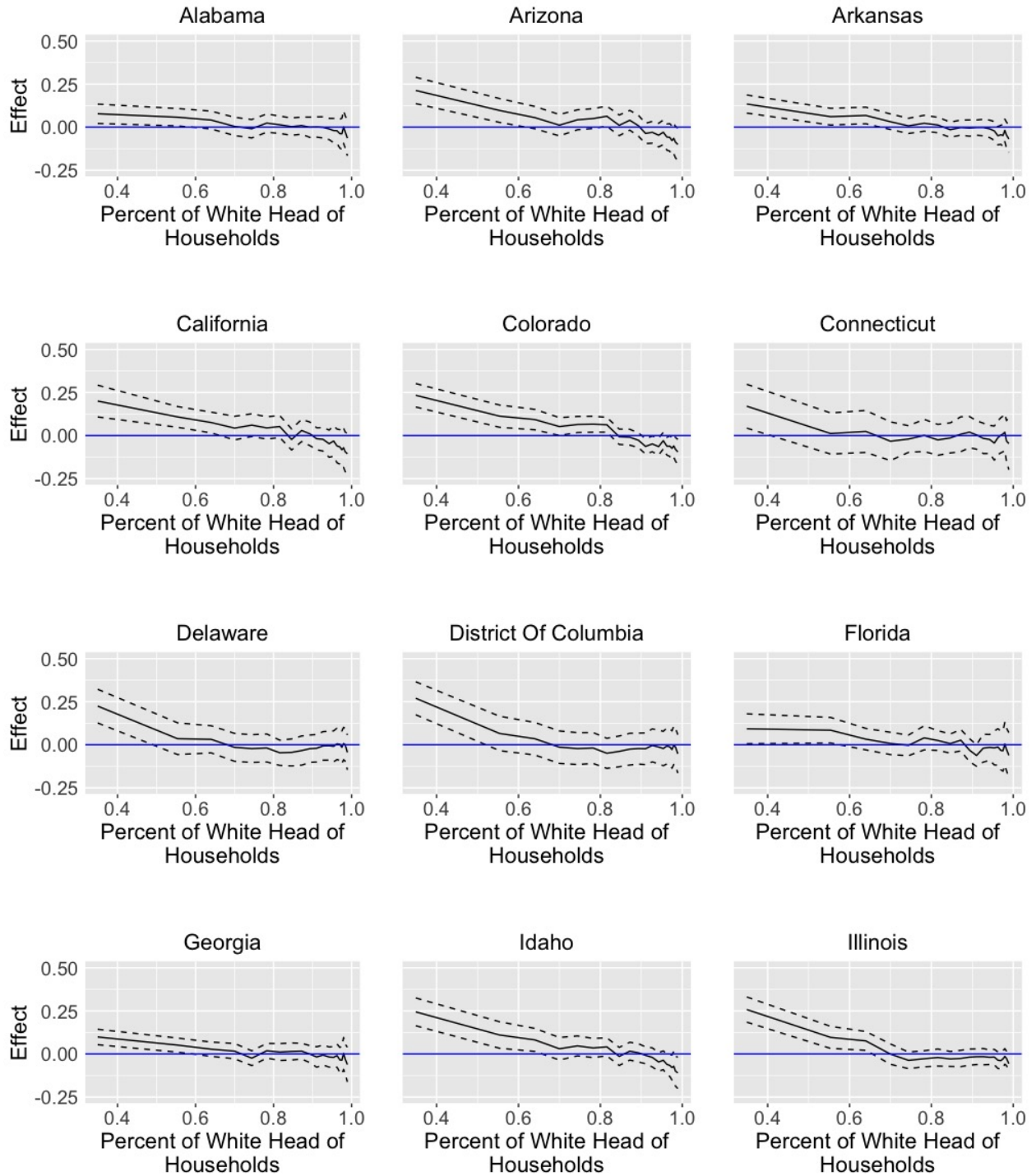


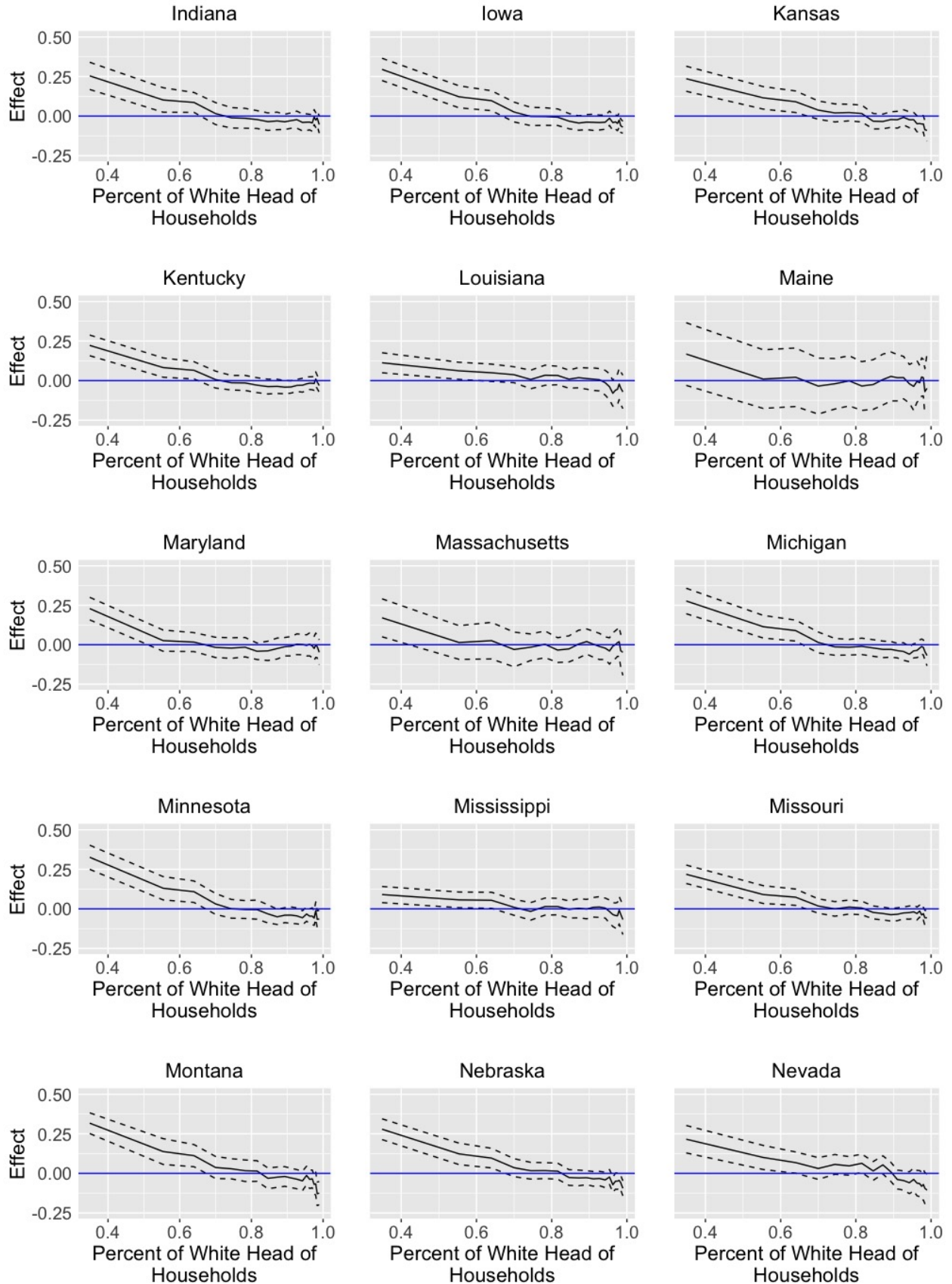


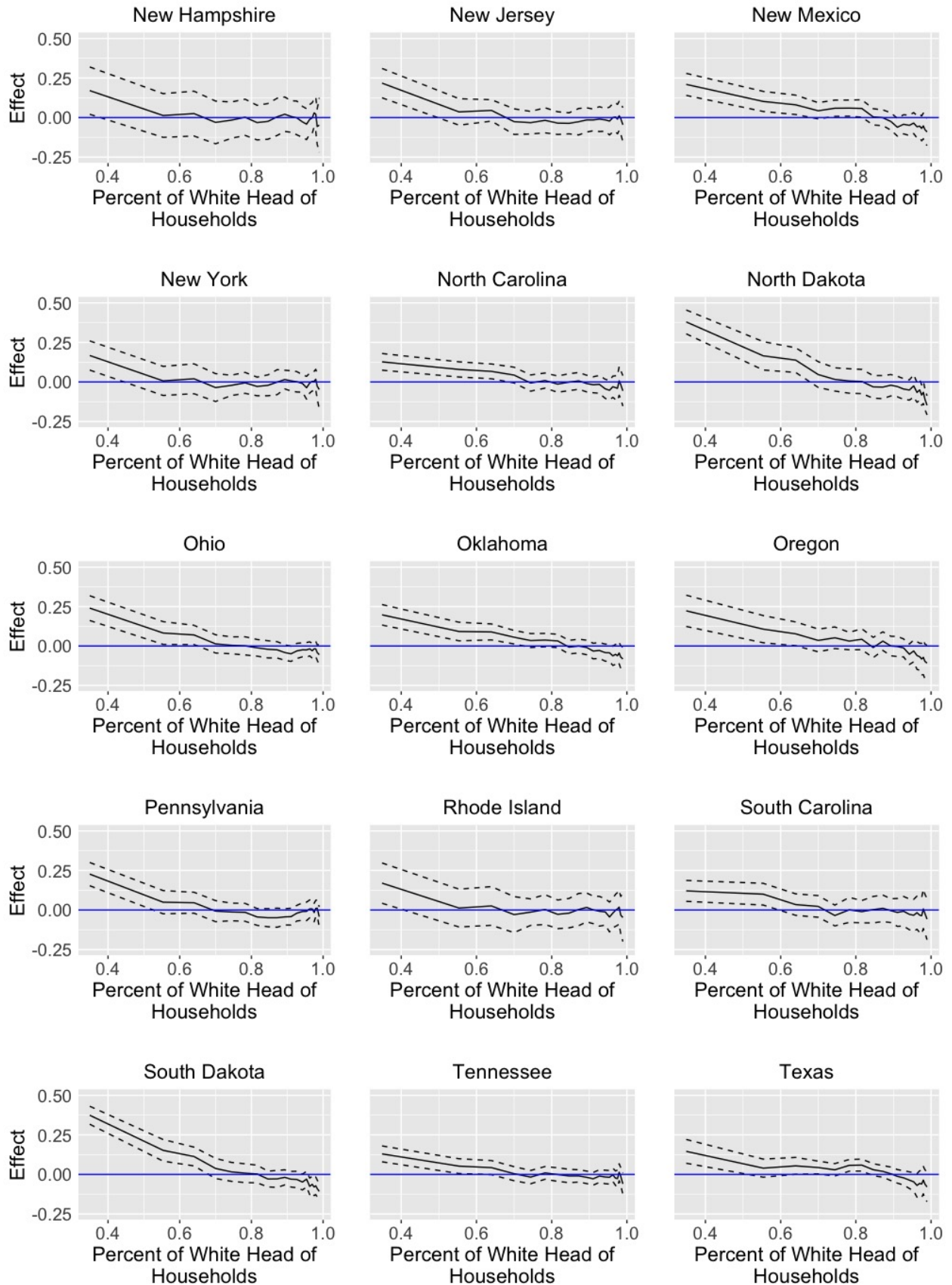
# Appendix E State Race Effects

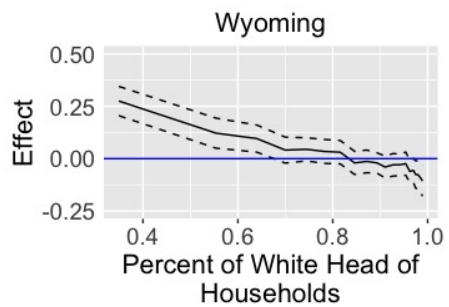
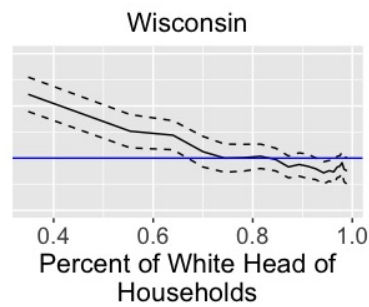
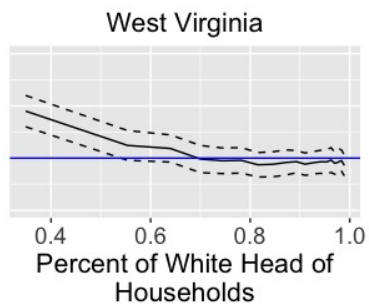
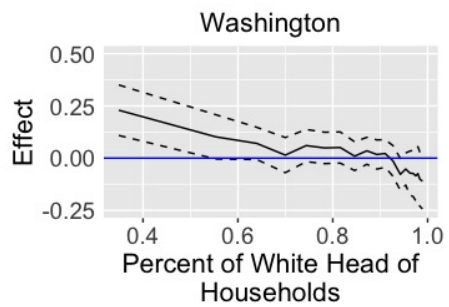
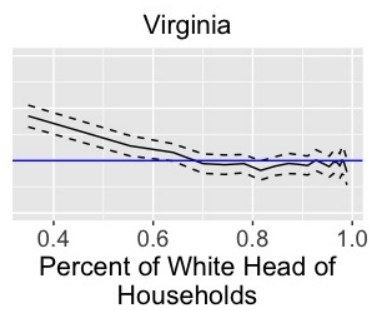
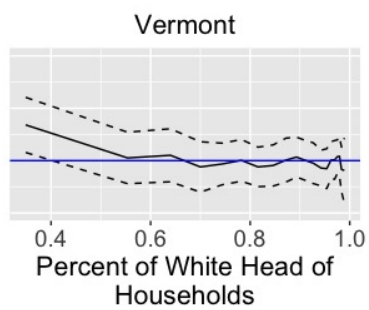
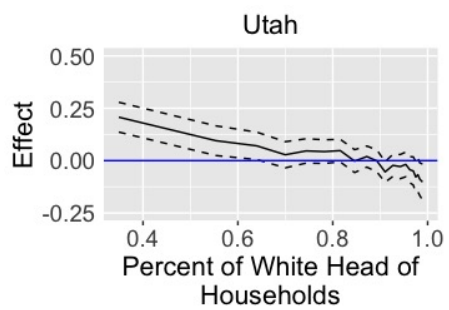
## E.1 Female Model

Here are the state race effects for all 49 states considered (the contiguous U.S. and Washington D.C.) for the Female model. We have the effect for each state plotted along with 95% credible interval.



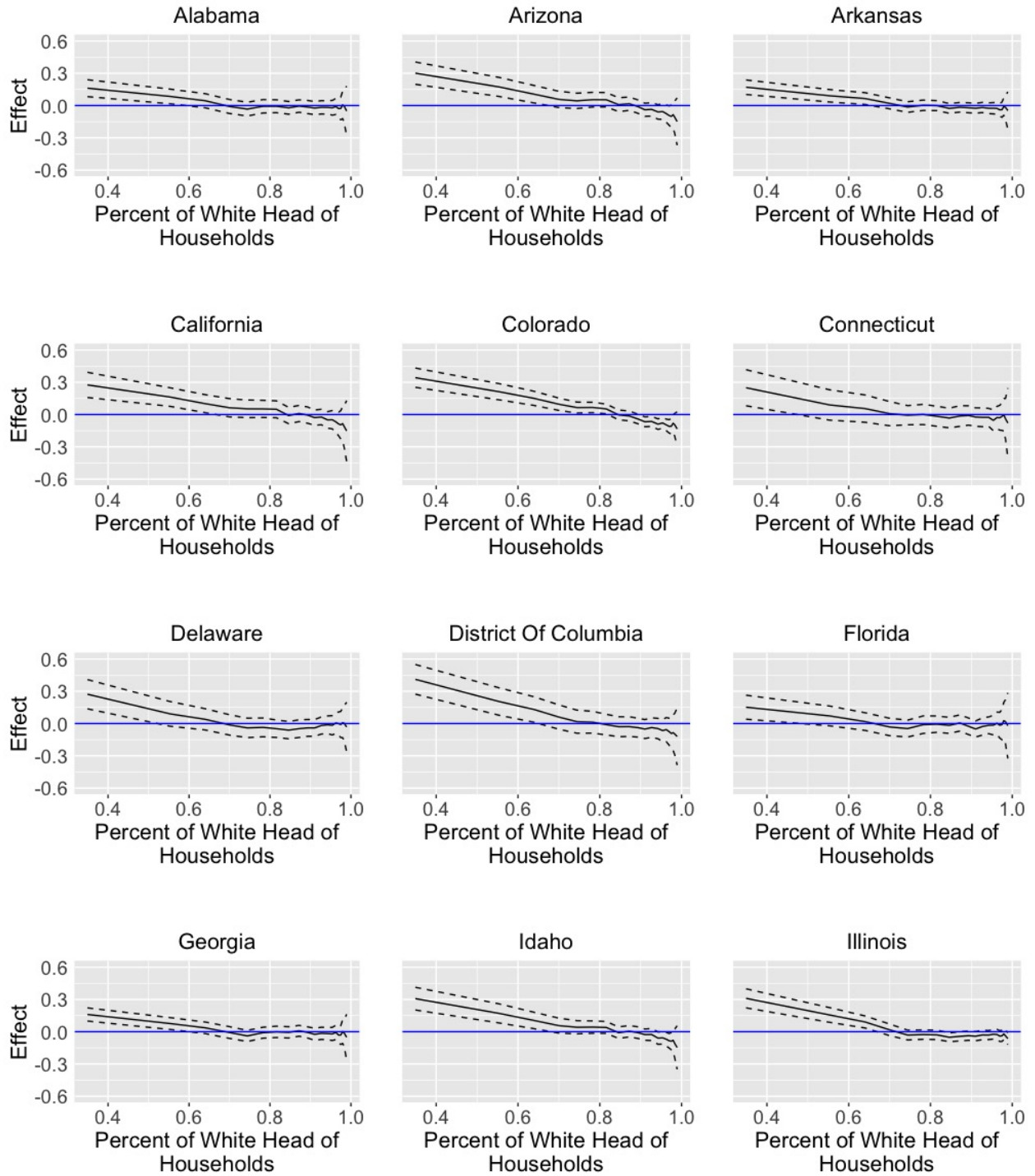


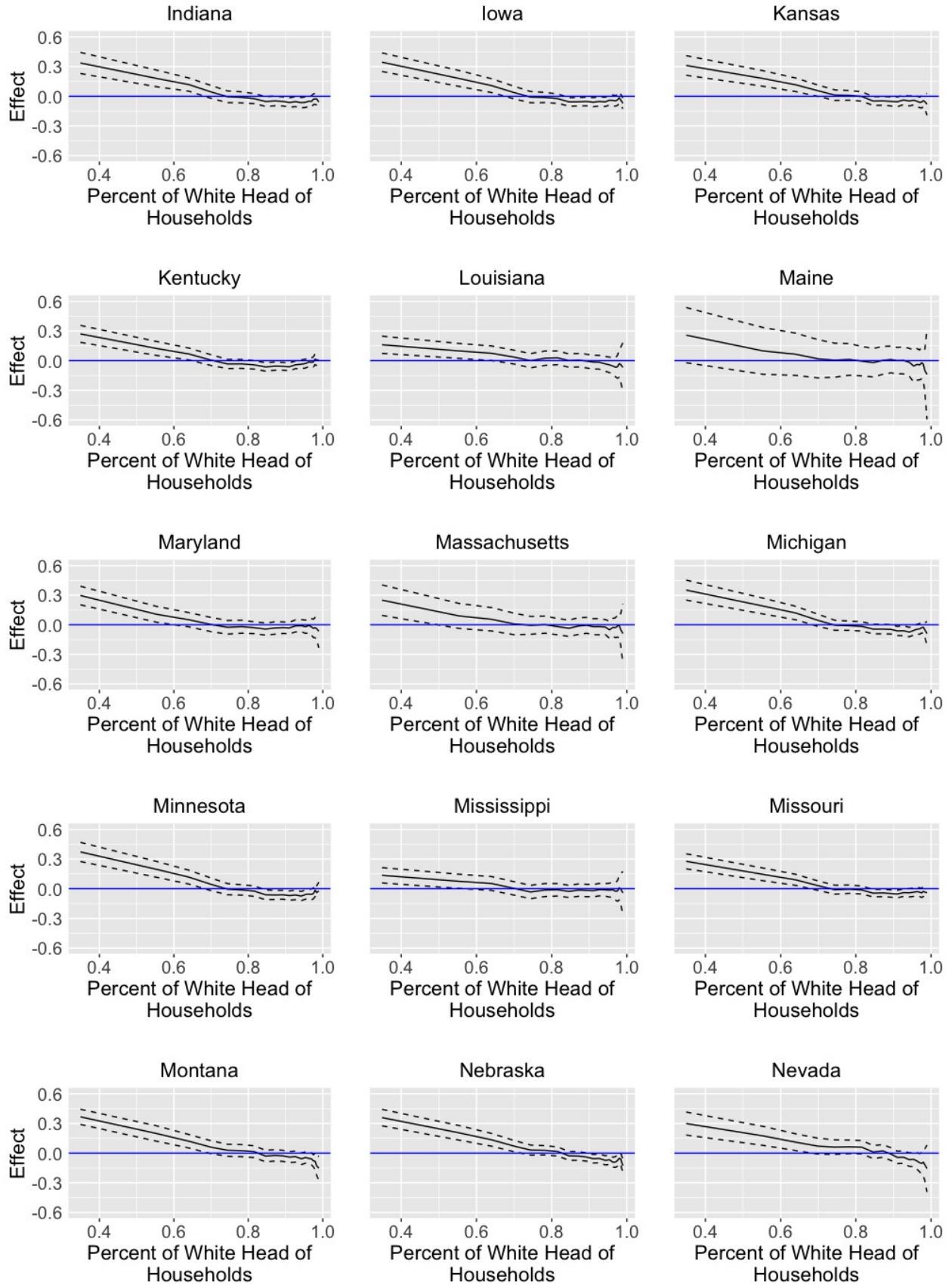


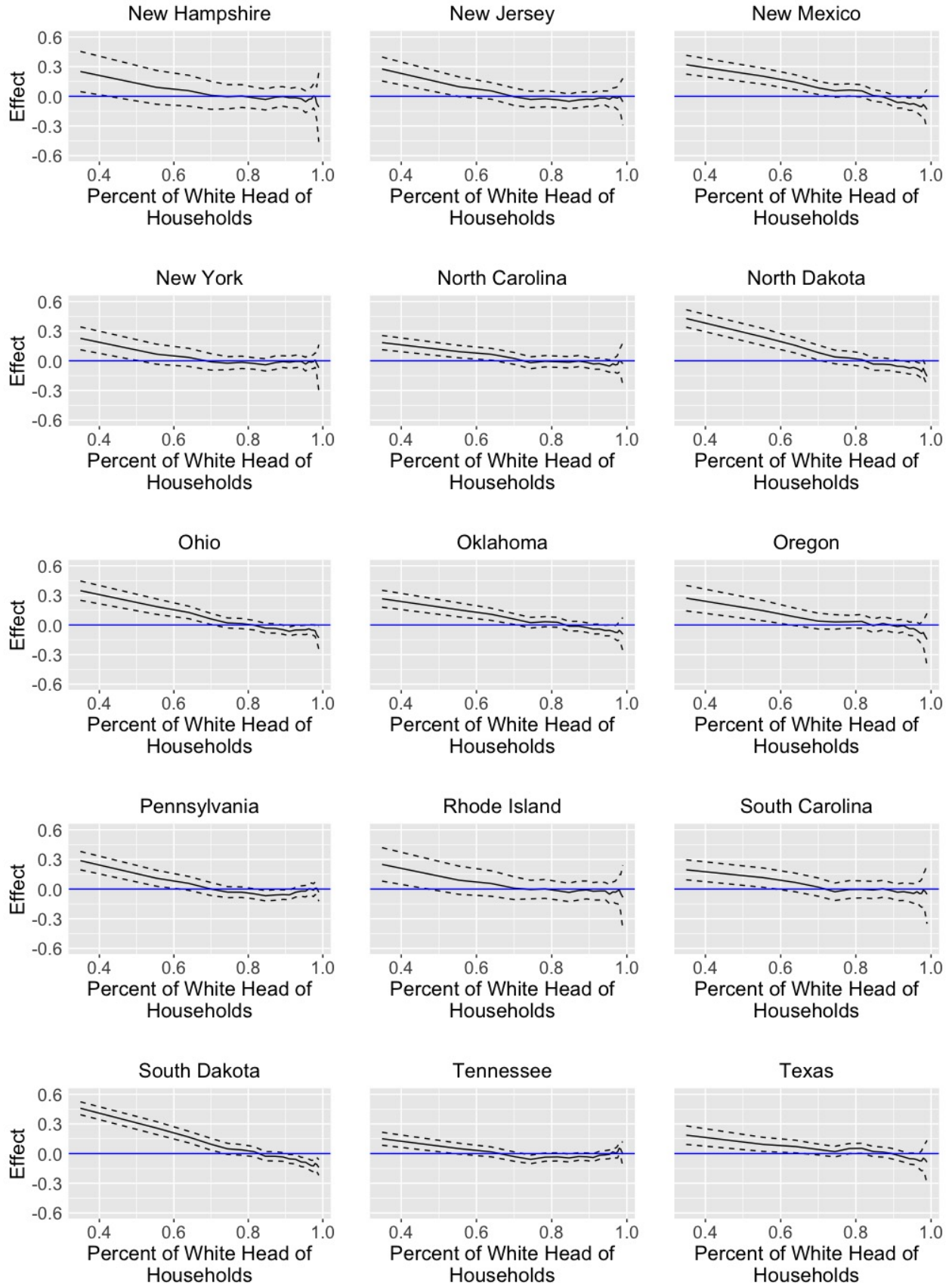


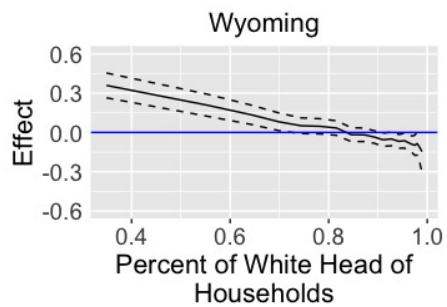
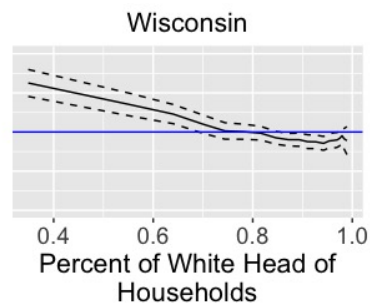
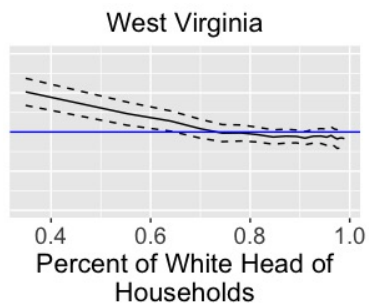
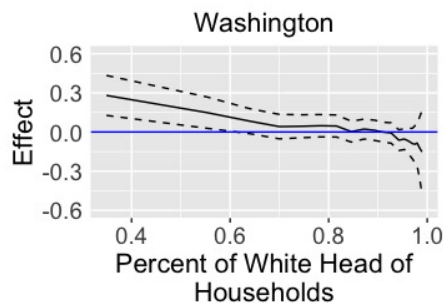
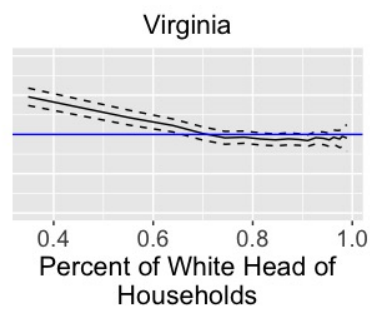
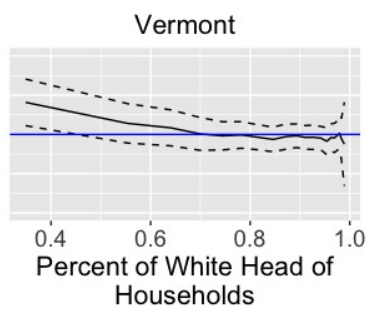
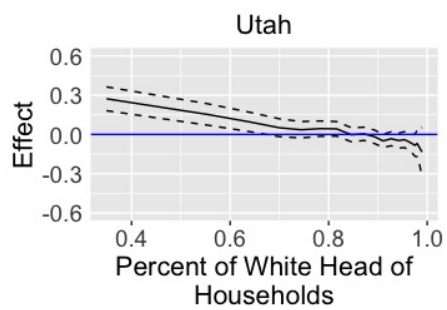
## E.2 Male Model

Here are the state race effects for all 49 states considered (the contiguous U.S. and Washington D.C.) for the Male model. We have the effect for each state plotted along with 95% credible interval.







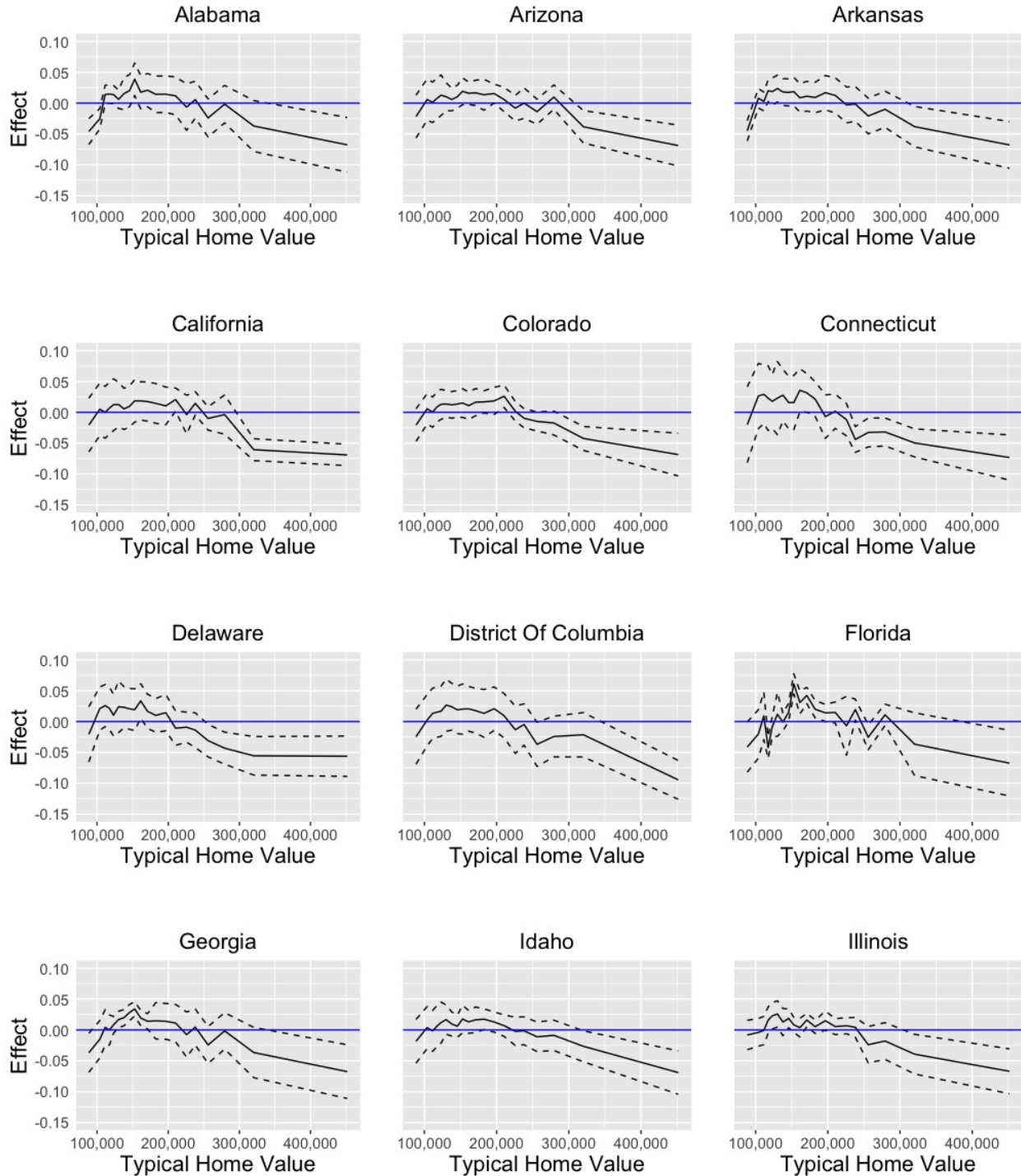


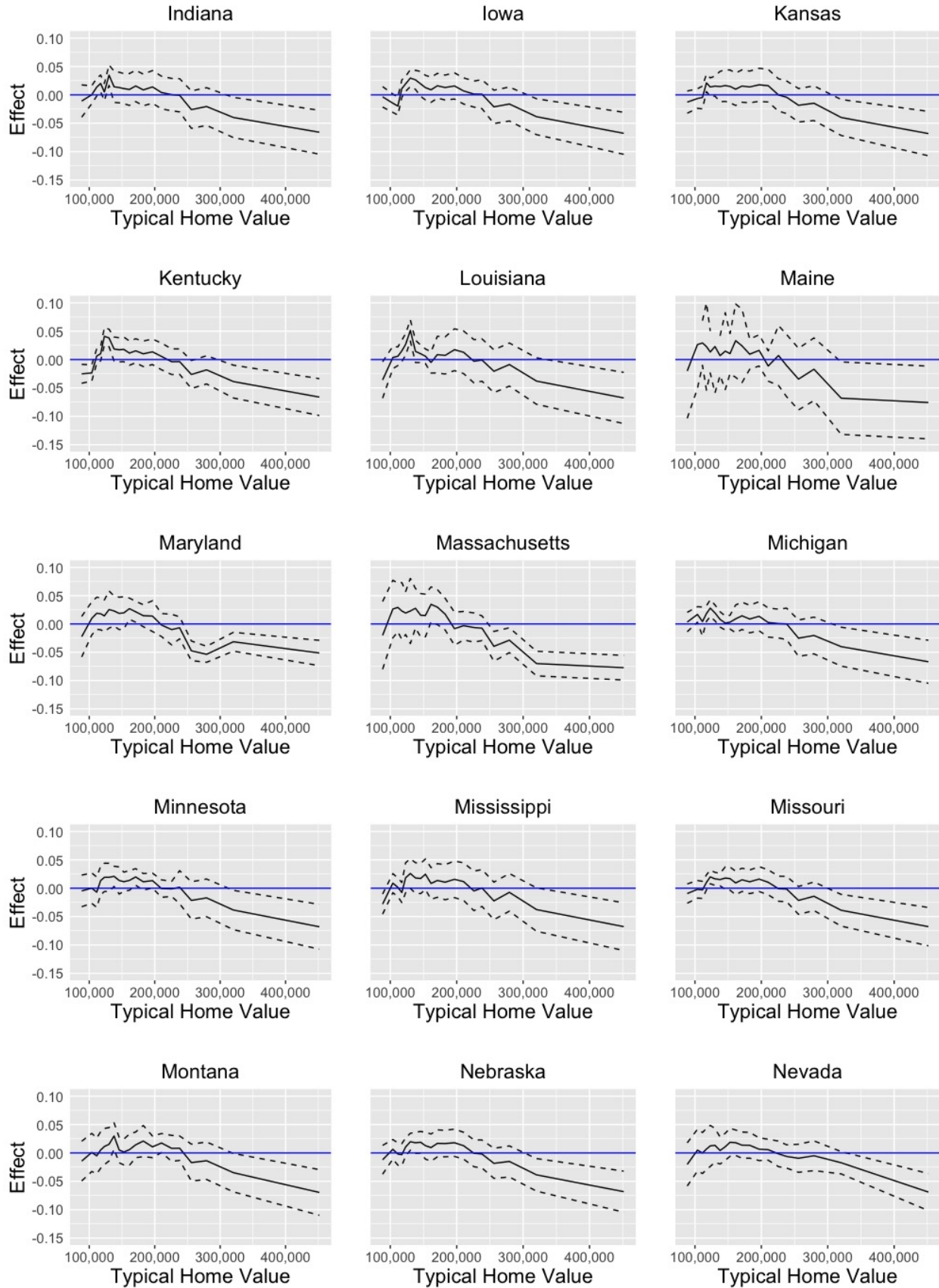


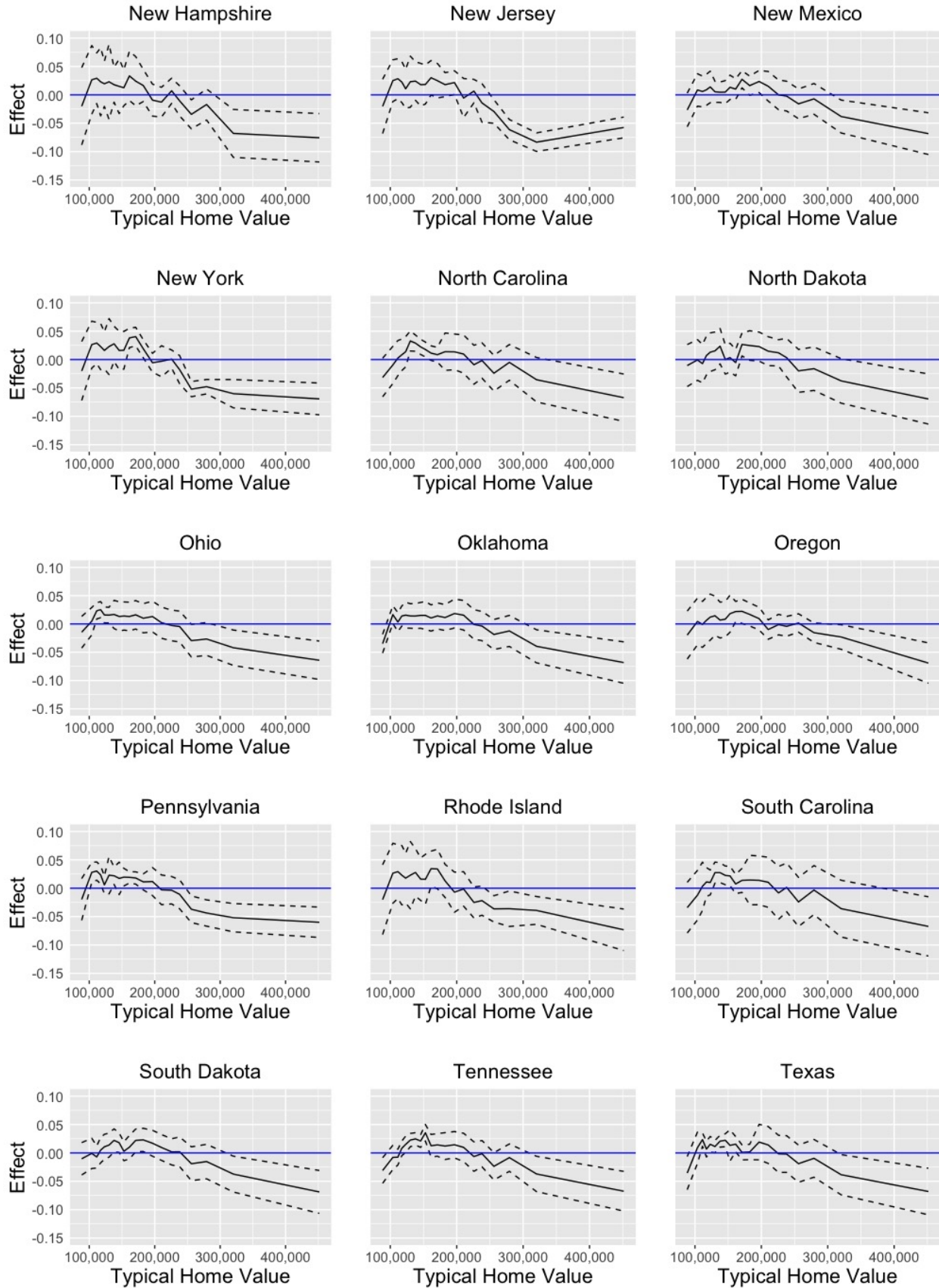
# Appendix F State Home Value Effects

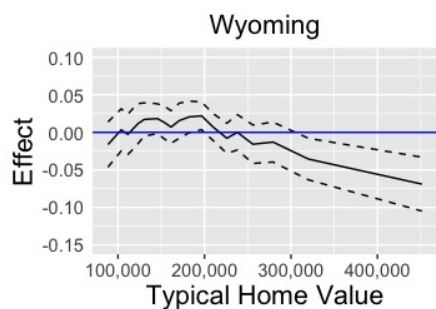
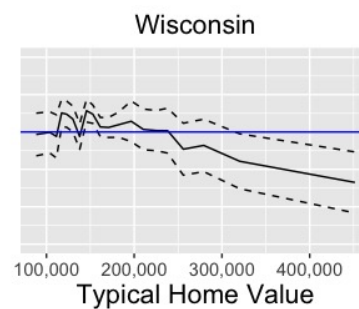
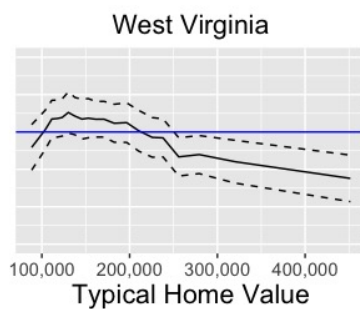
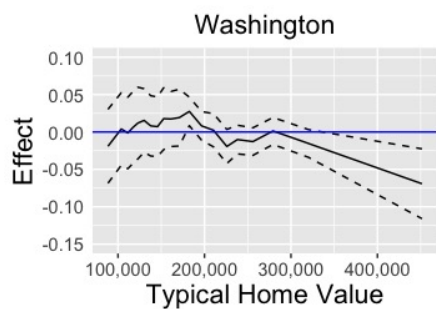
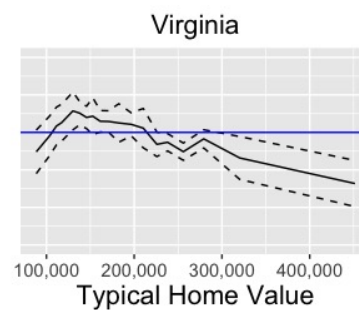
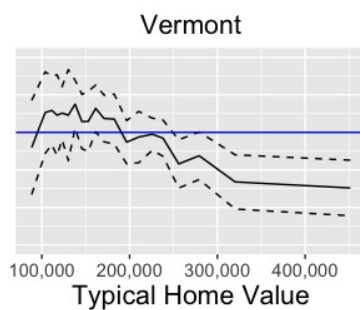
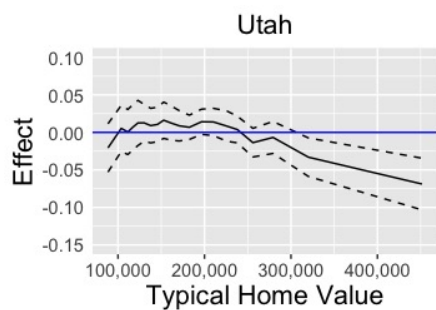
## F.1 Female Model

Here are the state home value effects for all 49 states considered (the contiguous U.S. and Washington D.C.) for the Female model. We have the effect for each state plotted along with 95% credible interval.









## F.2 Male Model

Here are the state home value effects for all 49 states considered (the contiguous U.S. and Washington D.C.) for the Male model. We have the effect for each state plotted along with 95% credible interval.

